Symmetry Energy in Nuclei

Outline:

1. ***Nuclear symmetry energy***
2. ***Two universal densities***
3. ***Half-infinite Matter***
    - ***a) Solution of Skyrme-Hartree-Fock Equations***
    - ***b) Correlations of Symmetry Coefficients & S(ρ)***
4. ***$a_a(A)$ from Isobaric Analog States***
    - ***a) Use of Charge Invariance***
    - ***b) Data Analysis & Interpretation***
5. ***Validity of the employed procedures ?***
6. ***Constraints of S(ρ) from structure and reactions***
7. ***Summary & Outlook***

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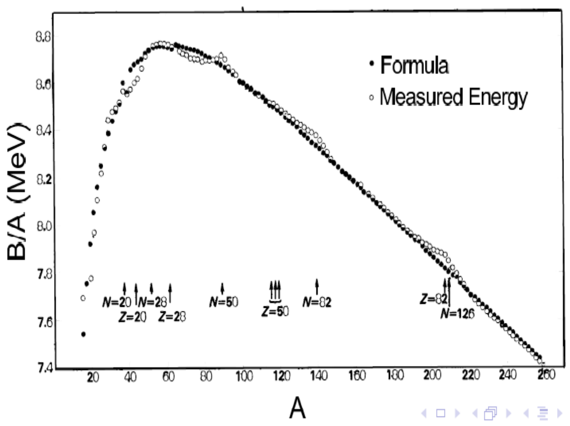
Nuclear Symmetry Energy

Bethe-Weizsäcker formula :

$$E = -a_V A + a_S A^{2/3} + a_C \frac{Z^2}{A^{1/3}} + \boxed{a_a \frac{(N-Z)^2}{A}} + E_{mic}$$

$a_v = 16$ MeV $a_S = 18$ MeV **a_a=21 MeV** a_c=0.7 MeV



In nuclear matter :

$$\delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p} = \frac{N-Z}{A}$$

Quadratic under n↔p symmetry

$$\frac{E}{A}(\rho,\delta) = \frac{E}{A}(\rho,0) + \frac{E}{A}(\delta) \cong \frac{E}{A}(\rho,0) + S(\rho)\delta^2$$

Symmetry energy

Expanded around normal density ρ_0 :

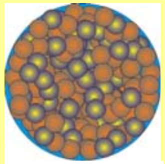
$$S(\rho) = a_a^v + \frac{L}{3}\frac{\rho-\rho_0}{\rho_0} + \frac{K_{sym}}{18}\left(\frac{\rho-\rho_0}{\rho_0}\right)^2 + \ldots$$

In neutron matter (neutron star $\delta \approx 1$): $E \approx E_0 + S$

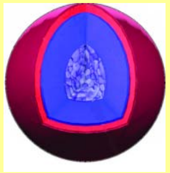
$$P \cong \rho^2 dS/d\rho \cong L\rho^2/(3\rho_0)$$

Physics of symmetry energy

- ✓ masses
- ✓ radii of n-rich nuclei
- ✓ n-skin of heavy nuclei (e.g. ^{208}Pb)
- ✓ ……
- ✓ neutron star properties
- ✓ supernova phenomena



^{208}Pb

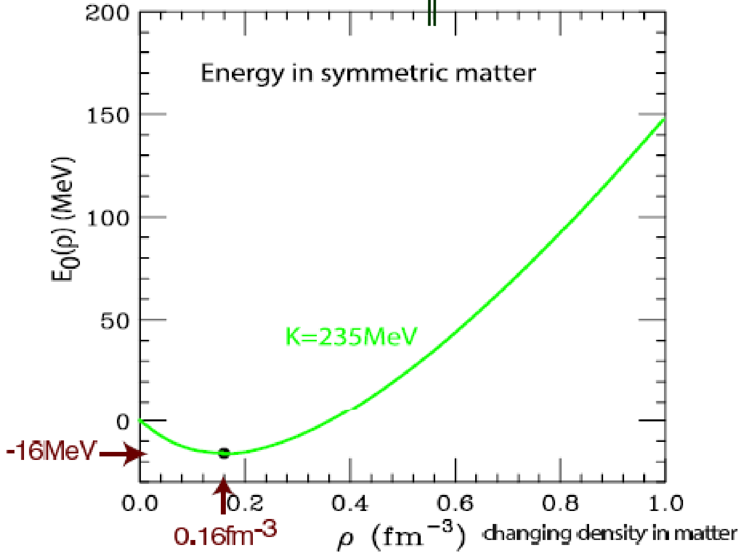


Neutron Star

~10^{-15} m **~10^4 m**

EOS: symmetric matter and neutron matter

$$E/A(\rho,\delta) = E/A(\rho,0) + S(\rho)\cdot\delta^2\;;\quad \delta = (\rho_n - \rho_p)/(\rho_n + \rho_p) = (N-Z)/A$$



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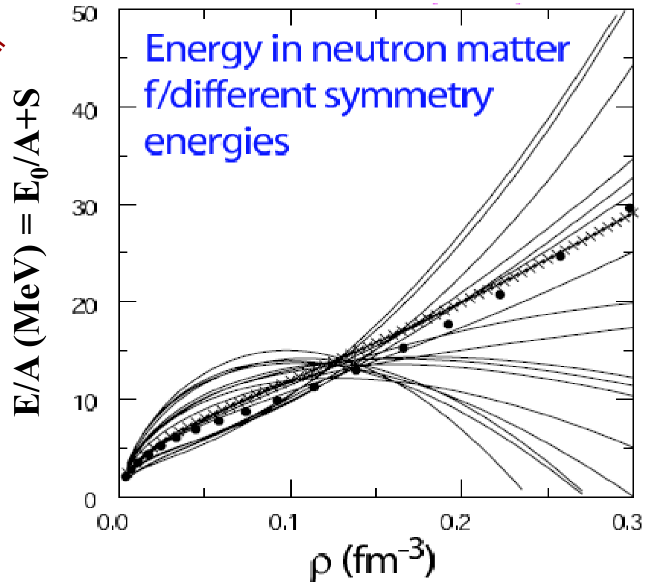
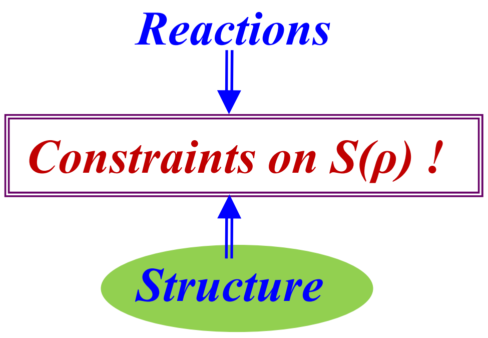
B.A. Brown, Phys. Rev. Lett. 85, 5296 (2001)



$$E_0(\rho_0) = -a_v = -16\ \mathrm{MeV}$$

Well constrained for the symmetric matter



Reactions

Constraints on S(ρ) !

Structure

- *The density dependence of symmetry energy is largely unconstrained*
- *Directions of the density dependence diverge at high densities*

Neutrons & Protons in Nucleus

Nuclear Energy

$$E = -a_v A + a_s A^{2/3} + \frac{a_a}{A}(N-Z)^2$$

$$= E_0(A) + \frac{a_a(A)}{A}(N-Z)^2$$

Asymmetry chemical potential

$$\mu_a = \frac{\partial E}{\partial(N-Z)} = \frac{1}{2}(\mu_n - \mu_p)$$

$$= \frac{2a_a(A)}{A}(N-Z)$$

Charge symmetry

Isoscalar density $\rho(r) = \rho_n(r) + \rho_p(r)$

Isovector density $\rho_a(r) = \frac{2a_a^V}{\mu_a}[\rho_n(r) - \rho_p(r)]$

both $\rho(r)$ & $\rho_a(r)$ have universal feature

→ *weakly depend on $\eta = (N-Z)/A$!*

In any nucleus:

$$\boxed{\rho_{n,p}(r) = \frac{1}{2}\left[\underbrace{\rho(r)}_{\text{Isoscalar}} \pm \frac{\mu_a}{2a_a^V}\underbrace{\rho_a(r)}_{\text{Isovector density}}\right]}$$

Universal Nuclear Densities

$$\boxed{\rho_{n,p}(r) = \frac{1}{2}\left[\rho(r) \pm \frac{\mu_a}{2a_a^V}\rho_a(r)\right]}$$

Net isoscalar density usually parameterized w/Fermi function

$$\rho(r) = \frac{\rho_0}{1 + \exp\left(\frac{r-R}{d}\right)} \qquad R = r_0\, A^{1/3}$$

Isovector density ρ_a ?? Related to $a_a(A)$ & to $S(\rho)$!!

$$\frac{A}{a_a(A)} = \frac{2(N-Z)}{\mu_a} = 2\int \mathrm{d}r\, \frac{\rho_{np}}{\mu_a} = \frac{1}{a_a^V}\int \mathrm{d}r\, \rho_a(r)$$

In uniform matter:

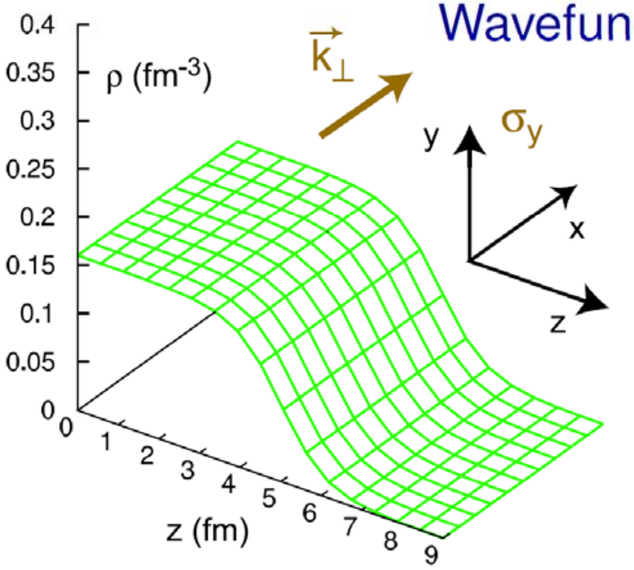
$$\mu_a = \frac{\partial E}{\partial(N-Z)} = \frac{2\,S(\rho)}{\rho}\rho_{np}$$

$$\boxed{\rho_a = \frac{2a_a^V}{\mu_a}\rho_{np} = \frac{a_a^V\,\rho}{S(\rho)}}$$

- ✓ ***Both n & p densities carry record of*** $S(\rho)$ → ***Hartree-Fock study of the surface***
- ✓ Mass formula: $a_a = 21\text{MeV}$; ***mass dependent*** $a_a(A)$ ***??***
  – ***simplest nuclear system w/ a surface*** → ***Half-infinite matter***

Half-Infinite Matter in Skyrme-Hartree-Fock

Direction of non-uniformity (z-axis); vacuum (+z) & infinite uniform matter (-z)



Wavefunctions: $\Phi(\boldsymbol{r}) = \phi(z)\, e^{i\boldsymbol{k}_\perp \cdot \boldsymbol{r}_\perp}$

matter interior/exterior:

$$\phi(z) \propto \sin\left(k_z z + \delta(\boldsymbol{k})\right)$$

$$\phi(z) \propto e^{-\kappa(\boldsymbol{k}) z}$$

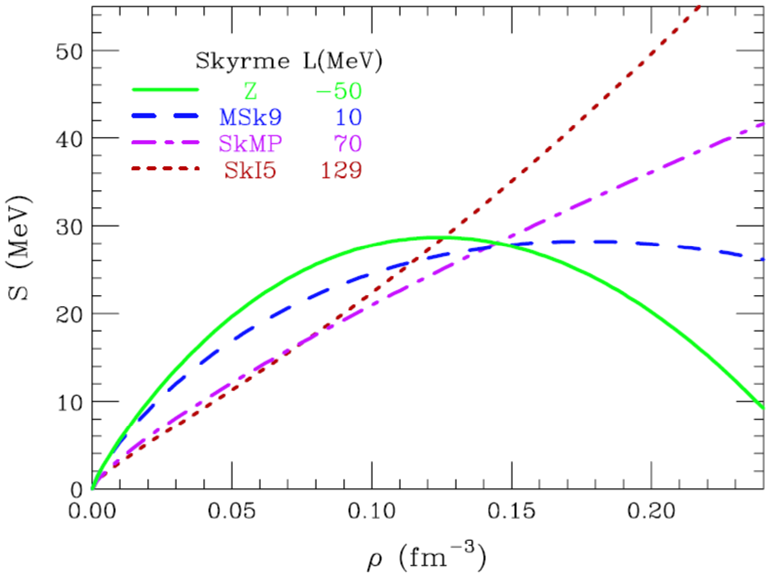
Discretization in k-space.
Set of 1D HF equations solved until self-consistency for different Skyrme interactions:

$$-\frac{d}{dz}\frac{\hbar^2}{2m^*(z)}\frac{d}{dz}\phi(z) + \left(\frac{\hbar^2 k_\perp^2}{2m^*(z)} + U(z)\right)\phi(z) = \epsilon(\boldsymbol{k})\,\phi(z)$$

P. Danielewicz & J. Lee, NPA 818, 36 (2009)

Isoscalar (Net) & Isovector Densities

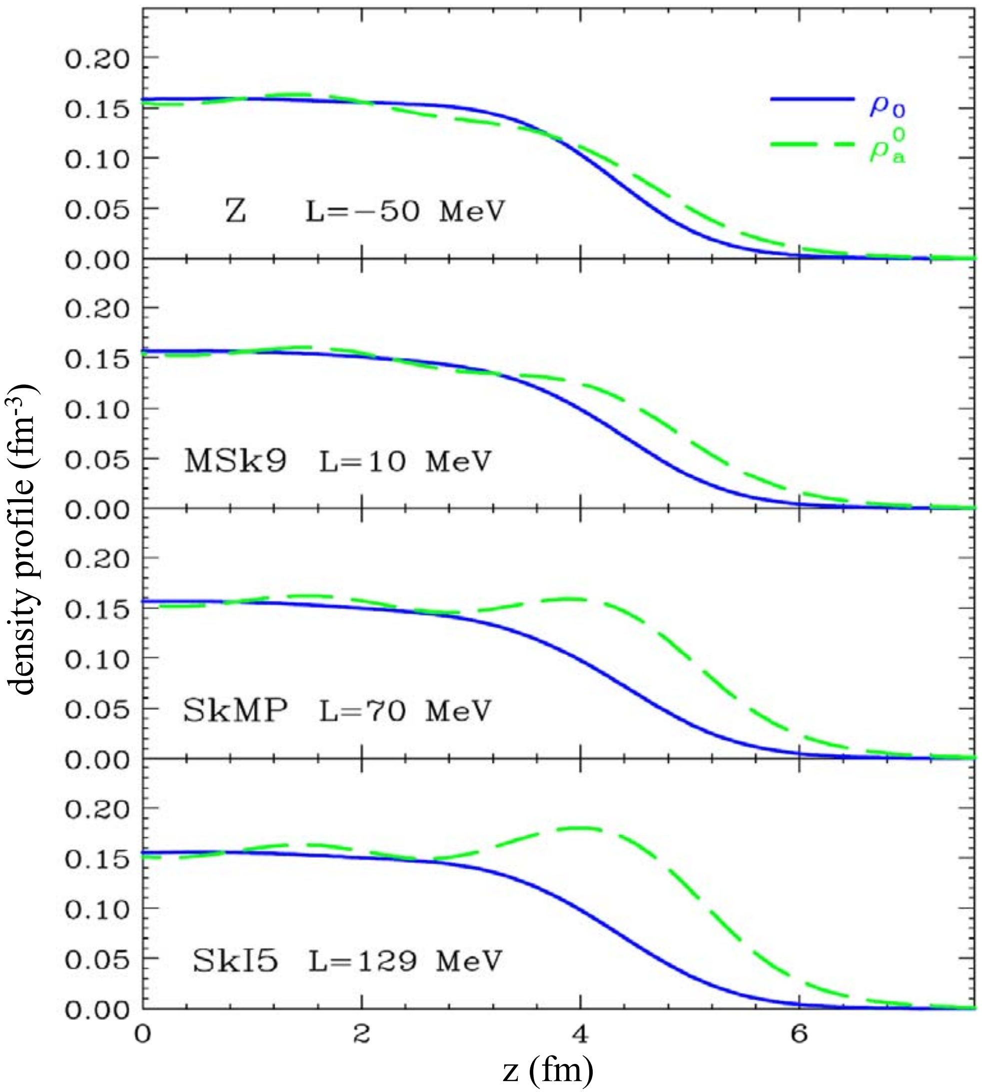
Results from four different Skyrme interactions in half-∞ matter



- ***Isovector density ρ_a changes significantly with interactions***
- ***Within matter, densities are fairly close to each other***
- ***Surface - difference is strongly correlated to the L-value – higher L → farther out ρ_a displaced relative to ρ***

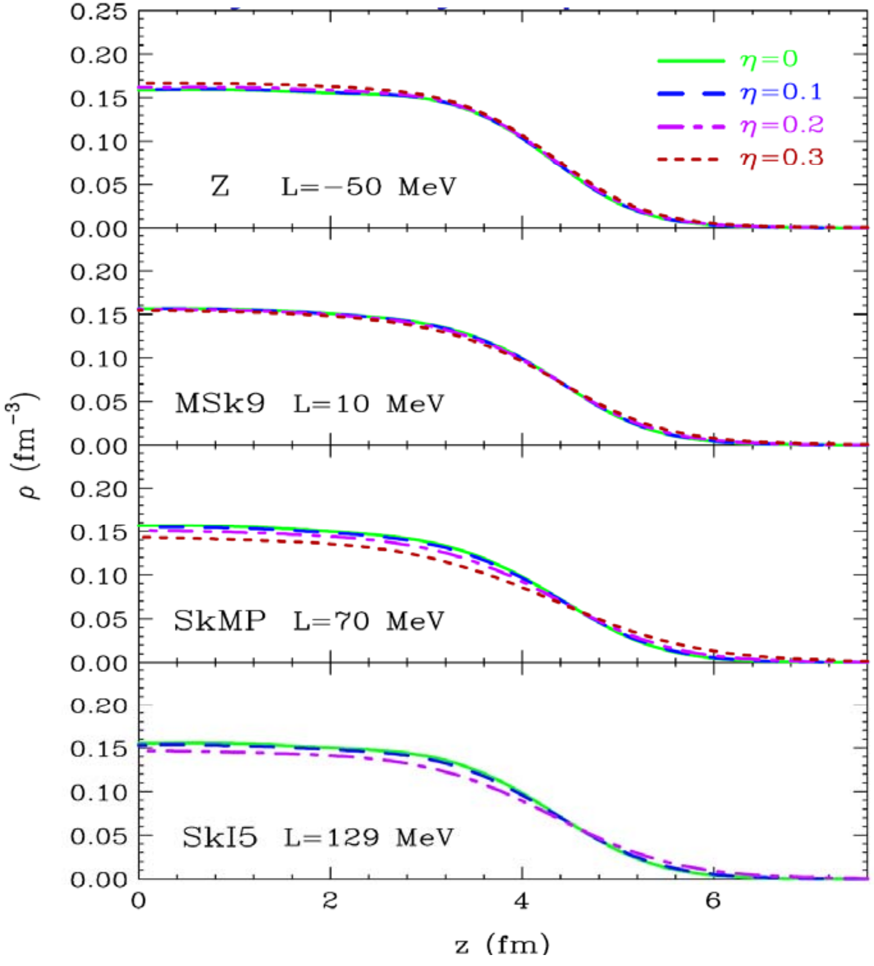
$$\rho_a = \frac{2a_a^V}{\mu_a}(\rho_n - \rho_p)$$

η = (N-Z)/A= 0

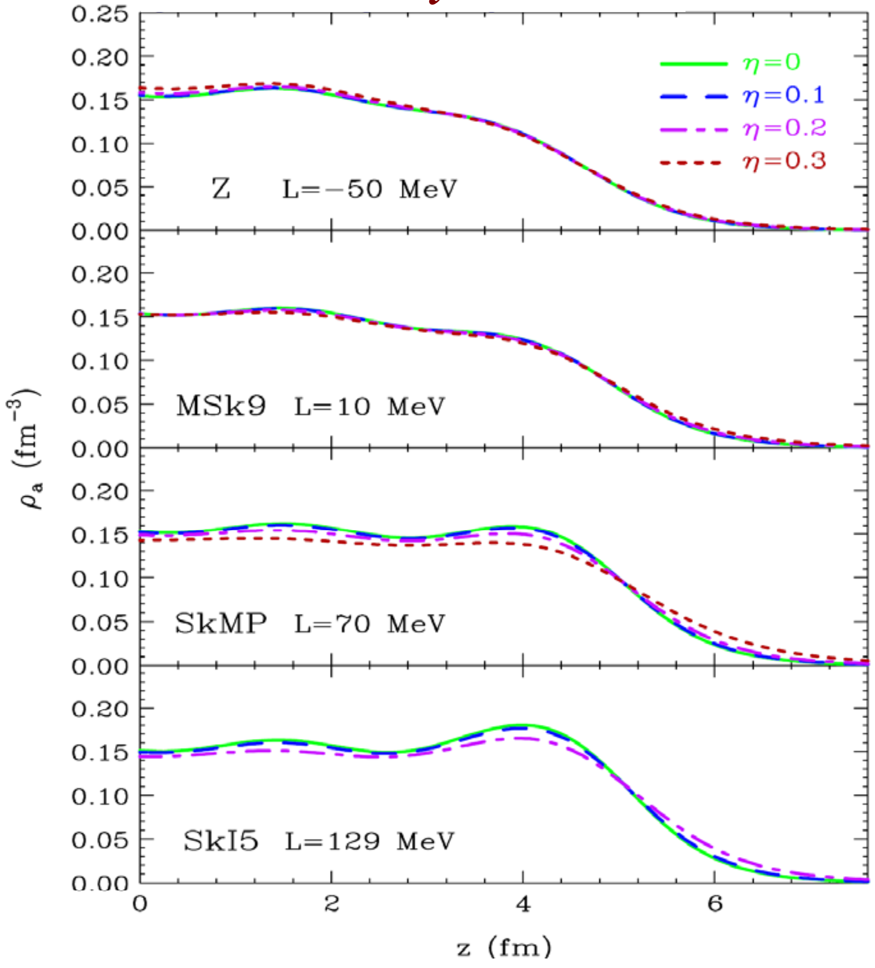


Asymmetry dependence of Densities

Isoscalar (Net) Density *$\eta = (N\text{-}Z)/A$*



Isovector Density $\rho_a = \frac{2a_a^V}{\mu_a}(\rho_n - \rho_p)$

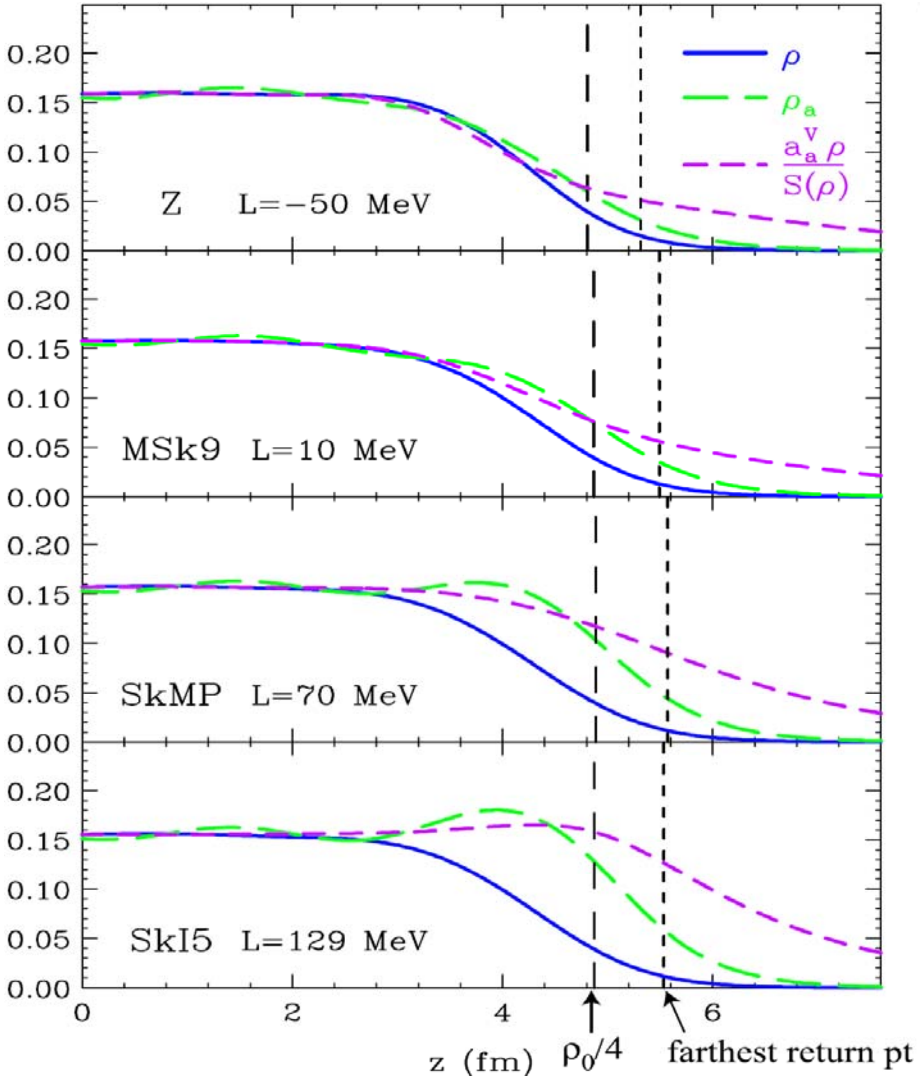


- ***Both isoscalar and isovector densities change little with asymmetry η***
- ***ρ & ρ_a with universal features → fundamental quantities in our framework !***

P. Danielewicz & J. Lee, NPA 818, 36 (2009)

Sensitivity to S(ρ)

η = (N-Z)/A= 0



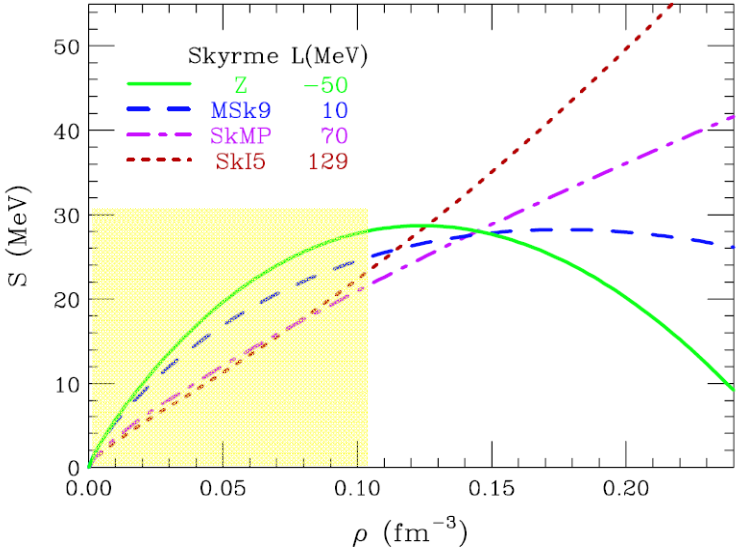
- *Uniform matter* $\rho_a = \rho\, a_a^V / S(\rho)$

→ also valid for weakly non-uniform matter (short-range of nuclear interactions)

- *Isovector density* ρ_a *follows the local approximation down to* $\rho \simeq \rho_0/4$

The more different between ρ_a & ρ *:*

- *higher L*
- *lower S(ρ) in the surface*



P. Danielewicz & J. Lee, NPA 818, 36 (2009)

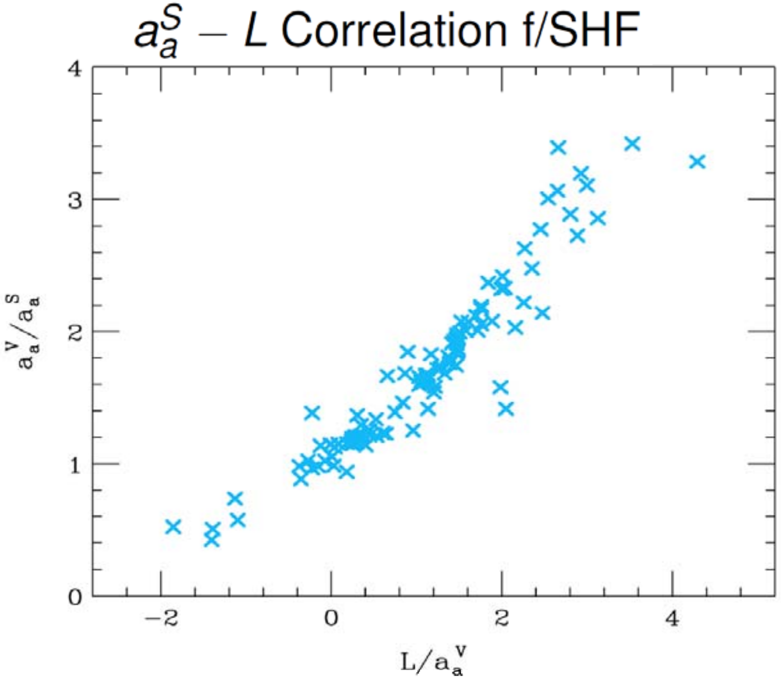
Correlations of Symmetry Coefficients & *S(ρ)*

~150 Skyrme interactions with force constants from J. Rikovska Stone

Bulk nuclear properties for different Skyrme interactions

P. Danielewicz & J. Lee, NPA 818, 36 (2009)

Name	a_V	m^*/m	K	a_a^V	L	a_S	a_a^S	ΔR
SkT	-15.40	0.602	333	24.8	28.2	14.2	17.5	0.477
SkT1	-15.98	1.000	236	32.0	56.2	18.2	14.6	0.799
SkT2	-15.94	1.000	235	32.0	56.2	18.0	14.7	0.794
SkT3	-15.94	1.000	235	31.5	55.3	17.7	15.3	0.776
SkT4	-15.95	1.000	235	35.4	94.1	18.1	11.5	0.986
SkT5	-16.00	1.000	201	37.0	98.5	18.1	10.9	1.084
SkM1	-15.77	0.789	216	25.1	-35.3	17.4	59.6	0.180



Surface region

$$\frac{A}{a_a(A)} = \frac{1}{a_a^V}\int d^3r\,\rho_a(r) = \frac{1}{a_a^V}\int d^3r\,\rho(r) + \frac{1}{a_a^V}\int d^3r\,(\rho_a - \rho)(r)$$

$$\simeq \frac{A}{a_a^V} + \frac{A^{2/3}}{a_a^S} \qquad \Rightarrow \qquad \boxed{\frac{1}{a_a(A)} \simeq \frac{1}{a_a^V} + \frac{A^{-1/3}}{a_a^S}}$$

nuclear volume *surface*

Symmetry coefficient -- $a_a(A)$

$$E = -a_V A + a_S A^{2/3} + a_C \frac{Z^2}{A^{1/3}} + a_a \frac{(N-Z)^2}{A} + E_{mic}$$

Determine ***$a_a(A)$*** by fitting directly to the ground-state nuclear energies ??

Not good ! Symmetry energy contribution is small + competition between different physics terms in the formula …

Best way → Study symmetry energy in isolation from the rest of Energy formula (impossible ??)

Charge invariance: invariance of nuclear interactions under rotations in *n-p* space

Symmetry term is a scalar in isospin space:

$$E_a = a_a(A)\,\frac{(N-Z)^2}{A} = 4\,a_a(A)\,\frac{T_z^2}{A}$$

Generalization

$$\boxed{E_a = 4\,a_a(A)\,\frac{T^2}{A} = 4\,a_a(A)\,\frac{T(T+1)}{A}}$$

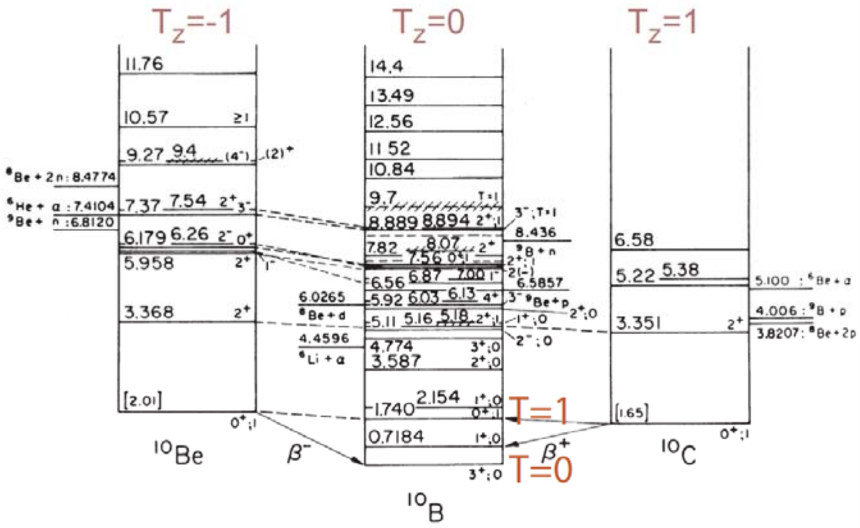
(Through +1, most of Wigner term from E_{mic} is absorbed to Symmetry term)

$a_a(A)$ Nucleus-by-Nucleus

$$\boxed{E_a = 4\, a_a(A)\, \frac{T(T+1)}{A}}$$

*In the ground state, **T** takes in the lowest possible value* $T = |T_z| = |N - Z|/2$

Low excited state of a given T – isobaric analog state (IAS) of the neighboring nucleus ground-state



With generalization, excitation energy to an IAS
(e.g. J. Jänecke et al., NPA 728, 23 (2003)):

$$E_2(T_2) - E_1(T_1) \;=\; \frac{4\, a_a}{A}\left\{T_2(T_2+1) - T_1(T_1+1)\right\}$$
$$+E_{\mathrm{mic}}(T_2, T_z) - E_{\mathrm{mic}}(T_2, T_z)$$

Corrections for microscopic effects + deformation

$$\boxed{a_A^{-1}(A) = \frac{4\, \Delta T^2}{A\, \Delta E}}$$

$$\overset{?}{=} (a_A^V)^{-1} + (a_A^S)^{-1}\, A^{-1/3}$$

✓ ***Data: Antony et al., ADNDT 66,1 (1997)***

✓ ***E_{mic}: Koura et al., ProTheoPhys 113, 305 (2005)***
Groote et al., ADNDT 17, 418 (1976)
Moller et al., ADNDT 59, 185 (1995)

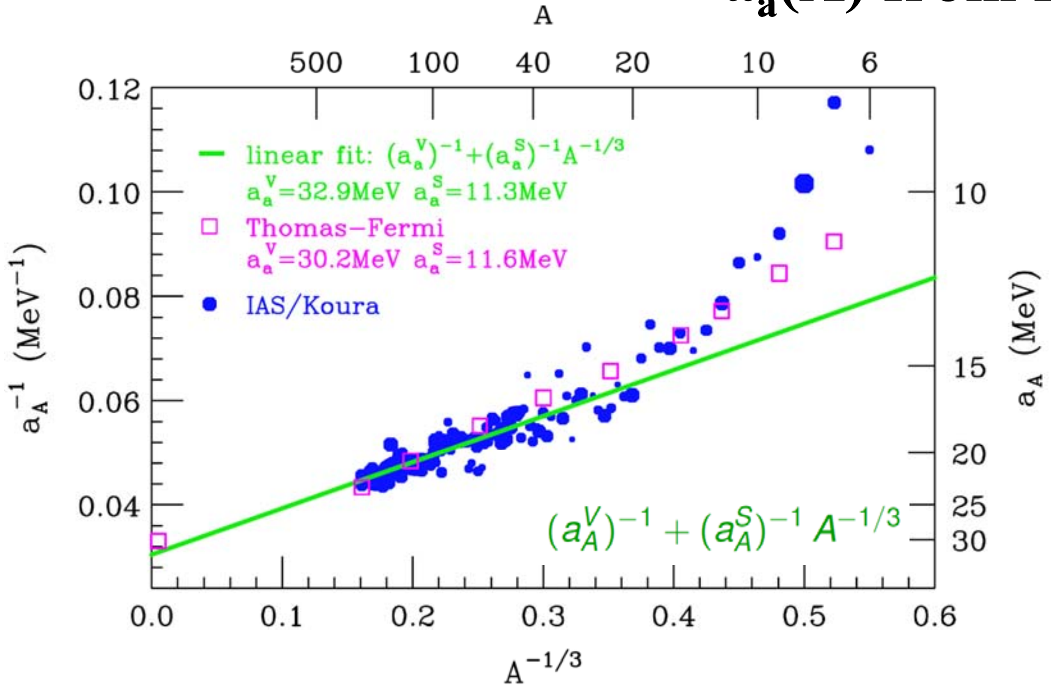
P. Danielewicz , NPA 727, 233 (2003)
P. Danielewicz & J. Lee, NPA 818, 36 (2009)
P. Danielewicz & J. Lee, Int. J. Mod. Phys. E 18, 892 (2009)

$a_a(A)$ from IAS

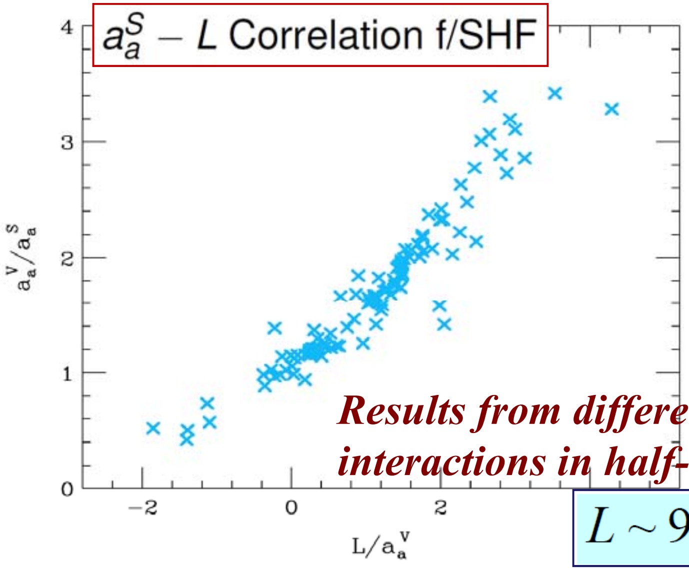
- ***Model -Independent approach !***
- ***Linear dependence from A > 20***

(lighter system– residual shell fluctuations are large)

$$a_A^V = 32.9\ \text{MeV},\ \ a_A^S = 11.3\ \text{MeV}$$



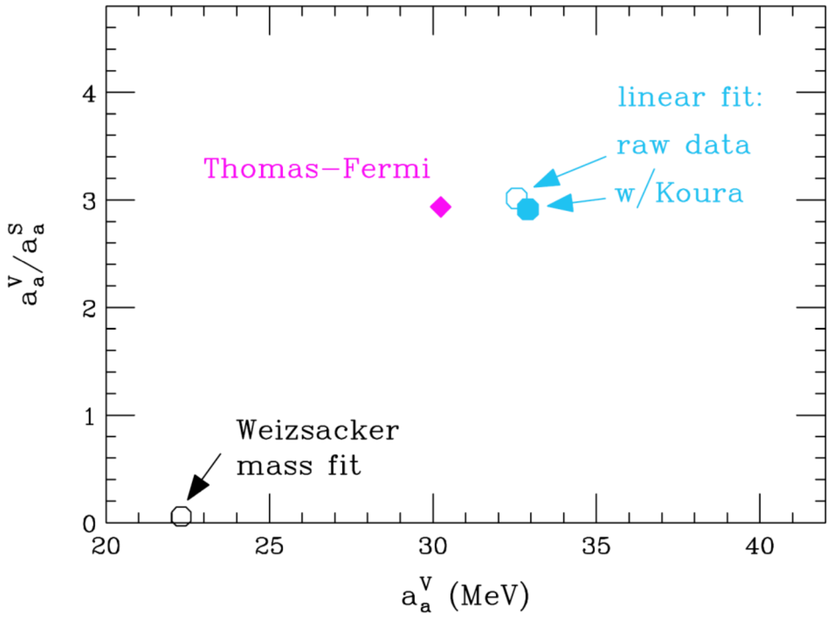
$a_a^S - L$ Correlation f/SHF



Results from different Skyrme interactions in half-∞ matter

$L \sim 95$ MeV

Symmetry-Energy Parameters



$$a_a^V = (31.5-33.5)\text{MeV},\ \ a_a^S = (9.5-12)\text{MeV}$$

P. Danielewicz & J. Lee, NPA 818, 36 (2009)

P. Danielewicz & J. Lee, Int. J. Mod. Phys. E 18, 892 (2009)

Consistency ?

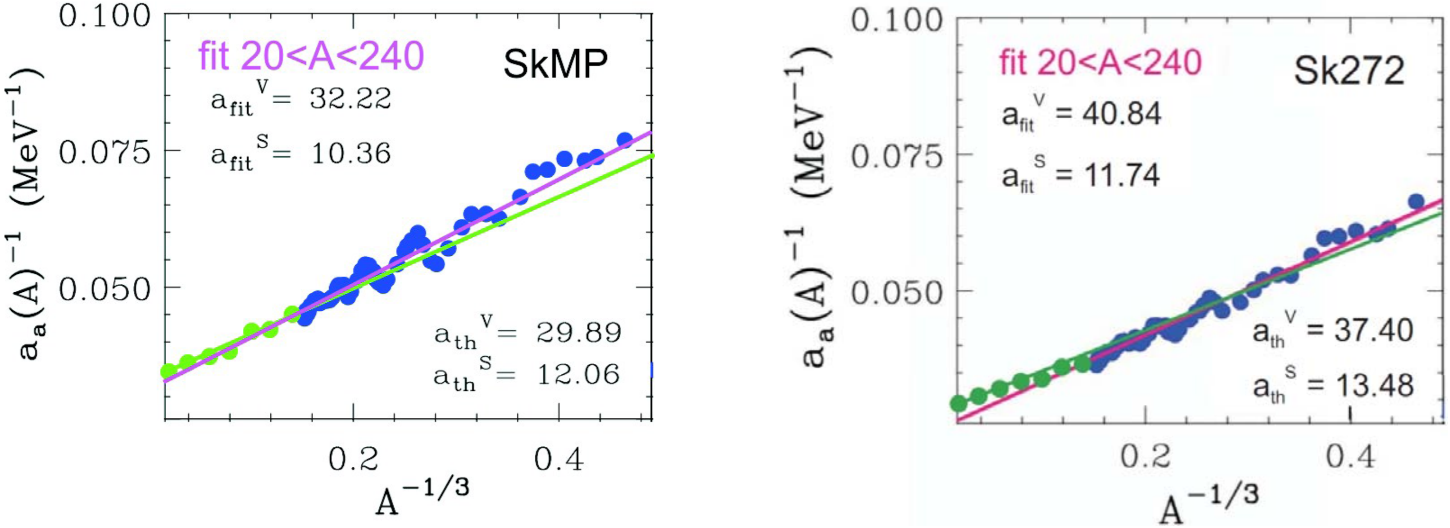
$$\frac{1}{a_a(A)} = \frac{1}{a_a^V} + \frac{A^{-1/3}}{a_a^S} \xrightarrow{A>20} \boxed{a_a^V, a_a^S, L}$$

Established method to be robust
→ analysis need to agree with those underlying the theory (half-∞ matter calc.)

Spherical SHF calculations for consistency check !

Coefficient-extraction :

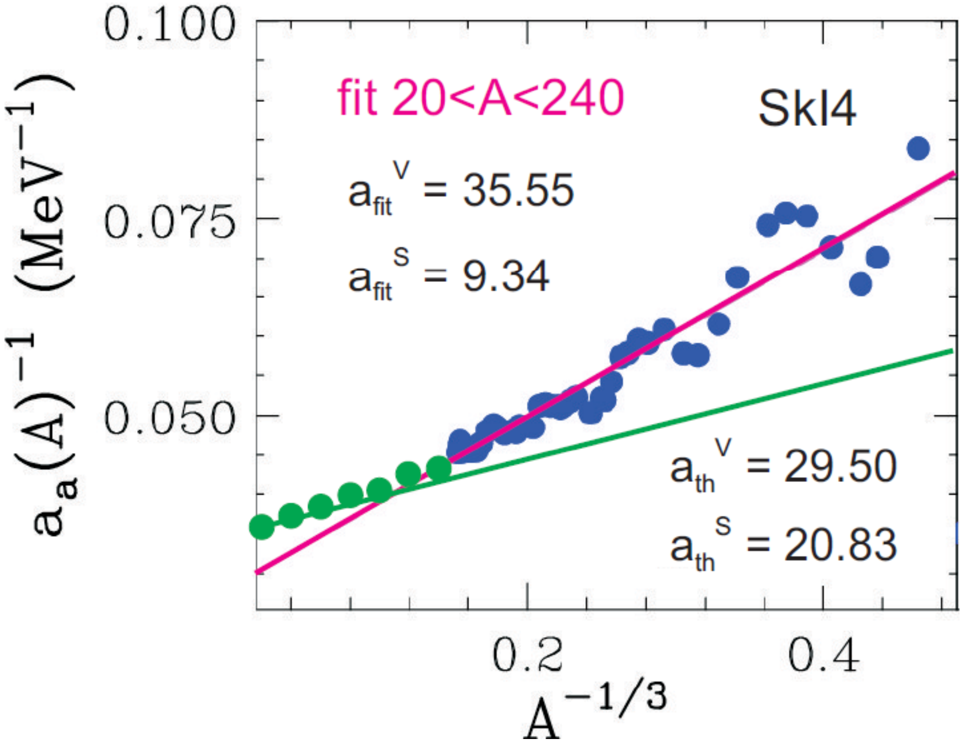
Realistic (blue) and ***unrealistically large (green) (up to A~10^6)*** nuclei *(code from P.-G. Reinhard)*



Red line: linear fit to nuclei in 20<A<240, green line: expectation from half-infinite matter Calc.

- ***Fitted values are close to the predictions***
- ***Systematic difference → Nuclei in Nature too small for clean surface/volume separation***
- ***Consequence:*** a_a^V a bit smaller, a_a^S a bit larger → ***L is a bit smaller than from the IAS analysis***

Outlier Skyrme Interactions



For some Skyrme interactions, dramatic difference between the coefficients from the fitted (20 < A < 240) values and predictions

Interactions with large deviations tend to have objectively unphysical features:

- *Unstable in long-wavelength limit*

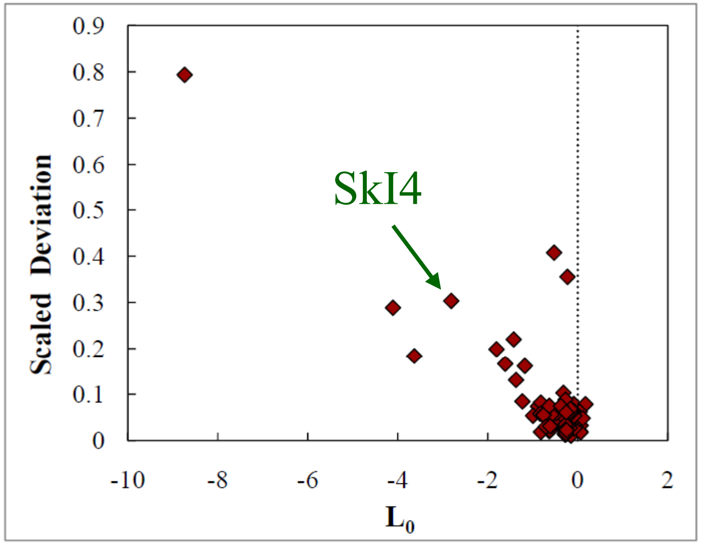
  and/or

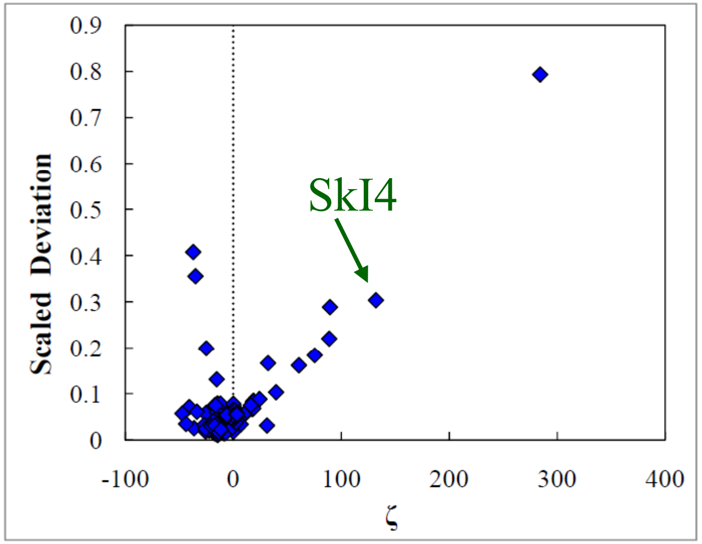
- *Strong non-locality in symmetry energy*

Deviations -- Skyrme Characteristics

✓ *Long-wavelength instability for the Skyrme interactions*
→ *lowest l=0 Landau parameter* $L_0 < -1$

✓ *Non-locality in symmetry energy quantified by* ζ*:* $\mathcal{H} = \ldots + \zeta\,(\nabla\rho_{np})^2$
excessive large ζ → *inter-nucleon interaction is senselessly long-range*





Before firmer conclusion a_a^V, a_a^S, L *, Skyrme interactions with non-physical features need to be filtered out!*

Constraints of *S(ρ)* from Different Sources

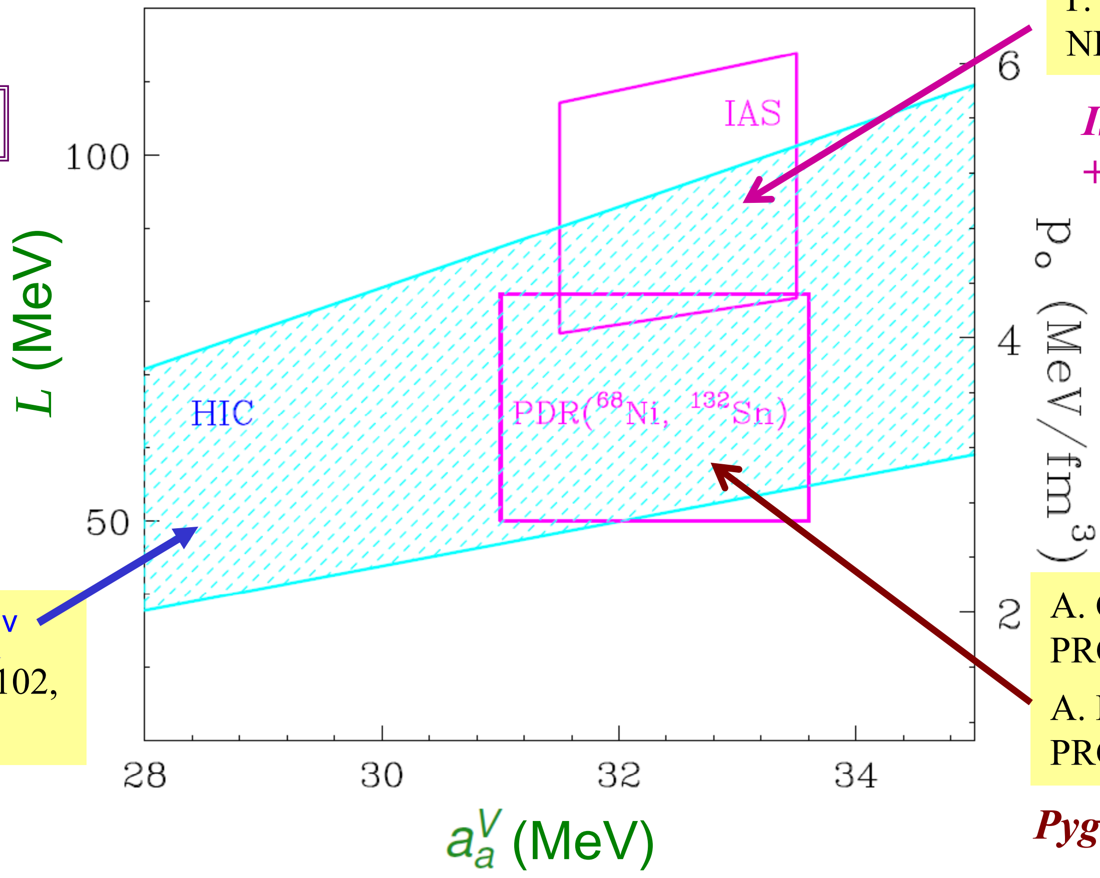
$$S(\rho) = a_a^V + \frac{L}{3}\frac{\rho - \rho_0}{\rho_0} + \dots$$

Structure

Constraints on S(ρ)

Reactions

M.B. Tsang et al., Phys. Rev. Lett 102, 122701 (2009)



P. Danielewicz & J. Lee, NPA 818, 36 (2009)

Isobaric analog state + half-∞ Calculations

Collisions involving ^{112}Sn, ^{124}Sn

ImQMB fits for a_a^V

M.B. Tsang et al., PRL 102, 122701 (2009)

A. Carbone et al., PRC 81, 041301(R) (2010)

A. Klimkiewicz et al., PRC 76, 051603 (2007)

Pygmy dipole resonances

More consistency → systematic errors need to be well understood and controlled in different sources

Summary and Outlook

- *Mass-dependent symmetry coefficient* $a_a(A) \simeq a_a^V/(1 + a_a^V/a_a^S A^{1/3})$

- *Two fundamental densities (isoscalar & isovector) characterize nucleon distributions*

- *Half-∞ HF calculations with ~150 skyrme interactions →* a_a^S *is strongly correlated with slope of symmetry energy (faster the drop of symmetry energy → smaller* a_a^S *)*

- *Parameters extracted from Isobaric Analogy States data (A>20):*

  $a_a^V = (31.5 - 33.5)\,\mathrm{MeV},\ a_a^S = (9.5 - 12)\,\mathrm{MeV}.$

  $L \sim 95\,\mathrm{MeV}$ *(w/ Half-∞ HF calculations)*

- *Spherical SHF calculations gives qualitative understanding of the validity of the employed procedures with IAS*

- *Constraint from IAS overlaps with those from reactions*

- *Outlook:*

  -- Better constraints on S(ρ) (interaction stability, Coulomb, shell & deformation effects)

  -- Feature of symmetry energy by systematics of proton distributions alone

  (benefited from the universal densities – isoscalar & isovector densities)