# Phase transitions in dilute stellar matter

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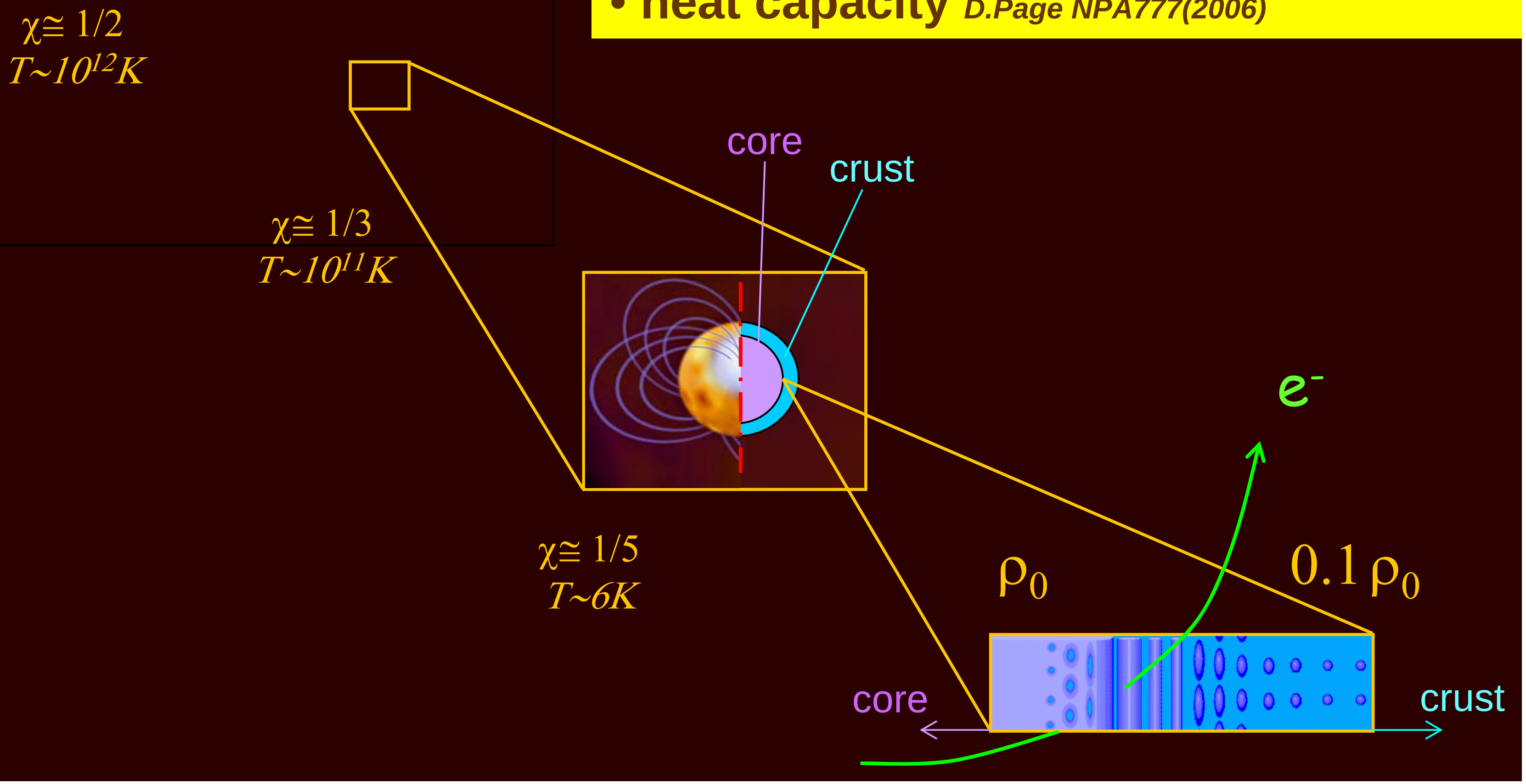
*Supernova* remnant
and *neutron star* in
Puppis A
(ROSAT x-ray)

**Motivation:** study of dishomogeneous phases in PNS crusts and supernova cores.

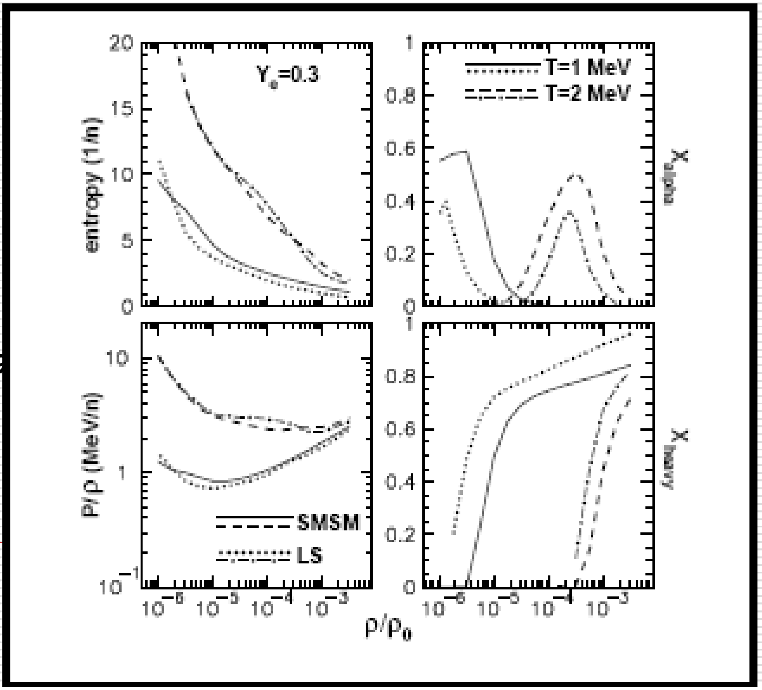
Influence on the explosion dynamics and the PNS cooling via:

- electron capture rates *Janka et al PR442 (2007)*
- neutrino opacity *H.Sonoda PRC75(2007)*
- heat capacity *D.Page NPA777(2006)*



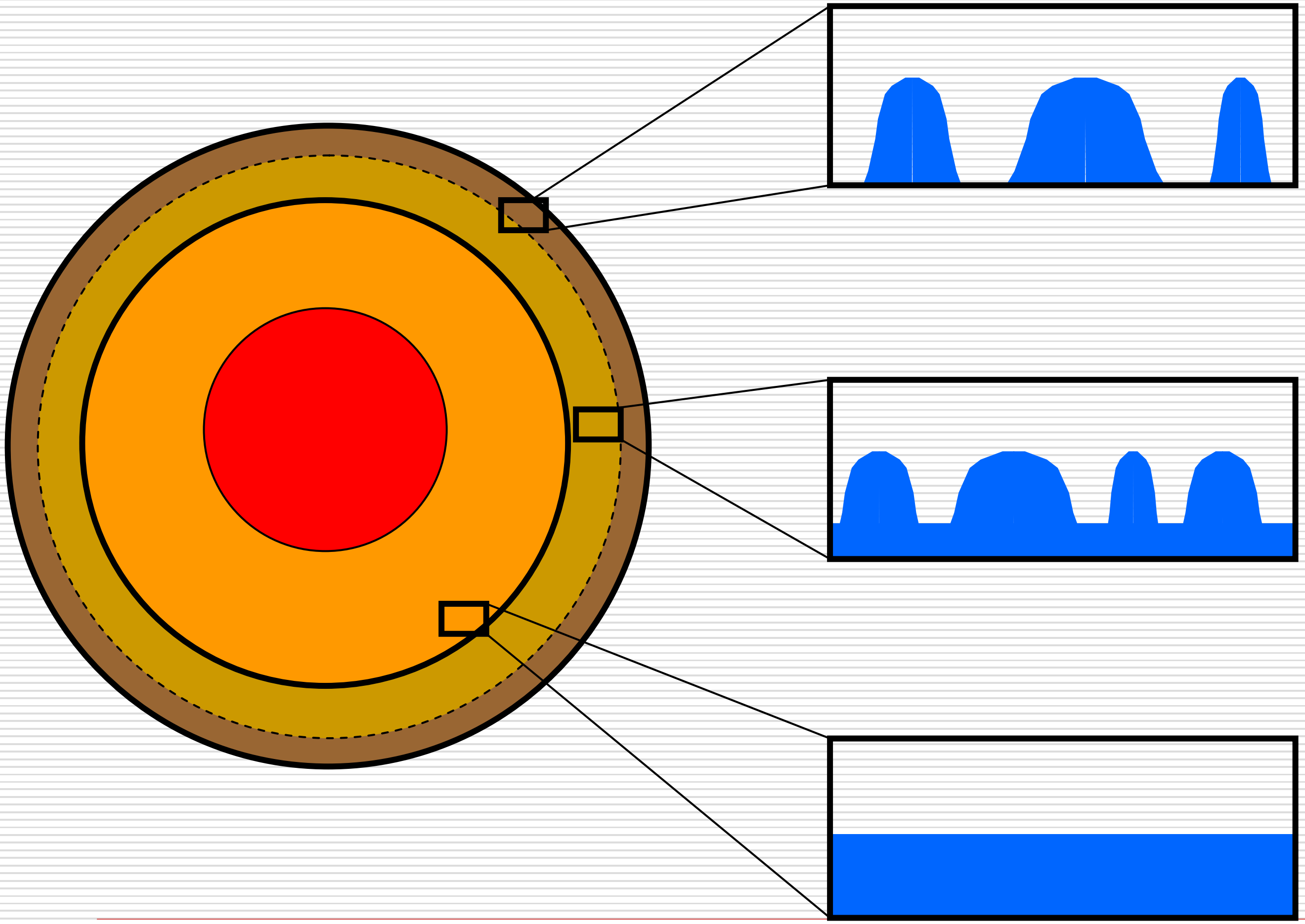
# Dilute stellar matter at T>0

- Standard treatment in supernova codes: statistical equilibrium of n,p,$\alpha$,1 heavy cluster + 1st order phase transition to uniform matter
  ***Lattimer-Swesty EOS, Shen EOS***
- **Improvement**: non-interacting ideal-gas of nuclei (NSE) ***R.S.Souza et al Astrophys.J.707:1495-1505,2009 A.Botvina et al Nucl. Phys. A:98-132,2010 S.I.Blinnikov et al 0904.3849***
- **Missing physics**: in medium corrections, inter-particle interactions
  - Interactions in the S matrix formalism
    ***S.K.Samaddar et al. Phys.Rev.C80:035803,2009***
  - Virial EOS A<5 ***A.Schwenk et al. Phys.Rev.C78:015806,2008***
  - Quasi-particle gas model
    ***Skyrme+A<14 S.Heckel et al Phys.Rev.C80:015805,2009***
    ***RMF+A<4*** S.***Typel et al Phys.Rev.C81:015803,2010***
  - Phenomenological models
    ***M.Hempel et al., astro-ph/0911.4073***



***A.Botvina I.Mishustin PRC 2005 & NPA 2010***

# Dilute matter at T>0: the phenomenology of the crust-core transition

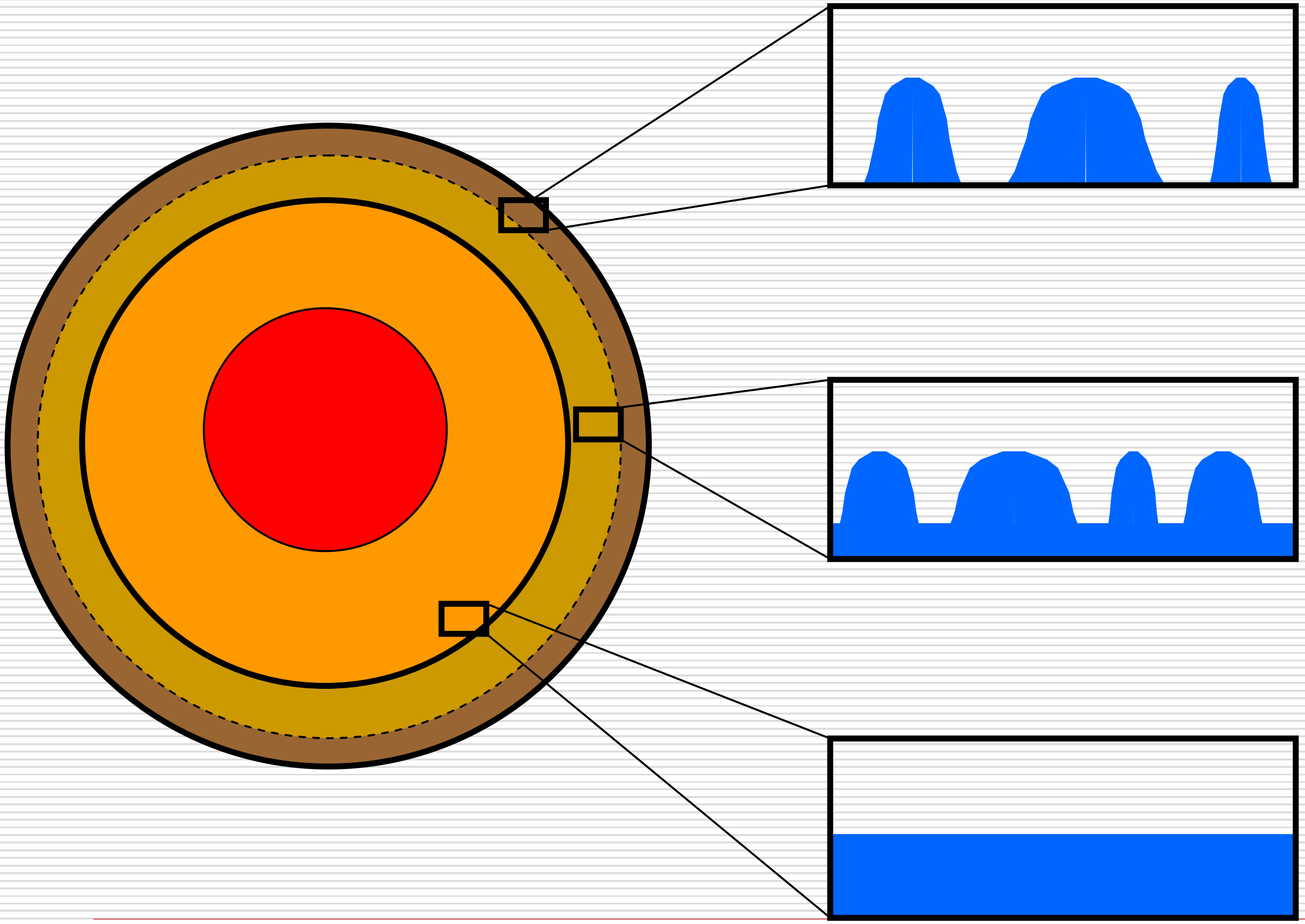


Nuclei in the outer crust

Neutron (proton) drip in the inner crust

Homogeneous matter in the core

# Dilute matter at T>0: the phenomenology of the crust-core transition



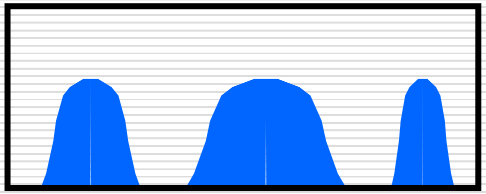
Nuclei in the outer crust

Neutron (proton) drip in the inner crust

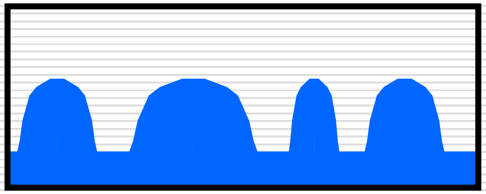
Homogeneous matter in the core

+ electrons $\rho_e=\rho_p$, $\gamma,\nu$

# An hybrid model for the crust-core transition

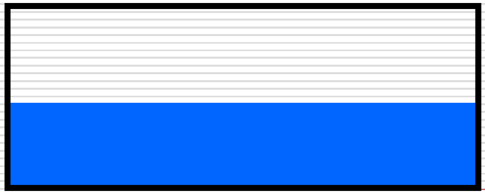


Nuclei in the outer crust



Neutron (proton) drip in the inner crust

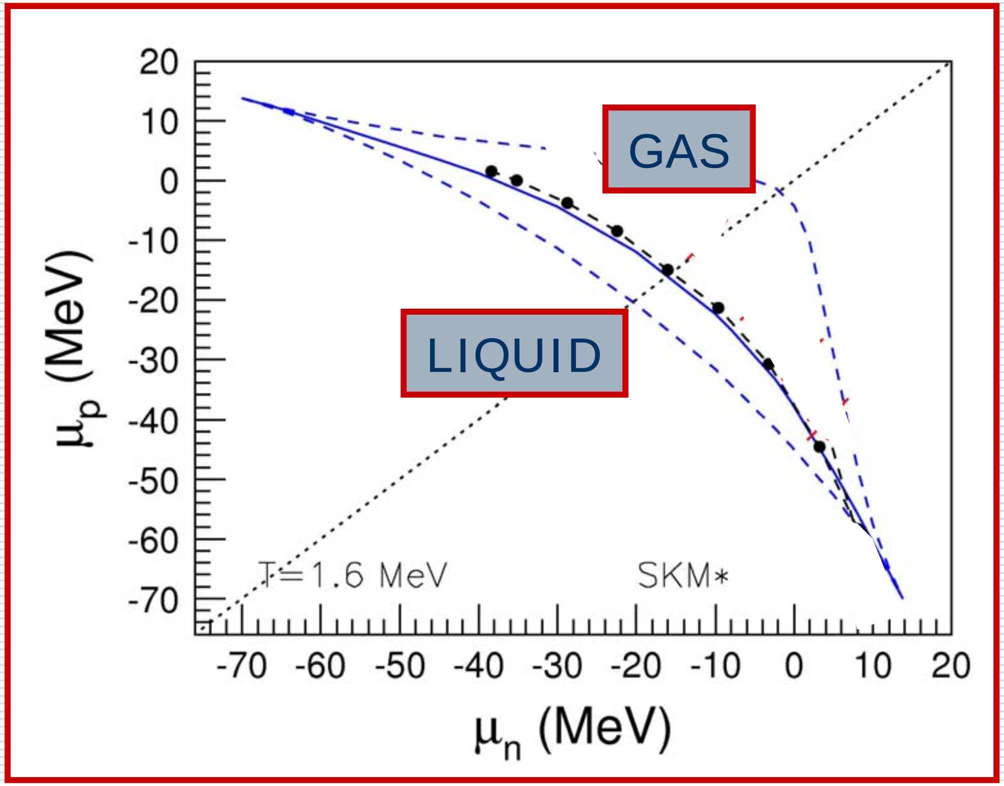
$$\rho_{p,n} = \frac{2}{h^3}\int_0^\infty \frac{4\pi p^2 dp}{1+e^{\beta\left(p^2/2m^*_{p,n}+U_{p,n}-\mu_{p,n}\right)}}$$



Homogeneous matter in the core → Finite temperature Hartree F
with Skyrme interactions (SI

+ electrons $\rho_e=\rho_p$, $\gamma,\nu$

# Thermodynamics of homogeneous matter in the MF approximation

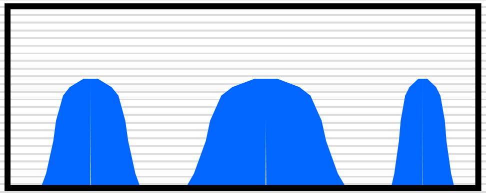


━ Coexistence of purely homogeneous NM

┅ Spinodal of purely homogeneous NM

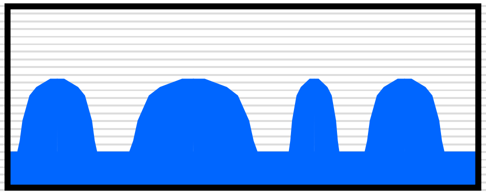
=> Homogeneous matter is unstable over a wide $\mu$,T region

# An hybrid model for the crust-core transition

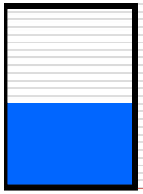


Nuclei in the
outer crust

→ Statistical ensemble of
interacting excited clusters



Neutron (proton)
drip in the inner crust



Homogeneous
matter in the core

→ Finite temperature Hartree Fock
with Skyrme interactions (SKM*, Sly230a)

+ electrons $\rho_e=\rho_p$, $\gamma,\nu$

# Statistical ensemble of interacting excited clusters

$$F_{i,n_i} = -Tn_i\left(\ln \mathbb{Z}_i + 1\right) \qquad \mathbb{Z}_i = \frac{g_i(T)V}{n_i}\left(\frac{m_i T}{2\pi}\right)^{3/2} e^{-m_i/T}$$

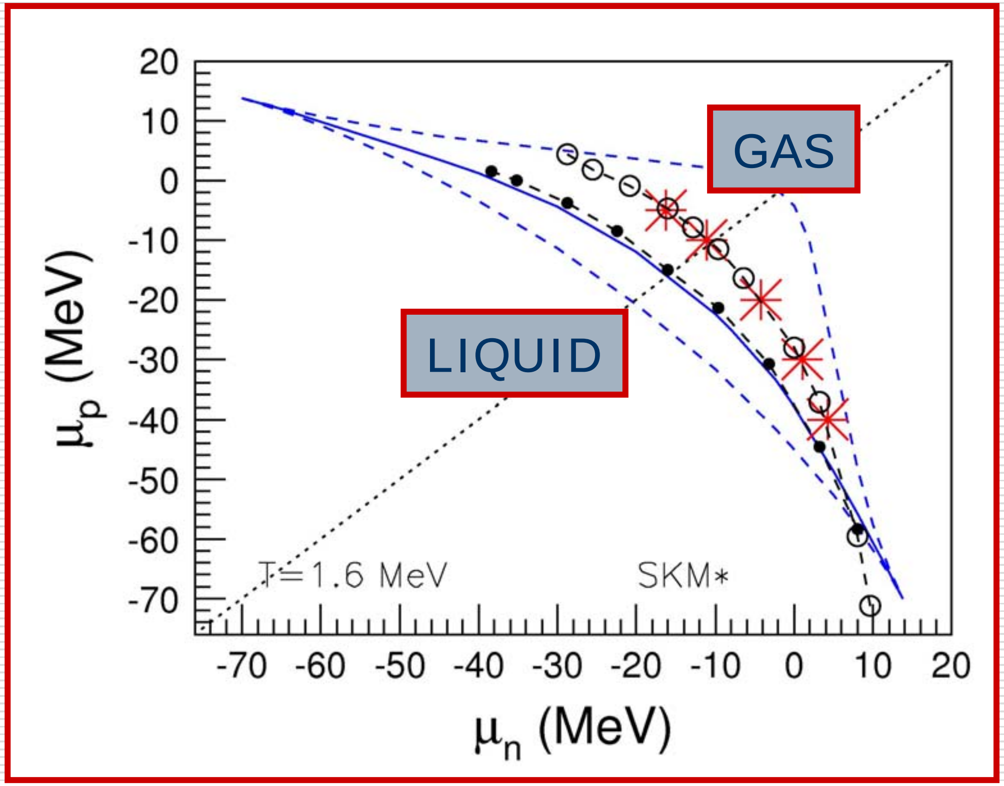
- **Standard NSE**
  - ☹ Statistical independence at the classical level
  - ☹ Non-interacting
  - ☺ Analytical calculations

$$i = \left\{A_i, Z_i\right\}$$

$$\mathbb{Z} = \sum_K \frac{\left(V - V_K\right)^{N_K} e^{-E_{coul}(K)/T}}{N_K\,!} \prod_{i=1}^{N_K} \mathbb{Z}_i$$

- **This work**
  - ☺ Exact quantum counting
  - ☺ Coulomb interaction + excluded volume
  - ☹ Expensive MC calculations
  - ☹ Convergence to be checked

# Thermodynamics of clusterized and unclusterized matter



— Coexistence of purely homogeneous NM

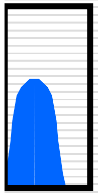
⋯ Spinodal of purely homogeneous NM

✳ Coexistence of purely clusterized NM

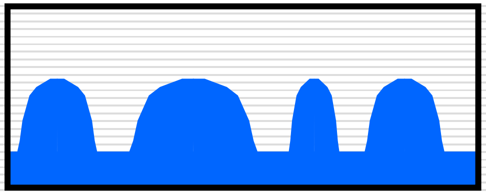
○○ ●-● Trajectories of purely clusterized NM

=> The mean-field instabilities are cured by cluster formation

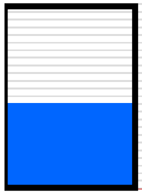
# An hybrid model for the crust-core transition



Nuclei in the outer crust → Statistical ensemble of interacting excited clusters



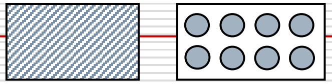
Neutron (proton) drip in the inner crust → the two components togeth



Homogeneous matter in the core → Finite temperature Hartree Fock with Skyrme interactions (Sly230a)

+ electrons $\rho_e=\rho_p$, $\gamma,\nu$

# Phase mixture versus phase coexistence

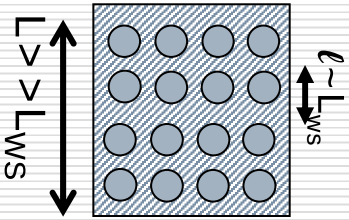


A system composed of heterogenous components I=HM, II=clus

☐ **Mixture**

(ex:atmosphere)

dishomogeneities on a microscopic scale



$$\beta^{HM} = \beta^{clus}$$

$$\mu_i^{HM} = \mu_i^{clus} \qquad i = n, p$$

$$P = P^{HM} + P^{clus}$$

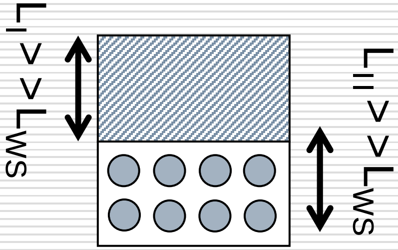
$$\rho_i = \rho_i^{HM} + \rho_i^{clus}$$ (+ excluded volume)

**=> Continuous EOS**

☐ **Coexistence**

(ex: Solid-Liquid)

dishomogeneities on a macroscopic scale



$$\beta^{HM} = \beta^{clus}$$

$$\mu_i^{HM} = \mu_i^{clus} \qquad i = n, p$$

$$P^{HM} = P^{clus}$$

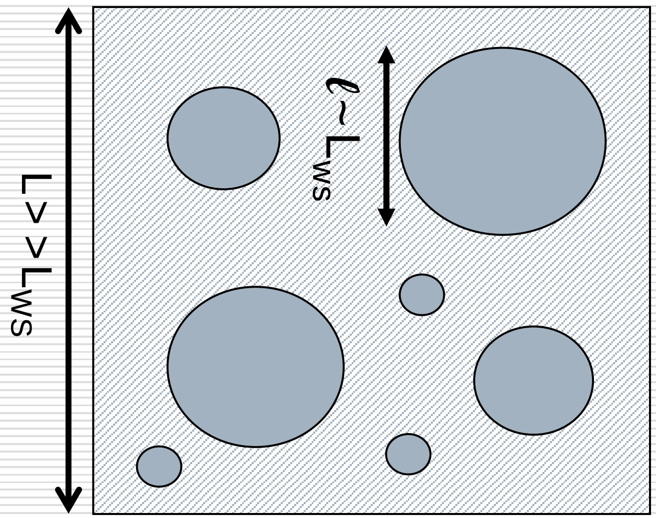
$$\rho_i = \rho_i^{HM} x + \rho_i^{clus}(1-x)$$ (Gibbs construction)

**=> jump in observables**

# No first order transition in dilute stellar matter

A first order crust-core transition (e.g. Lattimer-Swesty, Shen, etc.)

- Does not correspond to the physical structure of the crust *(microscopic fluctuations)*



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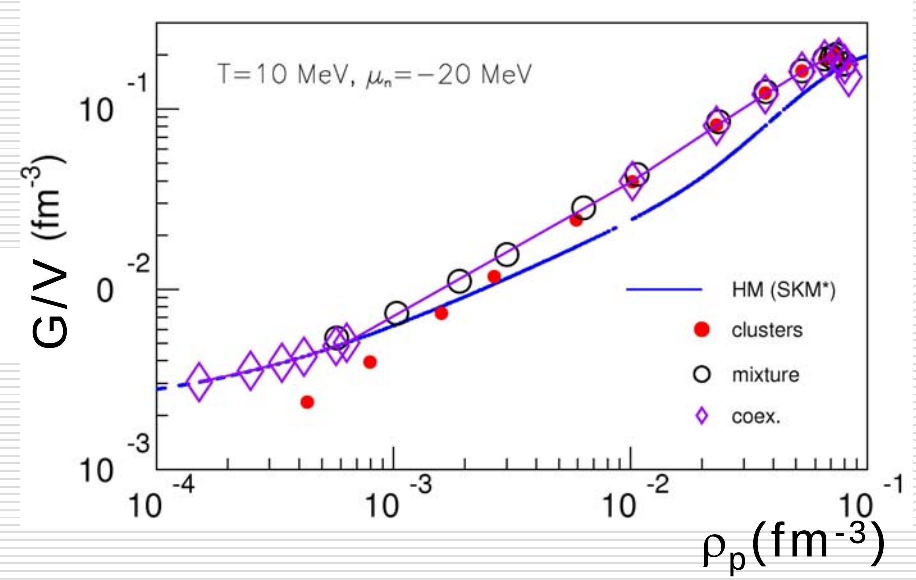

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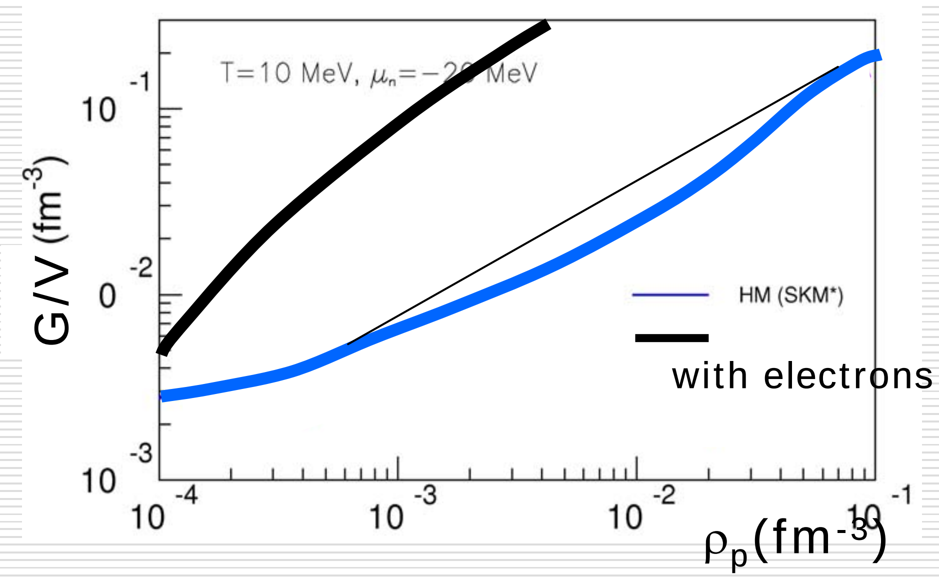
- ☐ Gives no entropy gain
- ☐ Ignores electron incompressibility!!!
  *(transition quenched because $\partial\mu_e / \partial\rho_e \approx GeV$ => concave entropy)*



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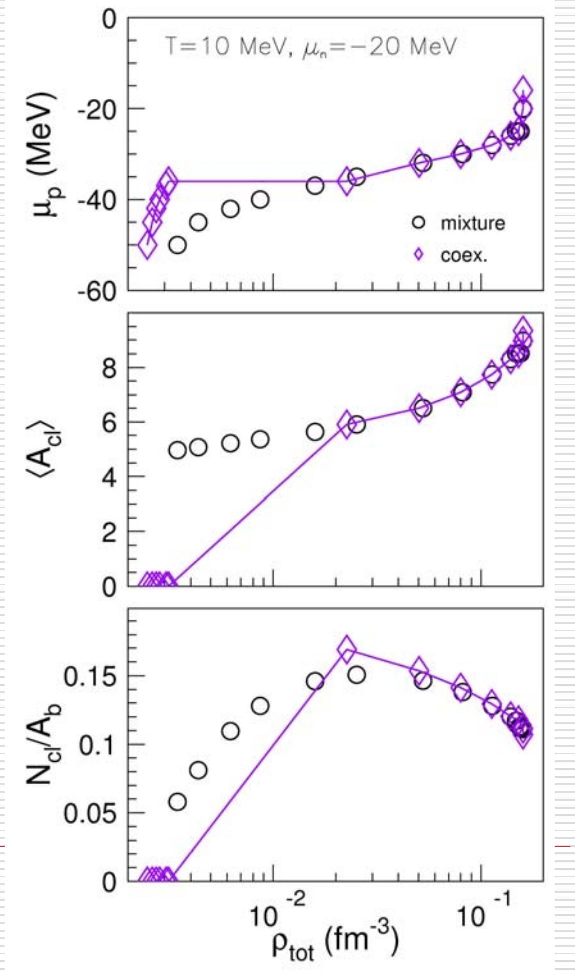
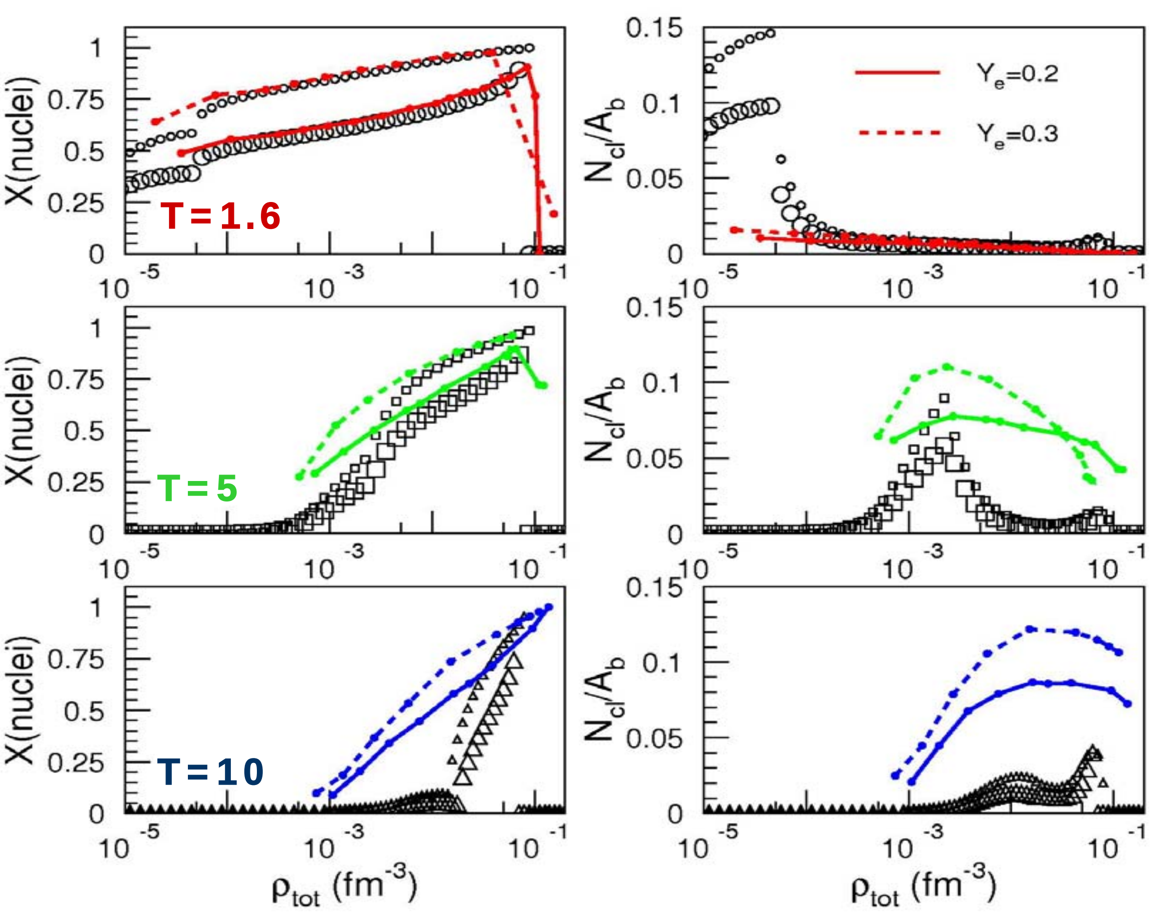
  *(microscopic fluctuations)*
- Gives no entropy gain
- Ignores electron incompressibility!!!
  *(transition quenched because* $\partial\mu_e / \partial\rho_e \approx GeV$
  *=> concave entropy)*
- Produces artificial discontinuities
  *(problem with SN codes)*



# Crust composition: cluster contribution

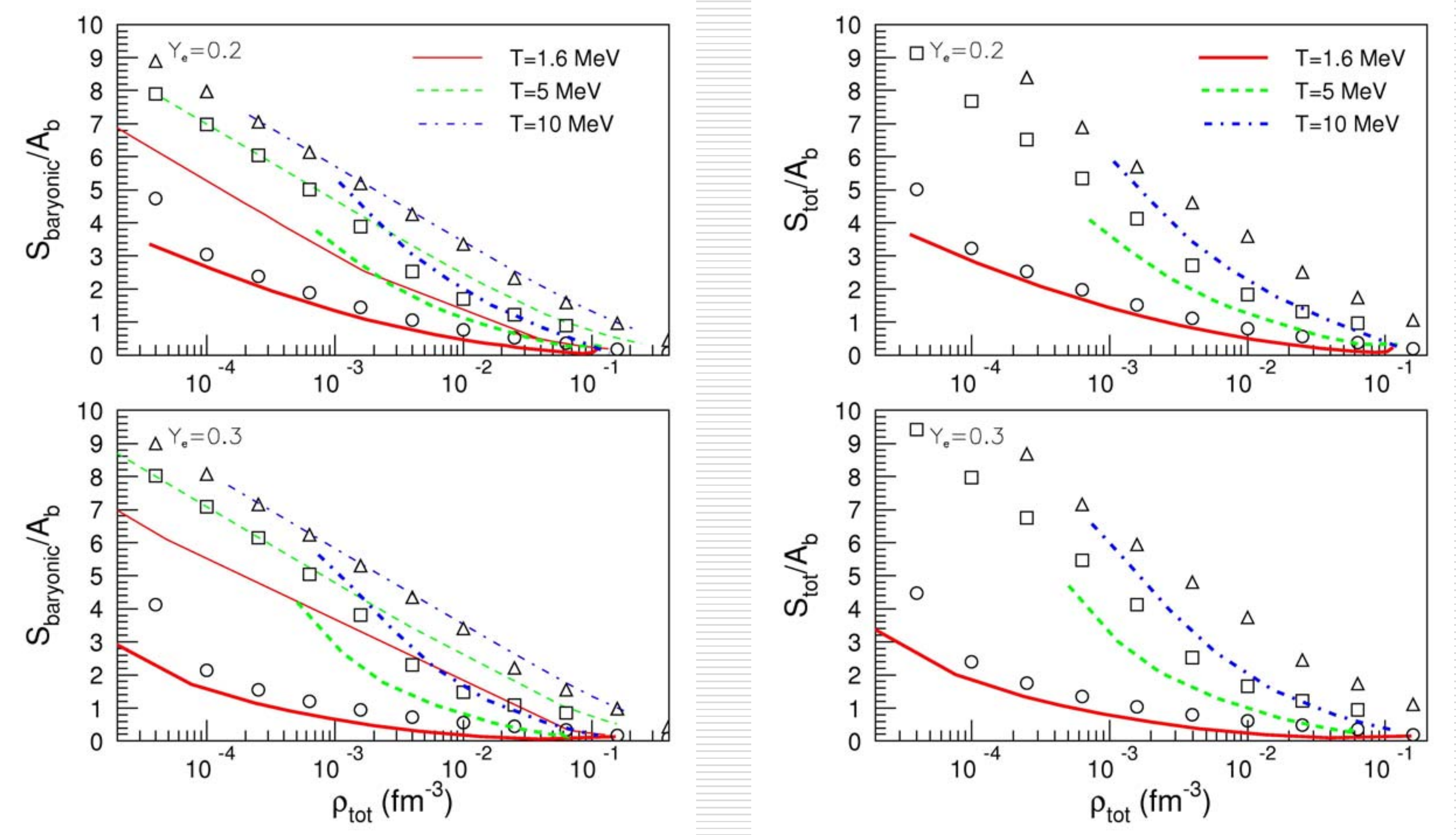
Lines: this work
Symbols: LS EOS



- Decreasing cluster size with increasing temperature
- Clusters still important at T=10 MeV

# Entropy density

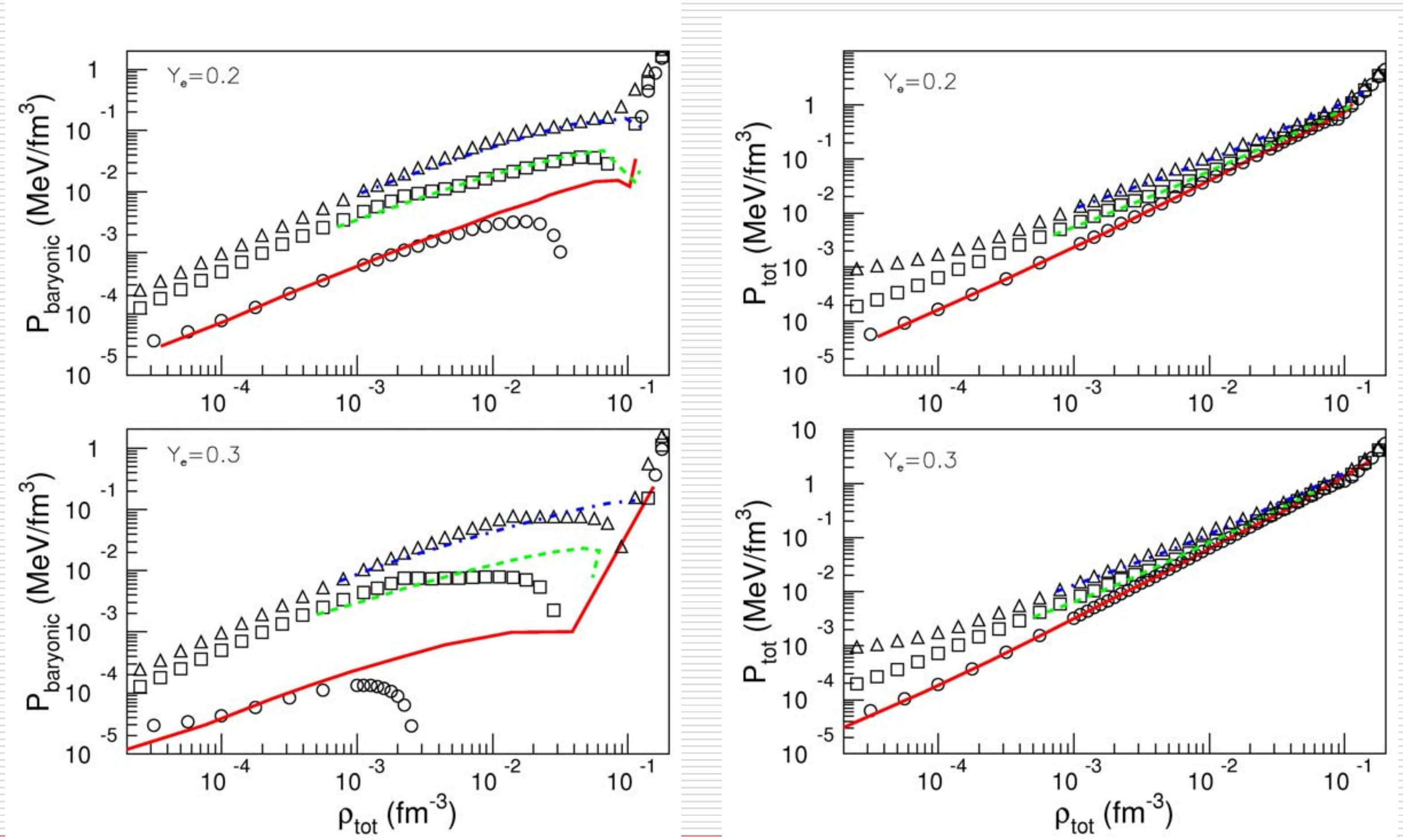
Symbols: LS EOS
Thick Lines: this work
Thin lines: clusters excluded



Differences with LS at high temperature even in the total entropy, due to the presence of clusters

# Pressure

Lines: this work
Symbols: LS EOS



Differences with LS at high density, due to the absence of a first order transition

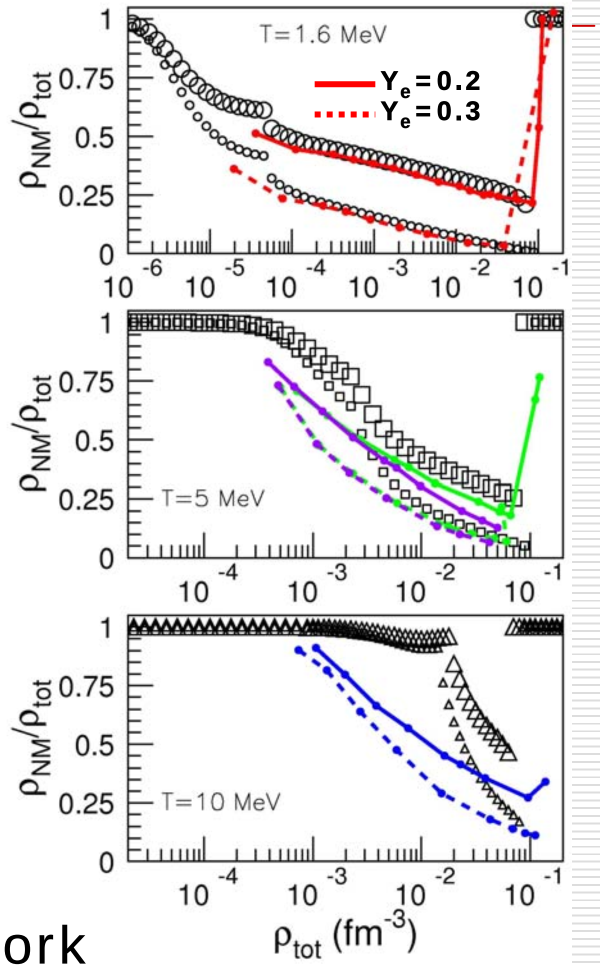
# Density and pressure at the crust-core transition

***J.M. Lattimer and M. Prakash, Phys. Rep. 442, 109 (2007).***

- Crustal fraction of the moment of inertia

$$\frac{\Delta I}{I}=\frac{28\pi P_t R^3}{3Mc^2\xi}\frac{1-1.67\xi-0.6\xi^2}{1+\dfrac{2P_t\left(1+5\xi-14\xi^2\right)}{\rho_t mc^2\xi^2}}$$

- Can be measured from pulsar glitches
- Puts constraints on the NS radius; ex: Vela pulsar
- Transition naturally obtained !



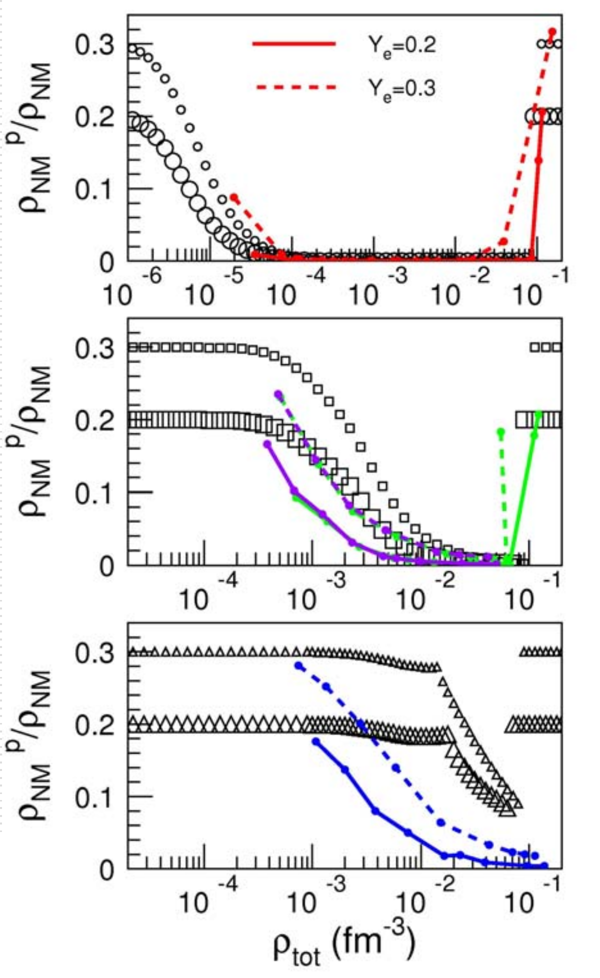
Lines: this work
Symbols: LS EOS

# Conclusion

- Specific thermodynamics for the dilute matter in NS crusts and SN cores
- Model-independent conclusion: no first-order phase transition
- Illustration within an improved NSE model combining nuclear matter properties and all-sized clusters

# Neutron versus proton drip

- Proton drip is negligible at low temperature
- Increases at high temperature, but much less than in LS



Lines: this work
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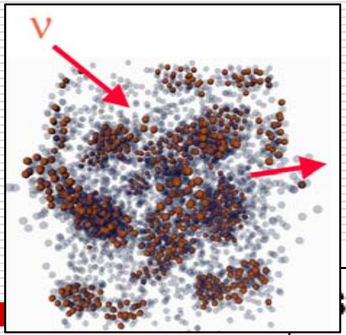
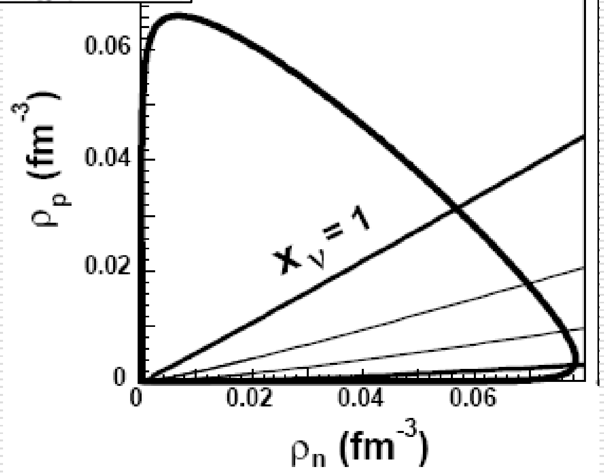
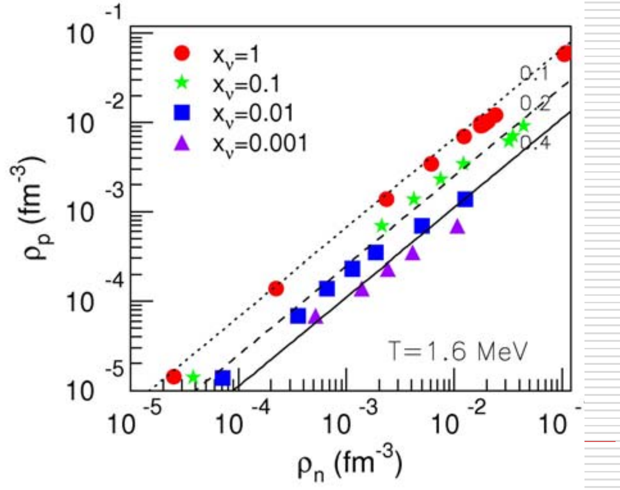
# Neutrino opacity



- $x_\nu$ percentage of trapped neutrinos in $\beta$ equilibrium
- Determines the leptonization rate => the size of the homologous core
- Similar results to MF calculation with the same effective interaction

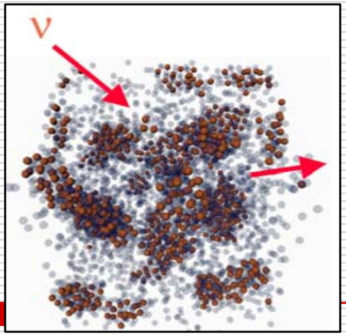
$$x_\nu = \frac{1}{2\pi^2\hbar^3\rho_\nu^{prod}} \int_0^{2\mu_\nu} de\, e^2 n_{\beta\mu}(e)$$



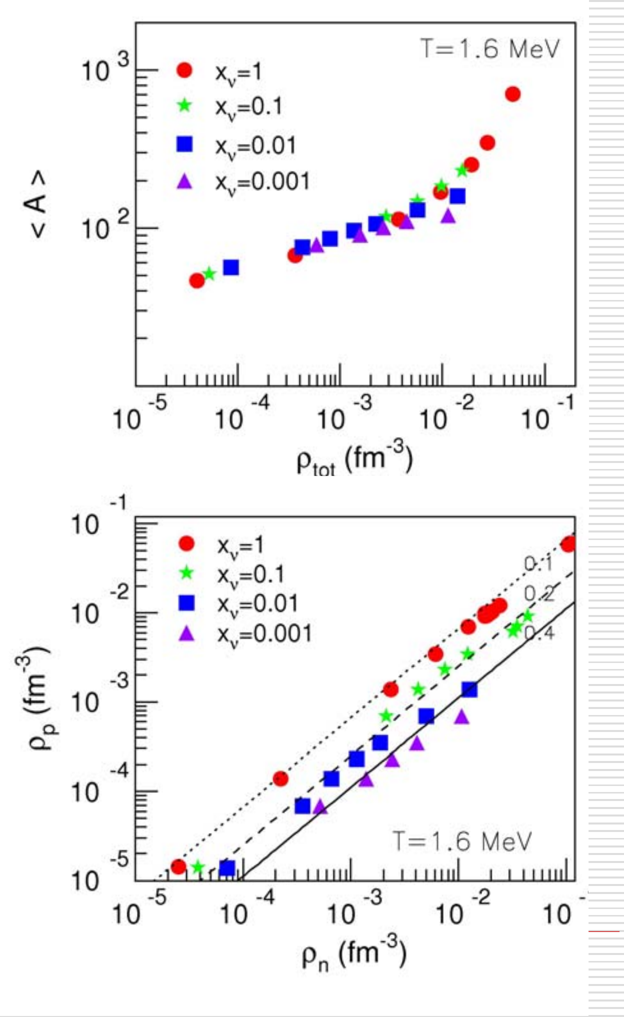


***C.Ducoin et al, NPA2007***

# Neutrino opacity

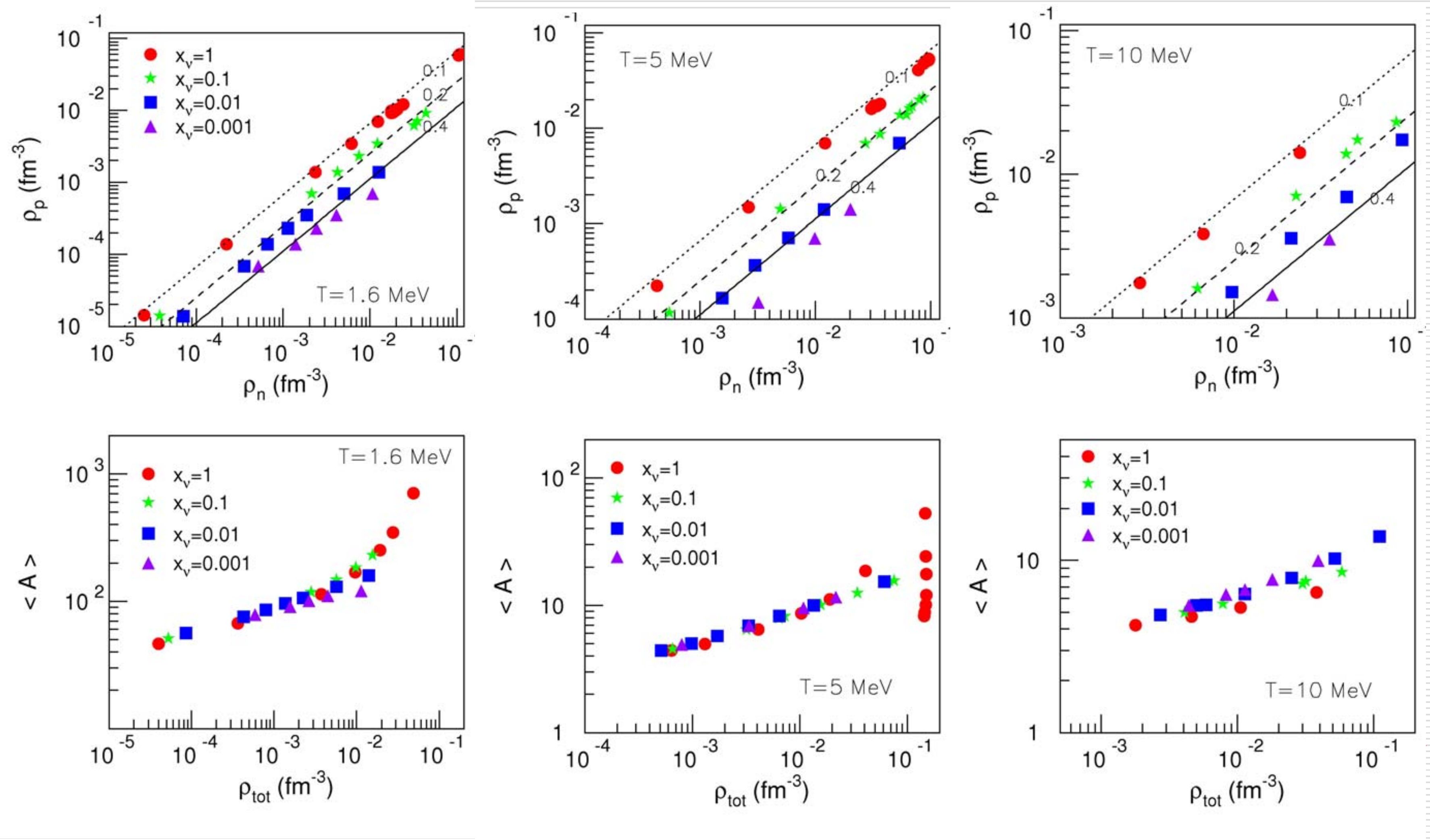


- $x_\nu$ percentage of trapped neutrinos
- Determines the leptonization rate => the size of the homologous core
- Similar results to MF calculation with the same effective interaction
- Can be correlated to the size of the clusters and number of free protons



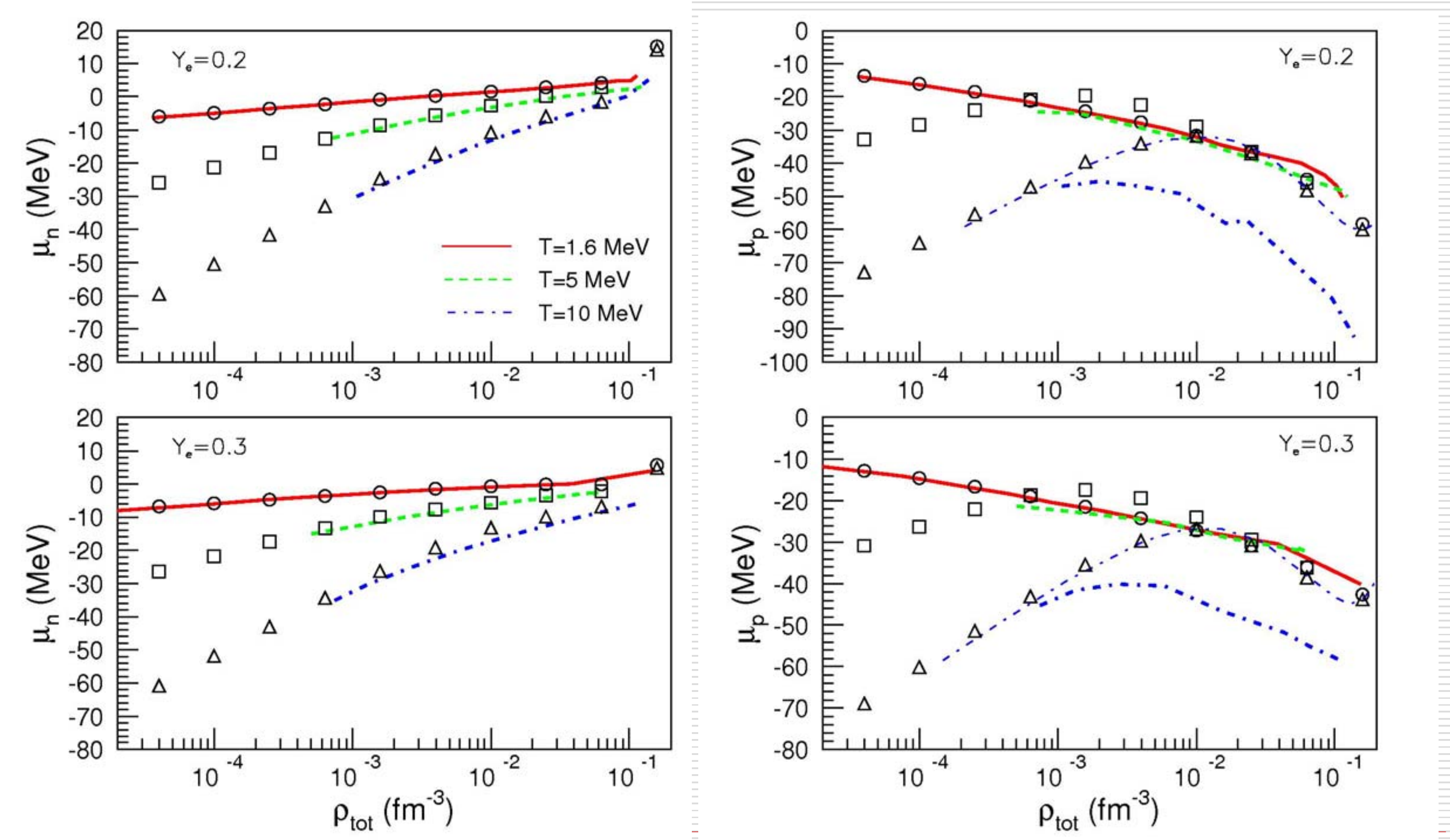
***A.Raduta, F.G., to be published***

# Opacity to neutrinos



# Chemical potentials

Symbols: LS EOS
Thick Lines: this work
Thin lines: clusters excluded



# Energy density

Symbols: LS EOS
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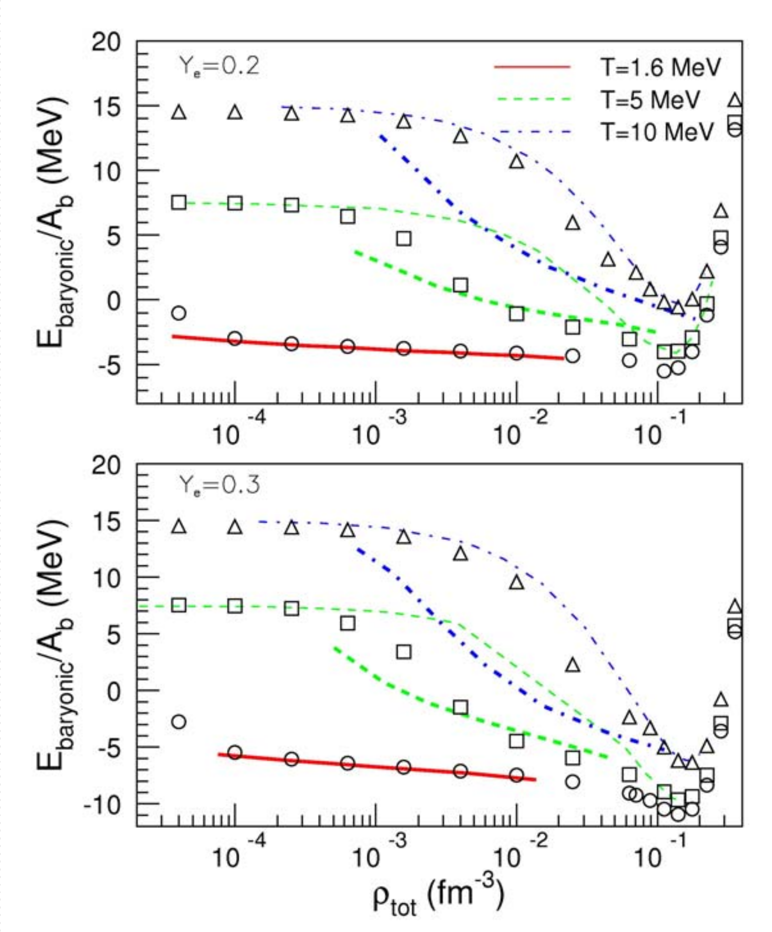
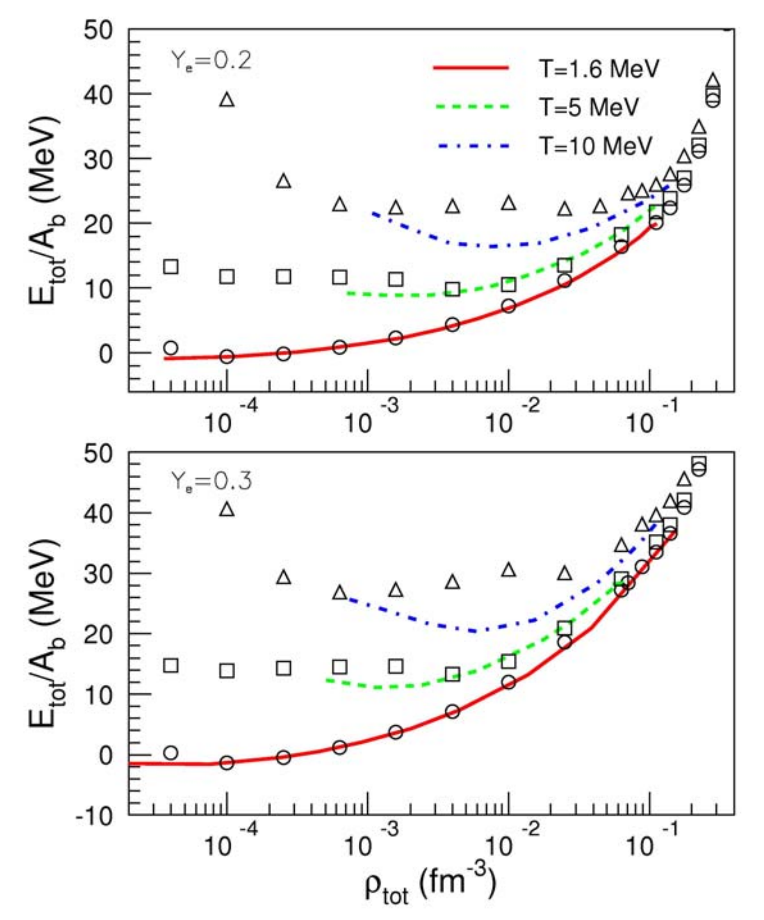
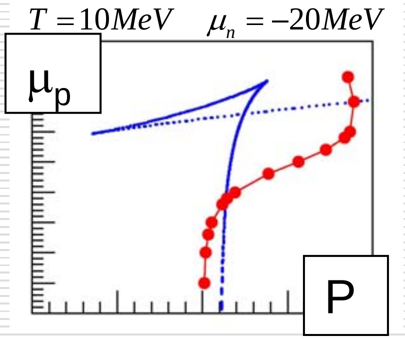




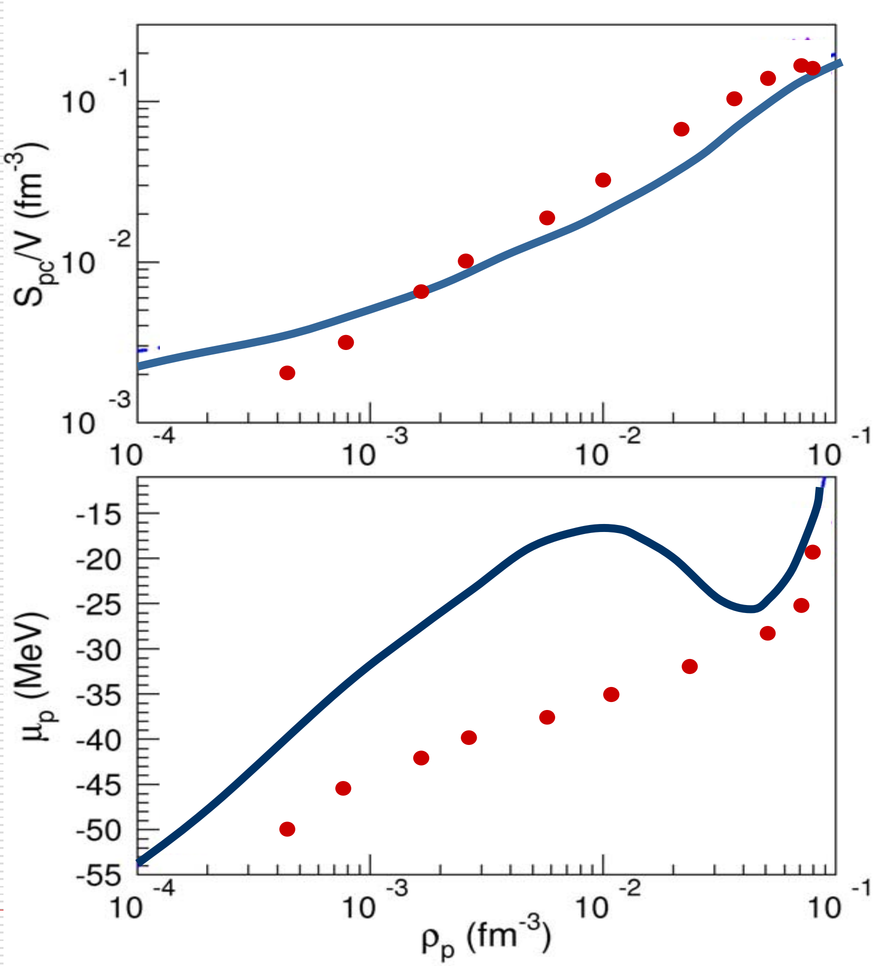
# The order of the crust-core transition



● **Clusters**

**HM**

- First order Φ transition:



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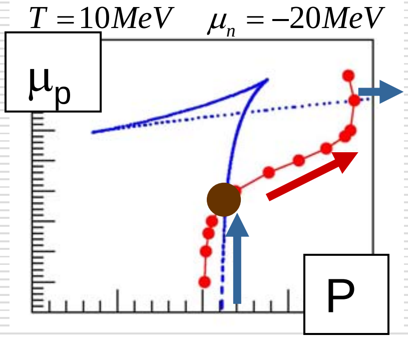
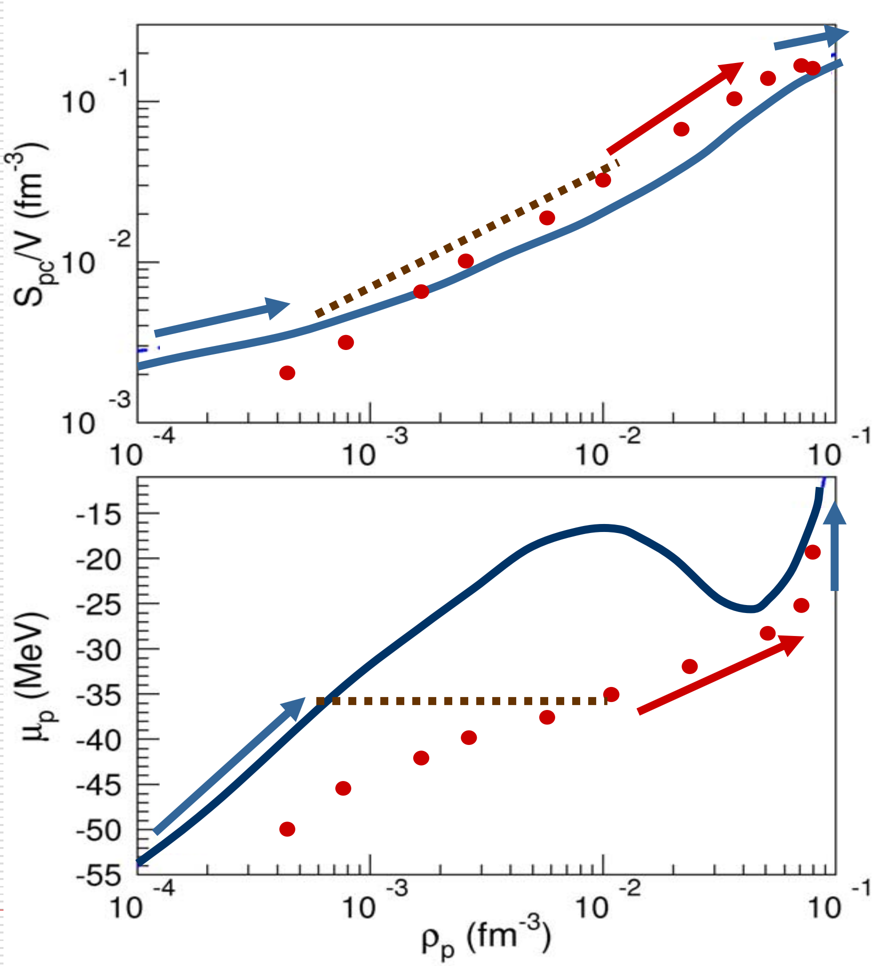


- First order Φ transition: discontinuous change from **HM**

  Through a coexistence point

  To **clusterized matter**

  To **Homogeneous Matter** again



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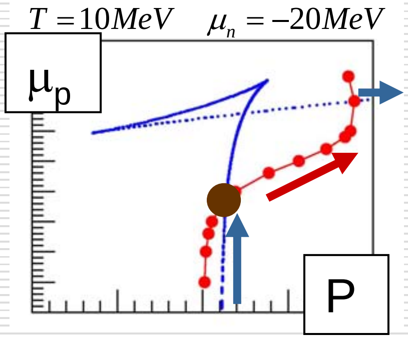

● **Clusters**

— **HM**

- First order Φ transition: discontinuous change from **HM**

  Through a coexistence point

  To **clusterized matter**

  To **Homogeneous Matter** again

- Mixture: ●

  Continuous EOS maximising the same entropy