Nuclear Symmetry Energy
in Compact Star Matter

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Work in progress with
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I. Compact Stars

- Stars (neutron stars, quark stars, ..)
  - ~ solar mass and ~10km or less in size
- Constituents of compact star
  - Hadrons responsible for mass of star
    - neutrons, protons, hyperons
    - pions, kaons, .......
  - Leptons
    - electrons, muons,
    - neutrinos(e-, mu-)
  - Exotics?
- ~ Zero temperature, Charge neutrality
Masses of Pulsars (Stairs 2006)

- more than 1800 pulsars known with 140 binary pulsars
- best determined mass:
  \[ M = (1.4414 \pm 0.0002) M_\odot \]
  Hulse-Taylor pulsar
  (Weisberg and Taylor, 2004)
- mass of PSR J0751+1807 corr.
  from \[ M = (2.1 \pm 0.2) M_\odot \]
  to \[ M = (1.14 - 1.40) M_\odot \]
  (Nice et al. 2008)
- mass of PSR J1903+0327 (not finalized yet):
  \[ M = (1.67 \pm 0.01) M_\odot \]
  (Freire et al. 2009)
• Stars and Gravity

• Equation of state (EOS) of compact star energy density and pressure

\[ P = n^2 \frac{\partial (\epsilon/n)}{\partial n} \]
• Higher nucleon number density inside $n = 0 \rightarrow n_0 \rightarrow 6n_0 \rightarrow$

1. Change of nucleon-nucleon interaction with density
2. Emerging of new hadrons with density kaons, hyperons (strange hadrons), ..

• Dense Hadronic Matter at the Core
  $\rightarrow$ Change of EOS with density
II. Hadronic Interactions

- Fermi pressure of nucleons
  + Strong interaction
- Nucleon-nucleon repulsion and attractions
- N-N interaction is known only upto nuclear density, $n_0 = 0.16 \text{ fm}^{-3}$
• Heavier nuclei

**Mass formula (Weizsäcker, 1935; Bethe and Bacher, 1936)**

Binding energy of a nucleus \( (A = Z + N) \)

\[
B(N, Z) = b_{\text{vol}}A - b_{\text{surf}}A^{2/3} - \frac{1}{2}b_{\text{sym}}\frac{(N - Z)^2}{A} - \frac{3}{5}\frac{Z^2e^2}{R_c}
\]

• Dense nuclear matter

\[
E(n, N_p) \simeq m_N + \frac{3}{5}E_F^0 \left(\frac{n}{n_0}\right)^{2/3} + S(n)(1 - 2N_p)^2 + V(n)
\]

\[
E_F^0 = \frac{(3\pi^2n_0/2)^{2/3}}{2m} = 37.2\text{MeV}
\]

• \( n > n_0 \),

• Lattice QCD, Effective theory for hadrons,
  Many body interactions, ...
III. Nuclear Symmetry Energy

- Measure of n-p asymmetry in nuclear interaction
- In nuclear matter

\[ E(n, N_p) \simeq m_N + \frac{3}{5} E_F \left( \frac{n}{n_0} \right)^{2/3} + S(n)(1 - 2N_p)^2 + V(n) \]

\[ S(n) = S_{\text{free}}(n) + S_{\text{int}}(n) \]

\[ S(n_0) \sim 30 \, \text{MeV} \]
• Non-interacting n-p system

$$\epsilon_n = \frac{8\pi}{(2\pi)^3} \int_0^{p_F} (p^2 + m_n^2)^{1/2} p^2 dp$$

$$n_n = \frac{8\pi}{(2\pi)^3} \int_0^{p_F} p^2 dp$$

$$S_{\text{free}}(n) = \left(2^{2/3} - 1\right) \frac{3}{5} E_F^0 \left(\frac{n}{n_0}\right)^{2/3}$$
Phenomenological forms of symmetry energy

\[
S_F(n) = (2^{2/3} - 1) \frac{3}{5} E_F^0 \left[ \left( \frac{n}{n_0} \right)^{2/3} - F(n) \right] + S_0 F(n)
\]

\[
S_\alpha = (2^{2/3} - 1) \frac{3}{5} E_F^0 \left( \frac{n}{n_0} \right)^{2/3} + A(\alpha) \frac{n}{n_0} + [18.6 - A(\alpha)] \left( \frac{n}{n_0} \right)^{B(\alpha)}
\]

\[
S_3(n) \approx S_0^* + L \rho + \frac{1}{2} K \rho^2
\]
• Up to nuclear density, \( n_0 = 0.16 \text{ fm}^{-3} \)

• Compact star, \( n > n_0 \) with central density \( n_{\text{center}} > 3 n_0 \)

\[ S(n) \ ? \ V(n) \ ? \]

• Simple extrapolation of what are known at low density nuclear matter?
IV. Nuclear symmetry energy in compact stars

• What is the role of symmetry energy (n-p asymmetry) in compact star?
• Symmetry energy provides a channel for new degrees of freedom in n-p system via weak interaction:
  muon,
  strange particles (kaon, hyperon), ..
Symmetry energy and weak equilibrium in compact star

- chemical potential of neutron and proton
  \[ \mu_n - \mu_p = 4(1 - 2N_p)S(n) \]

- beta equilibrium with electron
  \[ n \rightarrow p + e \implies \mu_n - \mu_p = \mu_e \]

- charge neutrality condition
  \[ n_p = n_e \]
• Strong interaction

\[ \mu_n - \mu_p = 4(1 - 2N_p)S(n) \oplus \{ k, \Sigma, \Xi \cdots \} \]

• Weak equilibrium with new degrees of freedom,

\[ \mu_n - \mu_p = \mu_\mu (m_\mu) \]
\[ \mu_{K^-} (m_{K^-}) \]
\[ \mu_e = \mu_{\Sigma^-} - \mu_n \]
\[ \mu_{\Xi^-} - \mu_\Lambda \]

• Kaon condensation, hyperon matter, ..
• Hadron interaction at high density
  \[ S(n) \quad \{ \kappa, \Sigma, \Xi, \ldots \} \quad m_k^* \]

• Chiral perturbation theory
• Tensor force with BR scaling
• Three body forces
• HLS
• Skyrmion on the lattice
• hQCD

......

piece-wise effective theories (or polytropic EOS)
  \[ \ldots, \, n(i) < n < n(i+1), \ldots \]
V. **Supersoft symmetric energy**

Transport model analysis using IBUU04
Z. Xiao, B.A. Li, L.W. Chen, G.C. Yong
and M. Zhang, PRL 102, 062502 (2009)
“Supersoft” is a disaster?

If Nature chose the “supersoft” $E_{SS}$

There could be NO stable neutron stars unless …!!

But Nature is full of neutron stars including the Hulse-Taylor binary pulsar.

Drastic wayout by D.H. Wen et al, PRL 103, 211102 (09): Modify Newtonian gravity, which could be emergent (e.g., a la E. Verlinde, arXiv:1001.0785).

But kaons will not condense,
VI. Possible structure with Strangeness

• When the difference between chemical potentials of proton and neutron becomes comparable to kaon mass or hyperon mass in medium, the corresponding strange particles begin populating and the EOS get changed significantly.

• Kaon condensation

  Kaplan and Nelson,
  Bethe and Brown,
  Brown, Rho and Kubodera, ...
Stars with kaon condensed matter

(Ⅰ) Self bound: \( n > n_c \)

(Ⅱ) \( n \leq n_K \)

(Ⅲ) \( n > n_K \)
Muto et al.
Strange quark star with kaon-condensed surface (K. Kim, M. Rho and HKL in progress)

• For a hadron system, where kaon chemical potential becomes much small about electron mass, there is no leptons to balance the positive charge of protons but kaons.

• This is equivalent to strange quark matter with high enough density where strange quark mass can be neglected.

• It can define the transition surface between hadronic phase with kaon condensation and strange quark matter.
Hadronic Matter  Quark Matter

\[ p \Rightarrow uuud \]
\[ n \Rightarrow uddd \]
\[ k^- \Rightarrow s\bar{u} \]

\[ \mu_K = 0 \Rightarrow \mu_s = \mu_u \]
\[ \mu_u = \mu_p \Rightarrow \mu_u = \mu_d \]

Kaon condensation
At zero chemical potential  SQM
• SQ star with self bound kaon surface
VII. Hadronic crystal-structure at higher density?

- Nucleon crystal structure at the core of compact star at higher density
- Solitonic description → Skyrmions on the lattice
- Density dependence of symmetry energy and effective mass of roaming particles through (kaon mass)
Appearance of fractionized skyrmions

\[ U = e^{2i\pi/f_\pi} \quad \rightarrow \quad \text{skyrmion} \]

\[ U = \xi_L\xi_R^\dagger, \quad \xi_{L,R} \quad \rightarrow \quad \text{half-skyrmion} \]

Simulate dense matter by putting skyrmions in FCC crystals and squeeze them: \( \frac{1}{2} \)-skyrmions in CC appear at \( n_{1/2} \)

B.Y. Park et al, 1999

\[ \mathcal{L}_\xi = \frac{f_\pi^2}{2} \left\{ \text{Tr} \left[ |D_\mu\xi_L|^2 + |D_\mu\xi_R|^2 \right] \right\} \]
“Phase” structure

The $\frac{1}{2}$-skyrmion phase at $n \geq n_{1/2}$ is characterized by

$$\langle \bar{q}q \rangle = 0, \quad f_\pi \neq 0, \quad a = 1, \quad \frac{m_\pi}{m_\rho} \ll 1$$

i.e. “vector mode”

Rough estimate: $n_{1/2} \sim (1.3 - 2) n_0$
What does the $\frac{1}{2}$-skyrmion phase do to $E_{\text{sym}}$?

Symmetry energy $\sim 1/N_c$

Collective-quantize the (neutron) skyrmion matter

$\rightarrow$ Isospin rotation

\[
U(\vec{r}, t) = A(t)U_0(\vec{r})A^\dagger(t)
\]

\[
E^{\text{tot}} = AM_{\text{cl}} + \frac{1}{2A\lambda_I}I^{\text{tot}}(I^{\text{tot}} + 1)
\]

\[
I^{\text{tot}} = \frac{1}{2}\hat{A}\delta \quad \delta = (n_p - n_n)/(n_n + n_p)
\]

\[
E = M_{\text{cl}} + \frac{1}{8\lambda_I}\delta^2 \quad \rightarrow \quad E_{\text{sym}} = \frac{1}{8\lambda_I}
\]

M.Rho, Trento, 2010

I. Klebanov, 1985

Moment of inertia
$E_{\text{sym}}$ from half-skyrmion matter

\[ E_{\text{sym}} = \frac{1}{8\lambda_I} \]

H.K. Lee, B.Y. Park, R. 2010
Anti-kaon “roaming” through $\frac{1}{2}$-skyrmion matter: Wess-Zumino term

How to measure isospin spirals?
How to understand the cusp at $n_{1/2}$ in “standard” nuclear physics?

H.K. Lee, B.Y. Park, R. 2010

In-medium scaling
How to concoct the $E_{ss}$

Possible mechanism: medium-scaling tensor forces

C. Xu & B.A. Li, arXiv:0910.4803

$$V_M^T(r) = S_M \frac{f_{NM}^2}{4\pi} m_M T_1 \cdot T_2 S_{12}$$

$$\left( \frac{1}{(m_M r)^3} + \frac{1}{(m_M r)^2} + \frac{1}{3m_M r} \right) e^{-m_M r}$$

$M = \pi, \rho, S_{\rho(\pi)} = +1(-1)$.

Tensor forces cancel: Exploit medium-enhanced cancelation to describe the C14 dating by J.W. Holt et al, PRL 100, 062501 (08)

Assumed scaling:

$$\frac{m^*_\rho}{m_\rho} \approx \frac{m^*_N}{m_N} \approx \frac{f^*_\pi}{f_\pi} \approx 1 - \alpha \frac{n}{n_0}$$
Apply to $E_{\text{sym}}$

M. Rho, Trento, 2010

C. Xu & B.A. Li, arXiv:0910.4803

$$\frac{m^*}{m_\rho} \approx \frac{m_N^*}{m_N} \approx \frac{f^*_\pi}{f_\pi} \approx 1 - \alpha \frac{n}{n_0} \quad \text{for all density (n)}$$

$$\Phi_i(n) = 1 - \alpha_i \left(\frac{n}{n_0}\right) \quad 0 = \alpha_0 < \alpha_1 < \alpha_2 < \alpha_3 \ldots < \alpha_m \approx 0.2$$
Tensor forces are drastically modified in the $\frac{1}{2}$-skyrmion phase

- For density $n < n_{1/2}$:
  $$\frac{m^*_\rho}{m_\rho} \approx \frac{m^*_N}{m_N} \approx \frac{f^*_\pi}{f_\pi} \approx 1 - \alpha \frac{n}{n_0}$$

- For density $n \geq n_{1/2}$:
  $$\frac{m^*_N}{m_N} \approx \frac{f^*_\pi}{f_\pi} \approx 1, \quad \frac{m^*_V}{m_V} \approx \frac{g^*_V}{g_V} \approx \frac{\langle \bar{q}q \rangle^*}{\langle \bar{q}q \rangle}$$

Above $n_{1/2}$, the $\rho$ tensor gets “killed,” enabling the pions ($\pi^0$’s) to condense → pionic crystal in dense neutron matter (e.g., Pandharipande and Smith 74).
This prediction could be checked or falsified at FAIR or even RIB (e.g., KoRia) machines.
Summary

• Higher density, \( n > n_0 \) at the core of compact star \( \rightarrow \) new physics

• Symmetry energy (\( n-p \) asymmetry) :
  new degrees of freedom into \( n-p \) system in weak equilibrium (crust and core)

• Emergence of strangeness at the core

• kaon condensation, hyperon degrees of freedom

Mass, radius and compositions, ....

Cooling, GW, GRB, .....