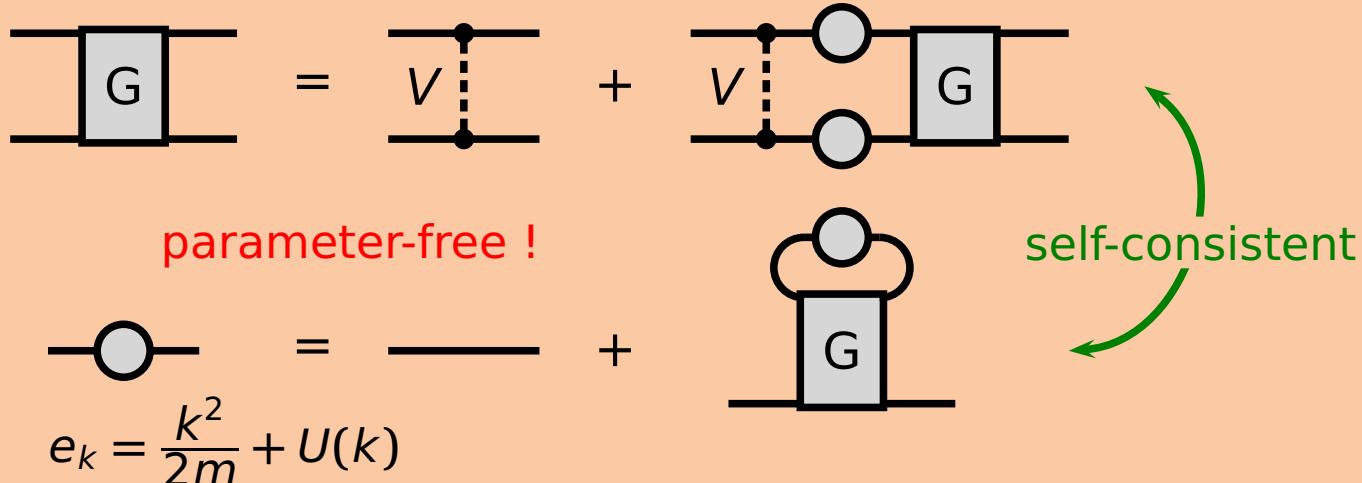


# Nuclear Symmetry Energy in the Brueckner-Hartree-Fock Approach

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# Brueckner Theory of Nuclear Matter:

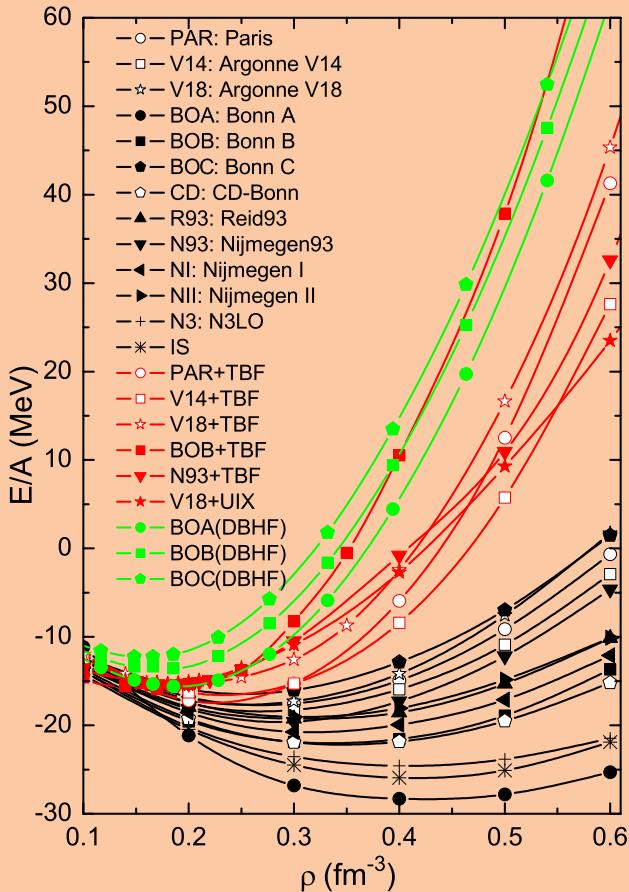
- Effective in-medium interaction  $G$  from potential  $V$ :



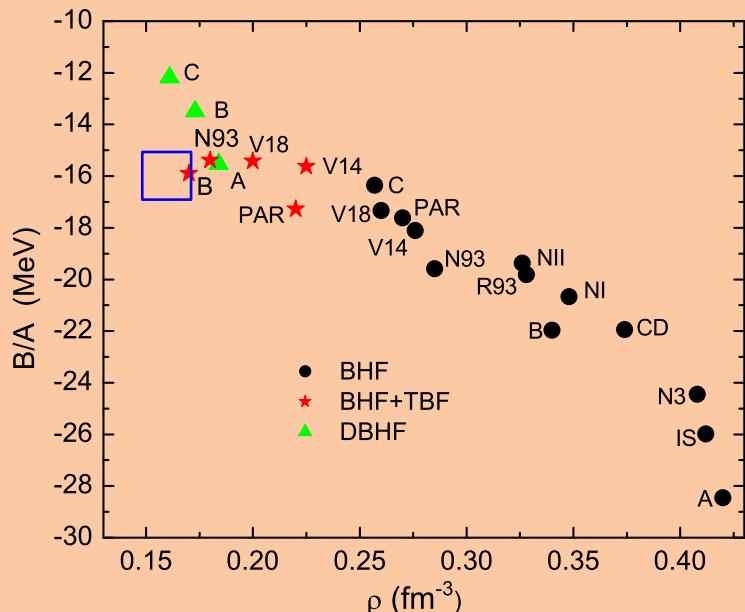
Compute: binding energy, s.p. properties, cross sections, ...

K.A. Brueckner and J.L. Gammel; PR 109, 1023 (1958) for nuclear matter

- BHF binding energy and saturation point of nuclear matter:



Coester band:



→ TBF substantially improve saturation properties

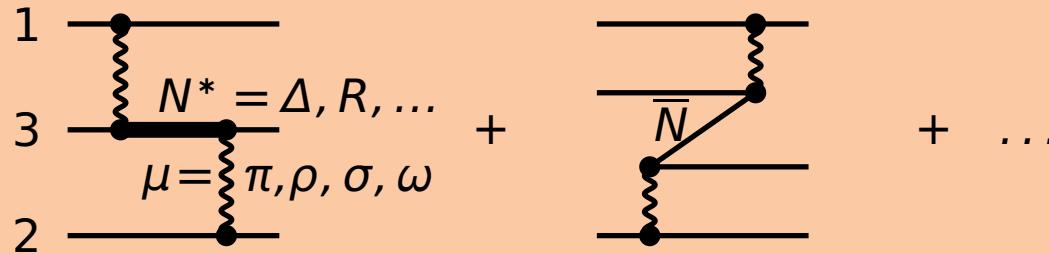
## Motivation for TBF:

- Structure of (light) nuclei, nucleon-deuteron scattering
- Saturation of nuclear matter
- Nuclear EOS at high density

## Goal:

- Construct nuclear TBF consistent with a given meson-exchange NN potential (Bonn B, Nijmegen 93)
- Use in microscopic BHF calculation of high-density nuclear matter
- Neutron star structure

# Three-Nucleon Forces:



- Only small effect required [ $\delta(B/A) \approx 1 \text{ MeV at } \rho_0$ ]
- Model dependent, no final theory yet
- Use and compare microscopic and phenomenological TBF...
  - Microscopic TBF of P. Grangé et al., PRC 40, 1040 (1989): Exchange of  $\pi, \rho, \sigma, \omega$  via  $\Delta(1232), R(1440), N\bar{N}$   
Parameters compatible with two-nucleon potential (Paris, V<sub>18</sub>, ...)
  - Urbana IX phenomenological TBF:  
Only  $2\pi$ -TBF + phenomenological repulsion  
Fit saturation point

# Microscopic Meson Exchange TBF:

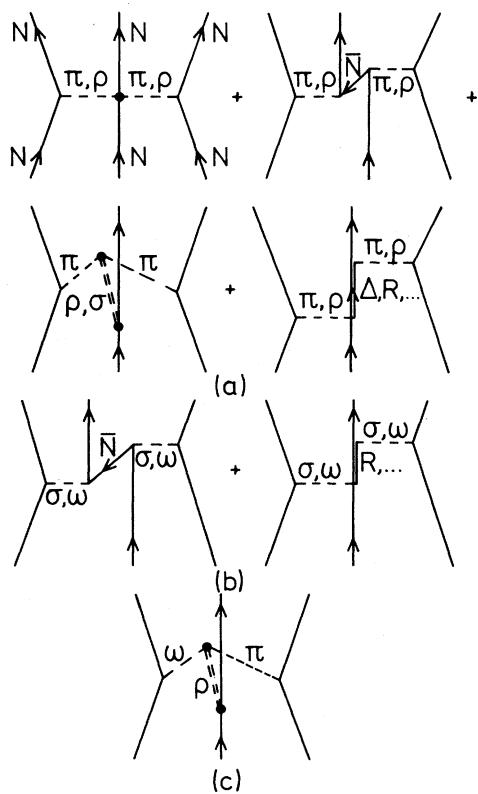


FIG. 3. Leading order contributions to the three-body force deduced from the meson-exchange current operators indicated in Fig. 2. See text for the explanation of the various groups (a)-(c).

P. Grangé, A. Lejeune, M. Martzolff,  
J.-F. Mathiot; PRC 40, 1040 (1989)

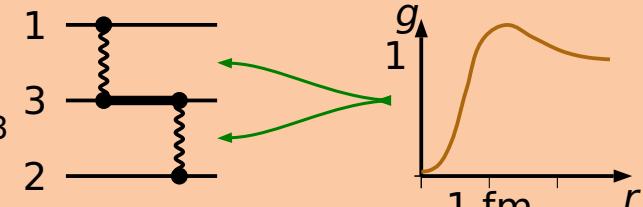
$\pi, \rho$  - part based on Tuscon-Melbourne TBF:  
S.A. Coon et al., NPA 317, 242 (1979);  
NPA 438, 631 (1985); PRC 48, 2559 (1993);

- Effects of  $\Delta(1232)$ ,  $R(1440)$ ,  $N\bar{N}$
- Parameters compatible with two-nucleon (Paris) potential

## Some Details:

- Average over spectator nucleon using BHF defect function:

$$\bar{V}_{12}(\mathbf{r}) = \rho \int d^3 r_3 \sum_{\sigma_3, \tau_3} g(r_{13}) g(r_{23}) V_{123}$$



$$= (\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) V_C(r) + (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) V_S(r) + V_I(r) \\ + S_{12}(\hat{\mathbf{r}}) [(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) V_T(r) + V_Q(r)] : \text{five components}$$

- Example:  $\rho\rho$  contribution:

$$V_O^{\rho\rho}(r) = -\frac{16 m_\rho^2 f_{\rho NN}^2 f_{\rho N\Delta}^2}{81 (m_\Delta - m_N)} \sum_3 \left[ 2 Y_x^\rho Y_y^\rho + P_r T_x^\rho T_y^\rho \right] \quad (O=C)$$

$$\underbrace{\left[ \frac{P}{2} T_x^\rho T_y^\rho - P_x Y_x^\rho T_y^\rho - P_y Y_y^\rho T_x^\rho \right]}_{\rho NN, \rho N\Delta \text{ form factors and kinematical factors}} \quad (O=T)$$

# Meson Exchange Parameters:

Table 1: Meson-exchange parameters of the Bonn B and Argonne  $V_{18}$  potentials.

The letter in brackets denotes the type of form factor: (M)onopole, (D)ipole, (R)oper.

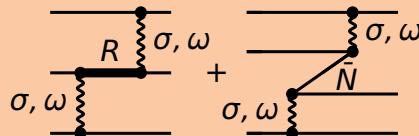
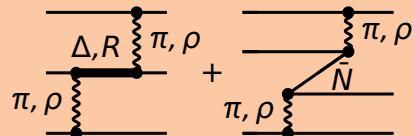
We use the baryon masses  $m_N = 938.4$  MeV,  $m_\Delta = 1232$  MeV,  $m_R = 1440$  MeV.

		$m$ (MeV)	$g^2/4\pi$	$\Lambda$ (MeV)	
Bonn B	$\pi$	138	14.4	1700 (M)	$a = 1.38, b = -2.80, c = 1.25$ : TM(99)
	$\rho$	769	0.90	1850 (D)	$\kappa = 6.1, g_{\pi N \Delta}/g_{\pi NN} = g_{\rho N \Delta}/g_{\rho NN} = 1.8$
	$\sigma NN$	550	8.94	1900 (M)	
	$\omega NN$	783	24.5	1850 (D)	
	$\sigma NR$	550	0.8	2000 (R)	$\alpha = 1$
	$\omega NR$	783	1.0	1850 (R)	$\alpha = 1$
$V_{18}$	$\pi$	138	14.43	1580 (M)	$a = 1.12, b = -2.49, c = 0.98$ : TM(81)
	$\rho$	776	0.55	1400 (M)	$\kappa = 6.6, g_{\pi N \Delta}/g_{\pi NN} = g_{\rho N \Delta}/g_{\rho NN} = 1.8$
	$\sigma NN$	540	11.9	1100 (M)	
	$\omega NN$	780	33.0	1300 (M)	
	$\sigma NR$	540	2.58	1450 (R)	$\alpha = -2.35$
	$\omega NR$	780	4.23	1550 (R)	$\alpha = -2.33$

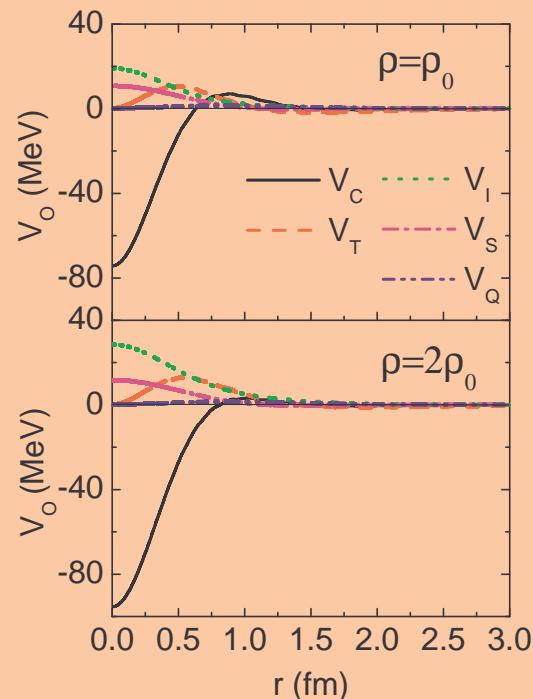
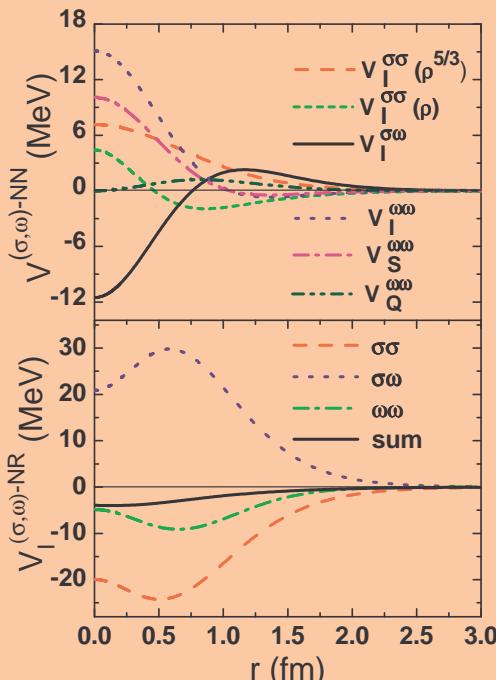
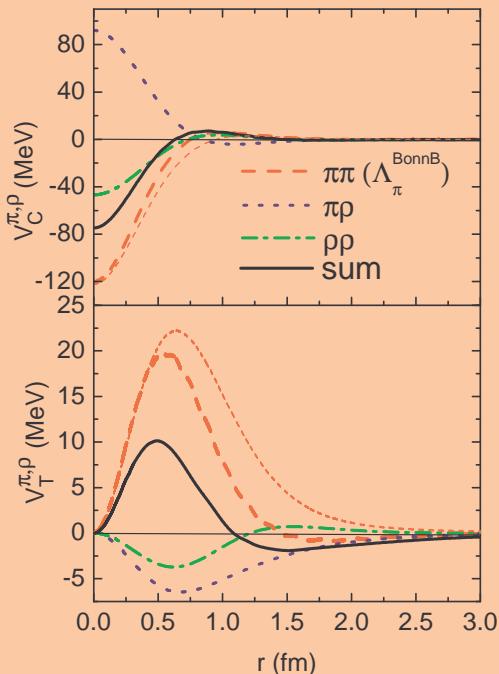
Nij93 has more parameters: 2 scalar mesons, wide  $\sigma$  and  $\rho$  mesons, “pomeron”

- Results with Bonn B potential ...

# Individual Meson Exchange Contributions:



Total



Strong compensation !

## Phenomenological TBF (Urbana Model):

- Two pion exchange + phenomenological repulsion:

$$V_{ijk} = \sum_{\text{cyc.}} \left[ \begin{array}{l} \textcolor{red}{A} \{X_{ij}, X_{jk}\} \{\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j, \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k\} \\ \textcolor{red}{A} \frac{1}{4} [X_{ij}, X_{jk}] [\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j, \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k] + \textcolor{blue}{U} T_{ij}^2 T_{jk}^2 \end{array} \right]$$

$$X_{ij} = Y(m_\pi r_{ij}) \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + T(m_\pi r_{ij}) S_{ij}$$

$$Y(x) = \frac{e^{-x}}{x} (1 - e^{-cx^2}), \quad T(x) = \left(1 + \frac{3}{x} + \frac{3}{x^2}\right) \frac{e^{-x}}{x} (1 - e^{-cx^2})^2$$

- Corresponds to micro TBF with only  $\pi\pi$  contribution and

$$\textcolor{red}{A} = -\frac{2}{81} \frac{(m_\pi f_{\pi NN} f_{\pi N\Delta})^2}{m_\Delta - m_N}$$

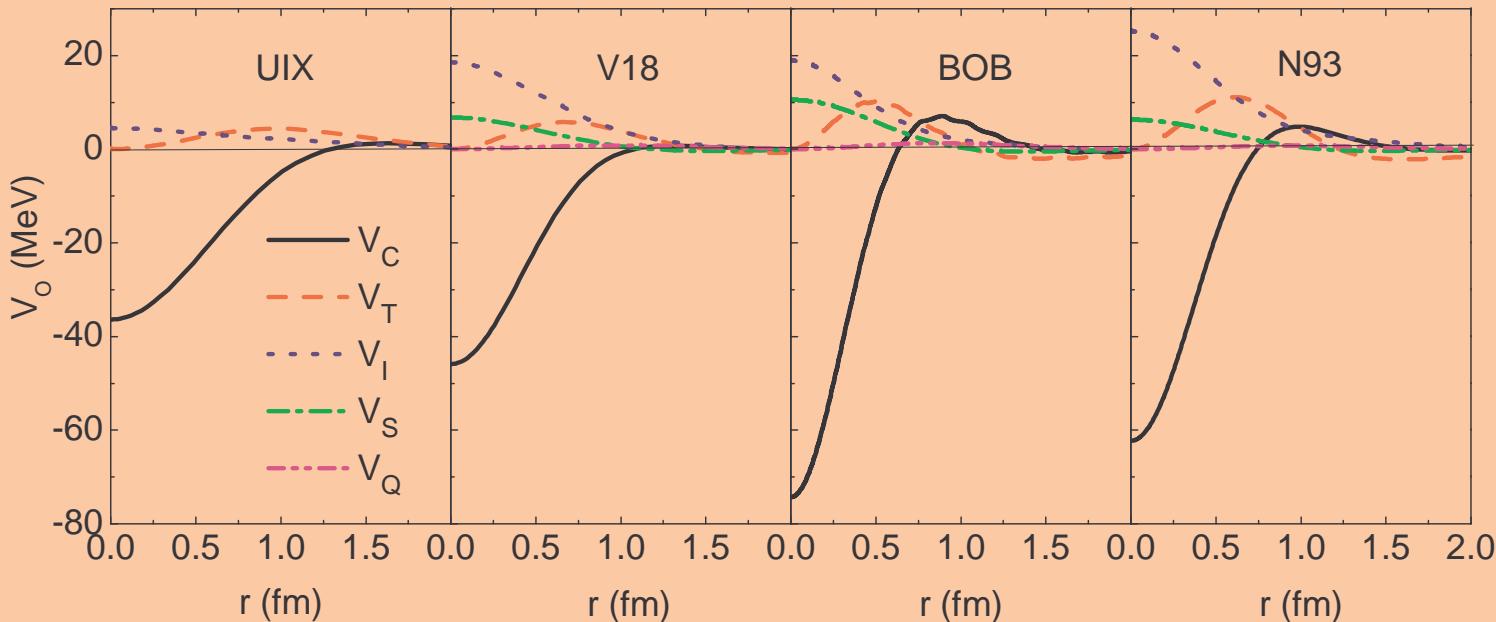
- Optimal parameters for BHF+TBF (with V18 2BF):

$$\textcolor{red}{A} \approx -0.0500 \text{ MeV}, \quad \textcolor{blue}{U} \approx 0.00126 \text{ MeV}$$

# Phenomenological vs. Microscopic TBF:

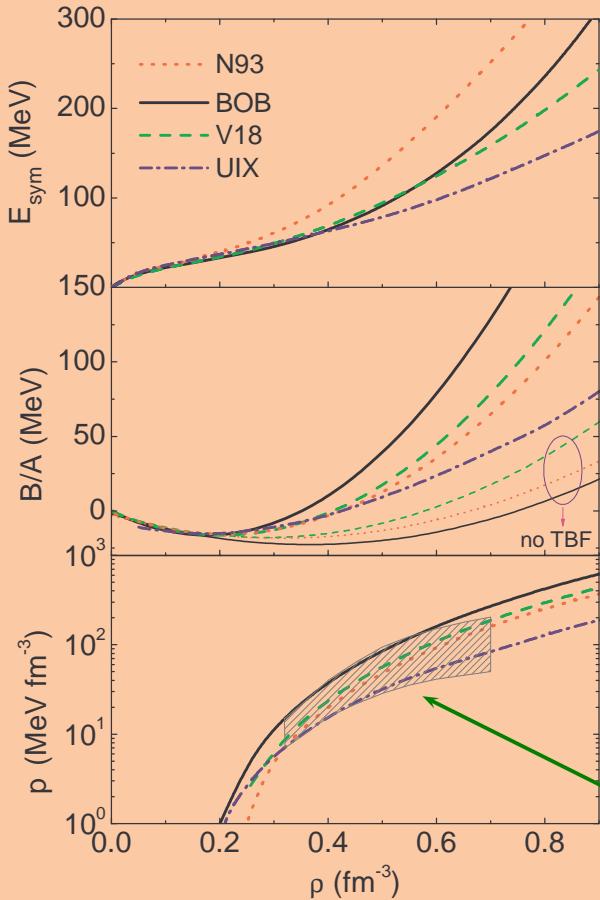
- Compare micro TBF with  $V_{18}$ , Bonn B, or Nijmegen 93 potential and UIX TBF (with  $V_{18}$ ):

$$\begin{aligned}\bar{V}_{ij}(\mathbf{r}) = & (\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)V_C(r) + (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)V_S(r) + V_I(r) \\ & + S_{ij}(\hat{\mathbf{r}})[(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)V_T(r) + V_Q(r)] \quad \text{at } \rho = \rho_0 : \end{aligned}$$



# Results of BHF+TBF Approach:

- Symmetry energy, EOS, saturation properties:

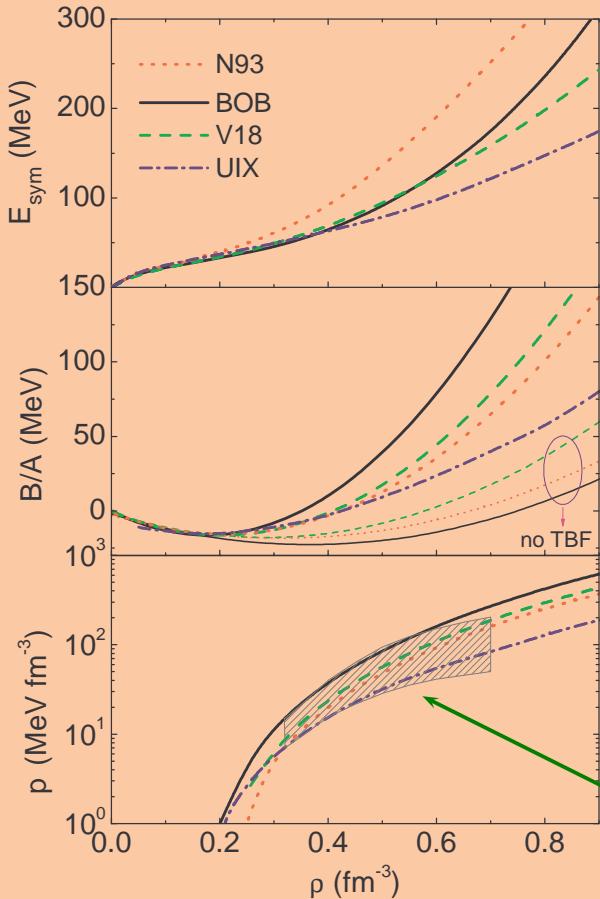


	$[\rho, B/A]_0$ [fm $^{-3}$ , MeV]	$K$ MeV	$E_{\text{sym}}$ MeV	$E'_{\text{sym}}$ MeV
N93	[0.18,-15.4]	216	34.0	35.5
BOB	[0.17,-15.9]	244	29.4	24.8
V18	[0.20,-14.7]	226	30.6	33.8
UIX	[0.18,-15.3]	192	33.5	24.5

Nuclear flow analysis of Science 298, 1592 (2002)

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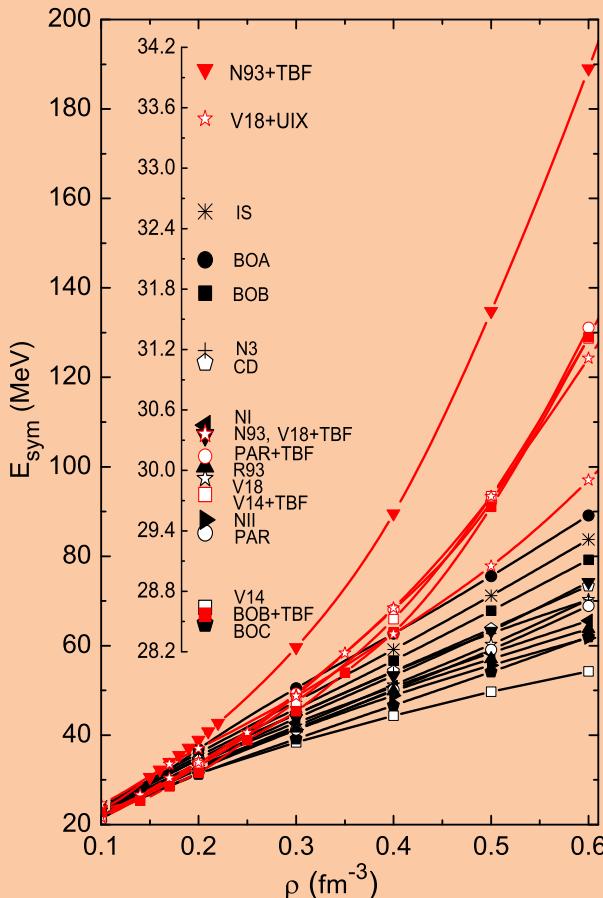


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➡ No asy-soft EOS !

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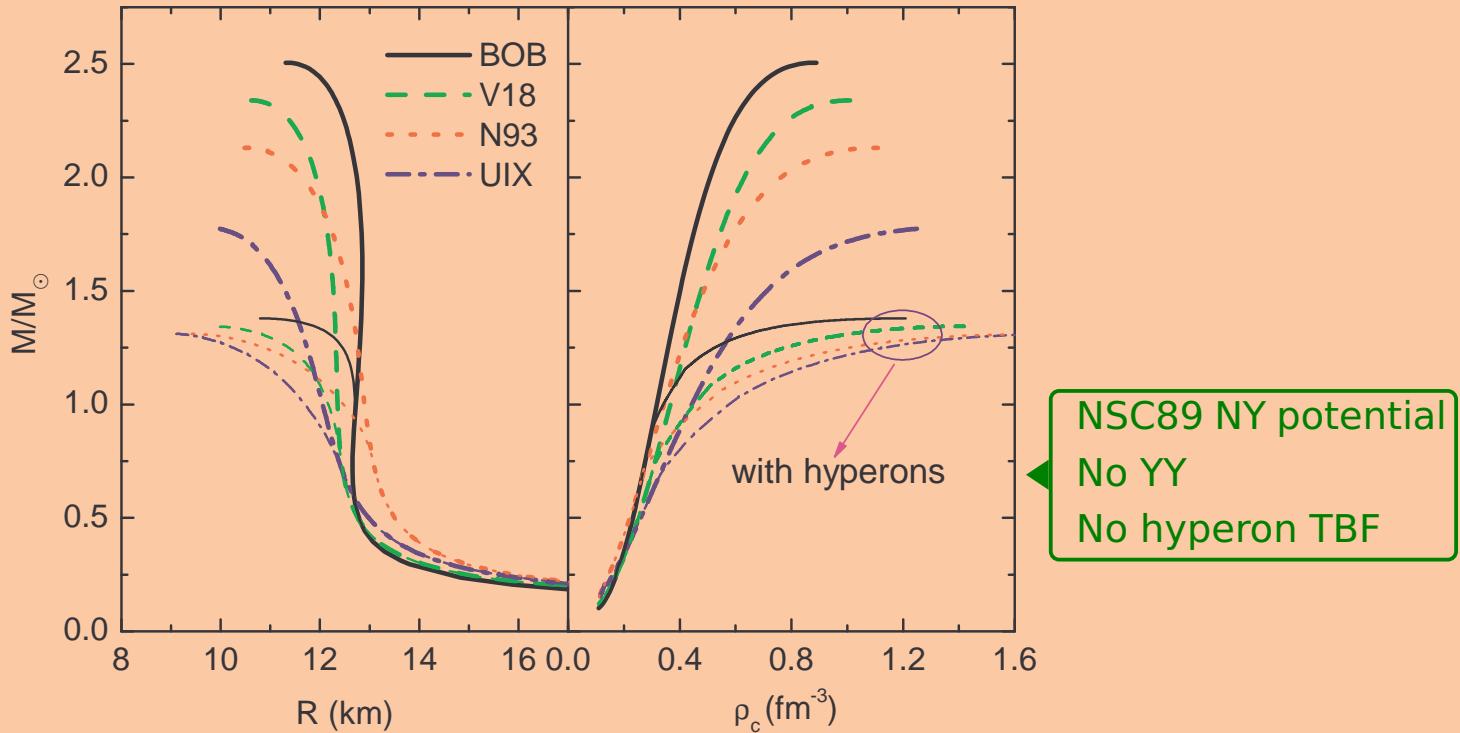
# Symmetry energy with and without TBF:



→ TBF increase slope of  $E_{\text{sym}}(\rho)$ !

# Neutron star structure:

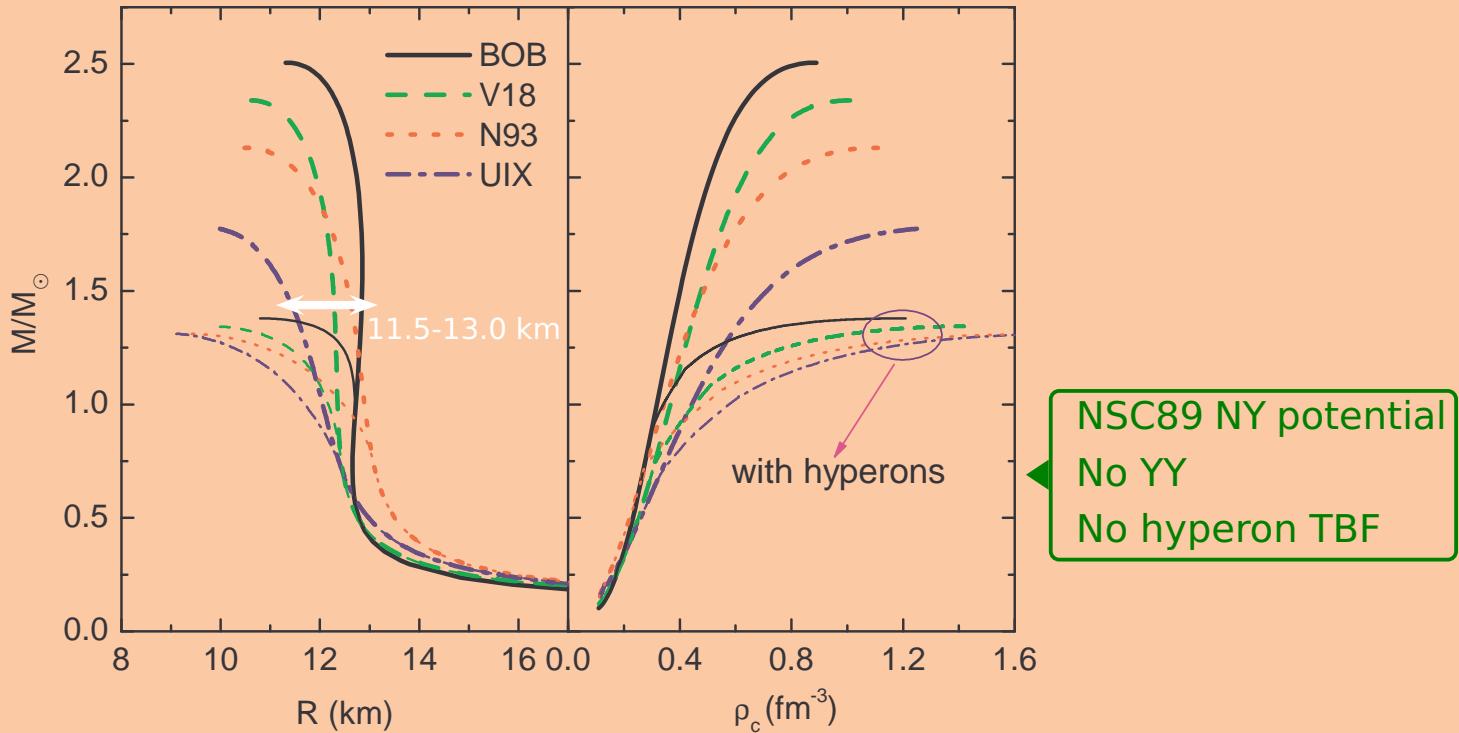
- Solve TOV equations:



→ Large variation of  $M_{\max}$  with nucleonic TBF  
Self-regulating softening due to hyperon appearance

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## Summary:

- Consistent microscopic TBF + BHF provide reasonable saturation properties
- $E_{\text{sym}}(\rho)$  rises faster than linear in all models
- Uncertain high-density behavior:  $M_{\text{max}} \approx 1.8 - 2.5 M_{\odot}$

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## Future:

- Technical improvements:  
3rd nucleon average, static approximation, ...
- BHF with TBF + 3 hole line corrections
- Micro TBF in light nuclei ?