Symmetry Energy in Nuclei

Outline:

1. Nuclear symmetry energy
2. Two universal densities
3. Half-infinite Matter
   a) Solution of Skyrme-Hartree-Fock Equations
   b) Correlations of Symmetry Coefficients & $S(\rho)$
4. $a_a(A)$ from Isobaric Analog States
   a) Use of Charge Invariance
   b) Data Analysis & Interpretation
5. Validity of the employed procedures?
6. Constraints of $S(\rho)$ from structure and reactions
7. Summary & Outlook

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Nuclear Symmetry Energy

Bethe-Weizsäcker formula:

\[ E = -a_V A + a_S A^{2/3} + a_c \frac{Z^2}{A^{1/3}} + a_a \frac{(N-Z)^2}{A} + E_{mic} \]

\[ a_V = 16 \text{ MeV} \quad a_S = 18 \text{ MeV} \quad a_a = 21 \text{ MeV} \quad a_c = 0.7 \text{ MeV} \]

In nuclear matter:

\[ \delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p} = \frac{N-Z}{A} \]

\[ \frac{E}{A}(\rho, \delta) = E_A(\rho, 0) + \frac{E_A(\delta)}{A} \approx E_A(\rho, 0) + S(\rho)\delta^2 \]

Symmetry energy

Expanded around normal density \( \rho_0 \):

\[ S(\rho) = a_V^* + \frac{L}{3} \frac{\rho - \rho_0}{\rho_0} + \frac{K_{sym}}{18} \left( \frac{\rho - \rho_0}{\rho_0} \right)^2 + \ldots \]

In neutron matter (neutron star \( \delta \approx 1 \)): \( E \approx E_0 + S \)

\[ P \approx \rho^2 \frac{dS}{d\rho} \approx L \rho^2 / (3 \rho_0) \]

Physics of symmetry energy:

- masses
- radii of n-rich nuclei
- n-skin of heavy nuclei (e.g. \( ^{208}\text{Pb} \))
- ……
- neutron star properties
- supernova phenomena

\[ ^{208}\text{Pb} \quad \text{Neutron Star} \]

\[ \approx 10^{-15} \text{ m} \quad \approx 10^4 \text{ m} \]
**EOS: symmetric matter and neutron matter**

\[ E/A(\rho, \delta) = E/A(\rho, 0) + S(\rho) \cdot \delta^2 ; \quad \delta = (\rho_n - \rho_p)/(\rho_n + \rho_p) = (N-Z)/A \]

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**E0/A+S/A (MeV) = \( E/\rho_0 \) (**\( \rho_0 \)**) = -av = -16 MeV**

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**Well constrained for the symmetric matter**

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**Reactions**

**Constraints on S(\( \rho \)) !**

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**Structure**

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**The density dependence of symmetry energy is largely unconstrained**

**Directions of the density dependence diverge at high densities**
Neutrons & Protons in Nucleus

### Nuclear Energy

\[ E = -a_v A + a_s A^{2/3} + \frac{a_a}{A} (N - Z)^2 \]

\[ = E_0(A) + \frac{a_a(A)}{A} (N - Z)^2 \]

**Asymmetry chemical potential**

\[ \mu_a = \frac{\partial E}{\partial (N - Z)} = \frac{1}{2} (\mu_n - \mu_p) \]

\[ = \frac{2a_a(A)}{A} (N - Z) \]

### Charge symmetry

**Isoscalar density**

\[ \rho(r) = \rho_n(r) + \rho_p(r) \]

**Isovector density**

\[ \rho_a(r) = \frac{2a_a^V}{\mu_a} [\rho_n(r) - \rho_p(r)] \]

Both \( \rho(r) \) & \( \rho_a(r) \) have universal feature

\[ \rightarrow \text{weakly depend on} \quad \eta = (N-Z)/A \]

### In any nucleus:

\[ \rho_{n,p}(r) = \frac{1}{2} [\rho(r) \pm \frac{\mu_a}{2a_a^V} \rho_a(r)] \]

Isoscalar Isovector density
Universal Nuclear Densities

\[ \rho_{n,p}(r) = \frac{1}{2} [\rho(r) \pm \frac{\mu_a}{2a_a^V} \rho_a(r)] \]

Net isoscalar density usually parameterized w/Fermi function

\[ \rho(r) = \frac{\rho_0}{1 + \exp\left(\frac{r-R}{d}\right)} \quad R = r_0 A^{1/3} \]

Isovector density \( \rho_a \) ?? Related to \( a_a(A) \) & to \( S(\rho) \) !!

\[ \frac{A}{a_a(A)} = \frac{2(N-Z)}{\mu_a} = 2 \int dr \frac{\rho_{np}}{\mu_a} = \frac{1}{a_a^V} \int dr \rho_a(r) \]

In uniform matter:

\[ \mu_a = \frac{\partial E}{\partial (N-Z)} = \frac{2 S(\rho)}{\rho} \rho_{np} \]

\[ \rho_a = \frac{2a_a^V}{\mu_a} \rho_{np} = \frac{a_a^V \rho}{S(\rho)} \]

✓ Both \( n \) & \( p \) densities carry record of \( S(\rho) \) \( \Rightarrow \) Hartree-Fock study of the surface

✓ Mass formula: \( a_a = 21 \text{MeV} \); mass dependent \( a_a(A) \) ??

– simplest nuclear system w/ a surface \( \Rightarrow \) Half-infinite matter
Half-Infinite Matter in Skyrme-Hartree-Fock

Direction of non-uniformity (z-axis); vacuum (+z) & infinite uniform matter (-z)

Wavefunctions: \( \Phi(r) = \phi(z) e^{i k_{\perp} \cdot r_{\perp}} \)

matter interior/exterior:

\[
\phi(z) \propto \sin(k_z z + \delta(k))
\]

\[
\phi(z) \propto e^{-\kappa(k)z}
\]

Discretization in k-space.
Set of 1D HF equations solved until self-consistency for different Skyrme interactions:

\[
- \frac{d}{dz} \left( \frac{\hbar^2}{2m^*(z)} \frac{d}{dz} \phi(z) \right) + \left( \frac{\hbar^2 k_{\perp}^2}{2m^*(z)} + U(z) \right) \phi(z) = \epsilon(k) \phi(z)
\]

P. Danielewicz & J. Lee, NPA 818, 36 (2009)
Results from four different Skyrme interactions in half-$\infty$ matter

- Isovector density $\rho_a$ changes significantly with interactions
- Within matter, densities are fairly close to each other
- Surface - difference is strongly correlated to the L-value – higher L $\Rightarrow$ farther out $\rho_a$ displaced relative to $\rho$

$$\eta = (N-Z)/A = 0$$

$$\rho_a = \frac{2a^V}{\mu_a} (\rho_n - \rho_p)$$
Asymmetry dependence of Densities

**Isoscalar (Net) Density**  \[ \eta = \frac{(N-Z)}{A} \]

**Isovector Density**  \[ \rho_a = \frac{2a^v}{\mu_a} (\rho_n - \rho_p) \]

- Both isoscalar and isovector densities change little with asymmetry \( \eta \)
- \( \rho \) & \( \rho_a \) with universal features \( \rightarrow \) fundamental quantities in our framework!

*P. Danielewicz & J. Lee, NPA 818, 36 (2009)*
Sensitivity to $S(\rho)$

$\eta = (N-Z)/A = 0$

- **Uniform matter** $\rho_a = \rho a^y_a / S(\rho)$
  - also valid for weakly non-uniform matter (short-range of nuclear interactions)

- **Isovector density** $\rho_a$ follows the local approximation down to $\rho \approx \rho_0/4$

The more different between $\rho_a$ & $\rho$:
- higher $L$
- lower $S(\rho)$ in the surface

*P. Danielewicz & J. Lee, NPA 818, 36 (2009)*
Correlations of Symmetry Coefficients & $S(\rho)$

~150 Skyrme interactions with force constants from J. Rikovska Stone

Bulk nuclear properties for different Skyrme interactions

*P. Danielewicz & J. Lee, NPA 818, 36 (2009)*

<table>
<thead>
<tr>
<th>Name</th>
<th>$a_V$</th>
<th>$m^*/m$</th>
<th>$K$</th>
<th>$a^V_\alpha$</th>
<th>L</th>
<th>$a_S$</th>
<th>$a^S_\alpha$</th>
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<td>SkT</td>
<td>-15.40</td>
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<td>59.6</td>
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\[
\frac{A}{a_a(A)} = \frac{1}{a^V_\alpha} \int d^3 r \rho_a(r) = \frac{1}{a^V_\alpha} \int d^3 r \rho(r) + \frac{1}{a^V_\alpha} \int d^3 r (\rho_a - \rho)(r)
\]

\[
\sim \frac{A}{a^V_\alpha} + \frac{A^{2/3}}{a^S_\alpha} \Rightarrow \frac{1}{a_a(A)} \sim \frac{1}{a^V_\alpha} + \frac{A^{-1/3}}{a^S_\alpha}
\]

Surface region
Symmetry coefficient -- $a_a(A)$

$$E = -a_v A + a_s A^{2/3} + a_c \frac{Z^2}{A^{1/3}} + a_a \frac{(N-Z)^2}{A} + E_{mic}$$

Determine $a_a(A)$ by fitting directly to the ground-state nuclear energies ??

*Not good!* Symmetry energy contribution is small + competition between different physics terms in the formula ...

Best way $\rightarrow$ Study symmetry energy in isolation from the rest of Energy formula (impossible ??)

**Charge invariance:** invariance of nuclear interactions under rotations in $n-p$ space

Symmetry term is a scalar in isospin space:

$$E_a = a_a(A) \frac{(N-Z)^2}{A} = 4 a_a(A) \frac{T_z^2}{A}$$

Generalization

$$E_a = 4 a_a(A) \frac{T_z^2}{A} = 4 a_a(A) \frac{T(T+1)}{A}$$

(Through +1, most of Wigner term from $E_{mic}$ is absorbed to Symmetry term)
$a_a(A)$ Nucleus-by-Nucleus

$$E_a = 4 a_a(A) \frac{T(T + 1)}{A}$$

In the ground state, $T$ takes in the lowest possible value $T = |T_z| = |N - Z|/2$

Low excited state of a given $T$ – isobaric analog state (IAS) of the neighboring nucleus ground-state

With generalization, excitation energy to an IAS (e.g. J. Jänecke et al., NPA 728, 23 (2003)):

$$E_2(T_2) - E_1(T_1) = \frac{4 a_a}{A} \left\{ T_2(T_2 + 1) - T_1(T_1 + 1) \right\} + E_{\text{mic}}(T_2, T_z) - E_{\text{mic}}(T_2, T_z)$$

Corrections for microscopic effects + deformation

$${a_A^{-1}(A)} = \frac{4 \Delta T^2}{A \Delta E}$$

$$= (a_A^V)^{-1} + (a_A^S)^{-1} A^{-1/3}$$

✓ Data: Antony et al., ADNDT 66,1 (1997)

✓ $E_{\text{mic}}$: Koura et al., ProTheoPhys 113, 305 (2005)

Groote et al., ADNDT 17, 418 (1976)

Moller et al., ADNDT 59, 185 (1995)
\( a_a(A) \) from IAS

- **Model-Independent approach**
- **Linear dependence from \( A > 20 \)**
  (lighter system– residual shell fluctuations are large)

\[
(a_A^V)^{-1} + (a_A^S)^{-1} A^{-1/3}
\]

\( a_A^V = 32.9 \text{ MeV}, \ a_A^S = 11.3 \text{ MeV} \)

**Symmetry-Energy Parameters**

Results from different Skyrme interactions in half-\( \infty \) matter

\( L \sim 95 \text{ MeV} \)

\( a_A^V = (31.5 - 33.5) \text{ MeV}, \ a_A^S = (9.5 - 12) \text{ MeV} \)

P. Danielewicz & J. Lee, NPA 818, 36 (2009)
Consistency ?

\[ \frac{1}{a_a(A)} = \frac{1}{a_a^V} + \frac{A^{-1/3}}{a_a^S} \quad A > 20 \]

Established method to be robust  
→ analysis need to agree with those underlying the theory (half-∞ matter calc.)

**Spherical SHF calculations for consistency check!**

Coefficient-extraction:

- **Realistic (blue)** and **unrealistically large (green)** (up to \( A \sim 10^6 \)) nuclei  
  (code from P.-G. Reinhard)

- **Red line**: linear fit to nuclei in \( 20 < A < 240 \),  
  **green line**: expectation from half-infinite matter Calc.

- **Fitted values are close to the predictions**
- **Systematic difference**: Nuclei in Nature too small for clean surface/volume separation
- **Consequence**: \( a_a^V \) a bit smaller, \( a_a^S \) a bit larger → \( L \) is a bit smaller than from the IAS analysis
For some Skyrme interactions, dramatic difference between the coefficients from the fitted $(20 < A < 240)$ values and predictions.

Interactions with large deviations tend to have objectively unphysical features:

- Unstable in long-wavelength limit and/or
- Strong non-locality in symmetry energy
Deviation -- Skyrme Characteristics

✓ Long-wavelength instability for the Skyrme interactions
  ➔ lowest $l=0$ Landau parameter $L_0 < -1$

✓ Non-locality in symmetry energy quantified by $\zeta$: $\mathcal{H} = \ldots + \zeta (\nabla \rho_{np})^2$

excessive large $\zeta$ ➔ inter-nucleon interaction is senselessly long-range

Before firmer conclusion $a^V, a^S, L$, Skyrme interactions with non-physical features need to be filtered out!
Constraints of $S(\rho)$ from Different Sources

$$S(\rho) = a_\alpha^\nu + \frac{L}{3} \frac{\rho - \rho_0}{\rho_0} + \ldots$$

- **Structure**
- **Constraints on $S(\rho)$**
- **Reactions**
- **Collisions involving $^{112}\text{Sn}, ^{124}\text{Sn}$**
- **ImQMB fits for $a_\alpha^\nu$**

More consistency $\rightarrow$ systematic errors need to be well understood and controlled in different sources
Summary and Outlook

• Mass-dependent symmetry coefficient $a_a(A) \approx a_a^V / (1 + a_a^V / a_a^S A^{1/3})$

• Two fundamental densities (isoscalar & isovector) characterize nucleon distributions

• Half-$\infty$ HF calculations with $\sim 150$ skyrme interactions $\rightarrow a_a^S$ is strongly correlated with slope of symmetry energy (faster the drop of symmetry energy $\rightarrow$ smaller $a_a^S$)

• Parameters extracted from Isobaric Analogy States data ($A>20$):
  
  $a_a^V = (31.5 - 33.5)$ MeV, $a_a^S = (9.5 - 12)$ MeV

  $L \sim 95$ MeV (w/ Half-$\infty$ HF calculations)

• Spherical SHF calculations gives qualitative understanding of the validity of the employed procedures with IAS

• Constraint from IAS overlaps with those from reactions

• Outlook:
  -- Better constraints on $S(\rho)$ (interaction stability, Coulomb, shell & deformation effects)
  -- Feature of symmetry energy by systematics of proton distributions alone
    (benefited from the universal densities – isoscalar & isovector densities)