

# Symmetry energy at subnuclear densities and macroscopic properties of neutron-rich nuclei

Kazuhiro Oyamatsu

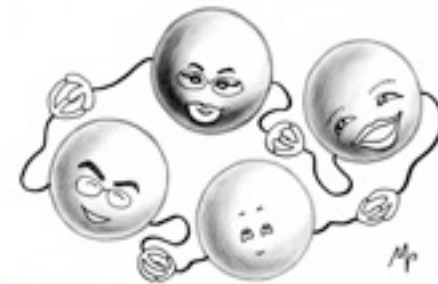
Department of Human Informatics,  
Aichi Shukutoku University

Nuclear matter EOS, laboratory nuclei and neutron-star matter  
Collaboration with

Iida(Kochi U.), Koura(JAEA), Kohama(RIKEN) since 2001  
Nakazato(Kyoto U.) since 2008

NuSYM10, Aug. 28, 2010

# Collaborator Meeting



## くじらの会 at 入野



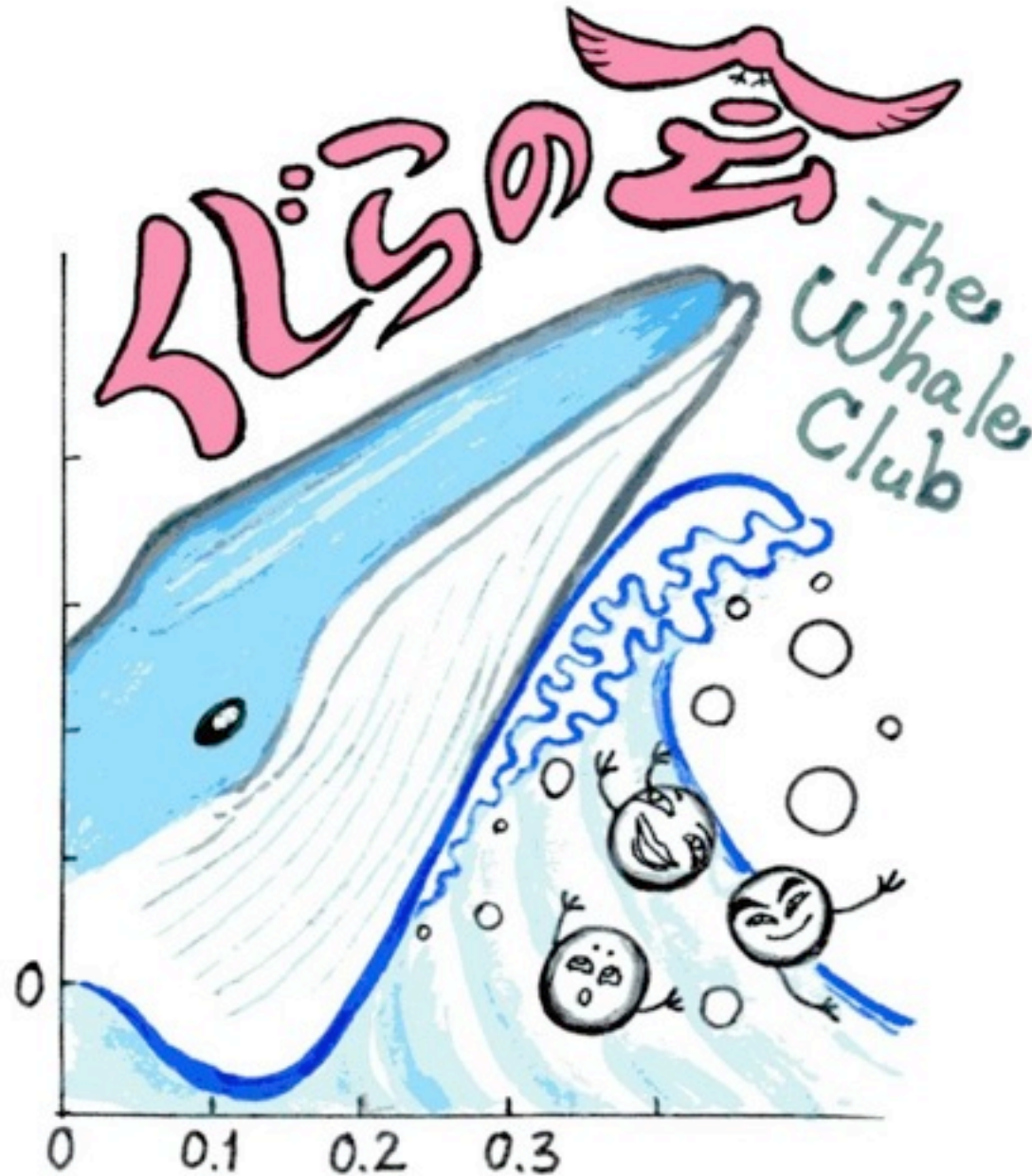
2008年10月29日(水)~31日(金)

Kohama  
( $\sigma_R$ :Kurotama)

Koura lida Oyamatsu  
(mass formula)

Today's  
talk

# We are focusing on the EOS



# Approaches to obtain the EOS of (uniform) nuclear matter

approach	starts from	ingredients	Theory/Model
empirical	the parametrized EOS	nuclear mass, size, ...	Liquid-Drop Model Droplet Model Thomas-Fermi Theory .....
Phenomenological	effective NN int. (Hamiltonian, Lagrangean)	nuclear mass, size, ...	Skyrme HF RMF AMD .....
microscopic	bare NN int. (AV18, Bonn, Paris,...)	NN scattering, ...	Variational Calc. DBHF .....

# Outline

We focus on macroscopic nuclear properties and adopt a macroscopic nuclear model.

1. From masses and radii of stable nuclei, we generate family of EOS and examine allowed regions of EOS parameter values.

2. We calculate neutron-rich nuclei in laboratories and identify key EOS parameter.

\*\*\* mass (2p, 2n separation energies), radius (matter, neutron skin) \*\*\*

\*\*\* neutron and proton drip line \*\*\*

3. We calculate nuclei in neutron-star crusts and identify key EOS parameter.

\*\*\* proton number and ratio \*\*\*

\*\*\* core-crust boundary density \*\*\*

\*\*\* existence of pasta nuclei \*\*\*

## Step 1

Generate all empirically allowed EOS's systematically

K. Oyamatsu and K. Iida, Prog. Theor. Phys. 109, 631 (2003).

## Adopted macroscopic model

Energy per cell (or Energy of a nucleus)

$$W = \int_{cell} d\mathbf{r} \left[ \underline{\varepsilon_0(n_n, n_p)} + m_n n_n + m_p n_p \right] + \int_{cell} d\mathbf{r} F_0 |\nabla n|^2 + \left( \text{electron kinetic energy} \right) + \left( \text{Coulomb} \right)$$

$n_n$  ( $n_p$ ) : local neutron (proton) density,  $n = n_n + n_p$  : total density

$\varepsilon_0(n_n, n_p)$  : EOS of uniform nuclear matter (energy density)

$F_0$  : surface energy parameter

Parametrization of the EOS (energy density)

$$\varepsilon_0(n_n, n_p) = \underbrace{\frac{3}{5} \left( 3\pi^2 \right)^{2/3} \left( \frac{\hbar^2}{2m_n} n_n^{5/3} + \frac{\hbar^2}{2m_p} n_p^{5/3} \right)}_{\text{Fermi kinetic energy density}} + \underbrace{\left[ 1 - (1 - 2Y_p)^2 \right] v_s(n) + (1 - 2Y_p)^2 v_n(n)}_{\text{potential energy density}}$$

Fermi kinetic energy density

potential energy density

potential energy densities of symmetric and neutron matter

$$v_s(n) = a_1 n^2 + \frac{a_2 n^3}{1 + a_3 n} \quad v_n(n) = b_1 n^2 + \frac{b_2 n^3}{1 + b_3 n}$$

★  $a_1 \sim b_2$  and  $F_0$  : masses and radii of stable nuclei ( $b_3 = 1.59 \text{ fm}^3$ , a fit to FP EOS)

★ very flexible function form:  $a_3$  can vary  $K_0$  widely. (better than Skyrme)

The function can be fitted to SIII and TM1 EOS very well.

# Simplified Thomas-Fermi calculation (The same method as Shen EOS)

energy minimization with respect to parameters of  $n_n(r)$  and  $n_p(r)$  (and lattice constant)

neutron (proton) density distribution  $n_n$  ( $n_p$ )

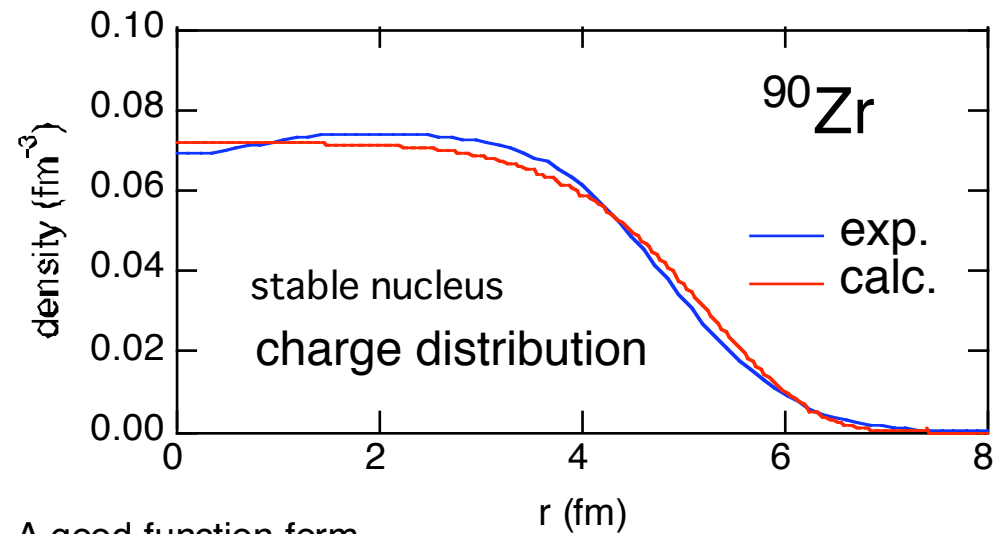
$$n_i(r) = \begin{cases} (n_i^{in} - n_i^{out}) \left[ 1 - \left( \frac{r}{R_i} \right)^{t_i} \right]^3 + n_i^{out} & r < R_i \\ n_i^{out} & r > R_i \end{cases}$$

$R_N$  ( $R_P$ ) : neutron (proton) radius parameter

$t_n$  ( $t_p$ ) : neutron (proton) surface thickness parameter

$n_i^{in}$  : central density

$n_n^{out}$  : neutron gas density ( $n_p^{out}=0$ )



A good function form

The n and p distributions are independent.

=> neutron skin

The empirical information is limited: radius and thickness.

The gradient term in Euler Eq. is continuous.

The density is zero beyond the classical turning point.

The values of parameters  $a_1 \sim b_3$  (EOS) and  $F_0$  are determined

to fit masses and radii of stable nuclei.

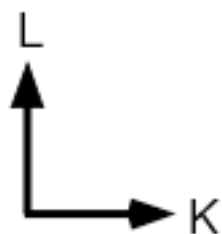
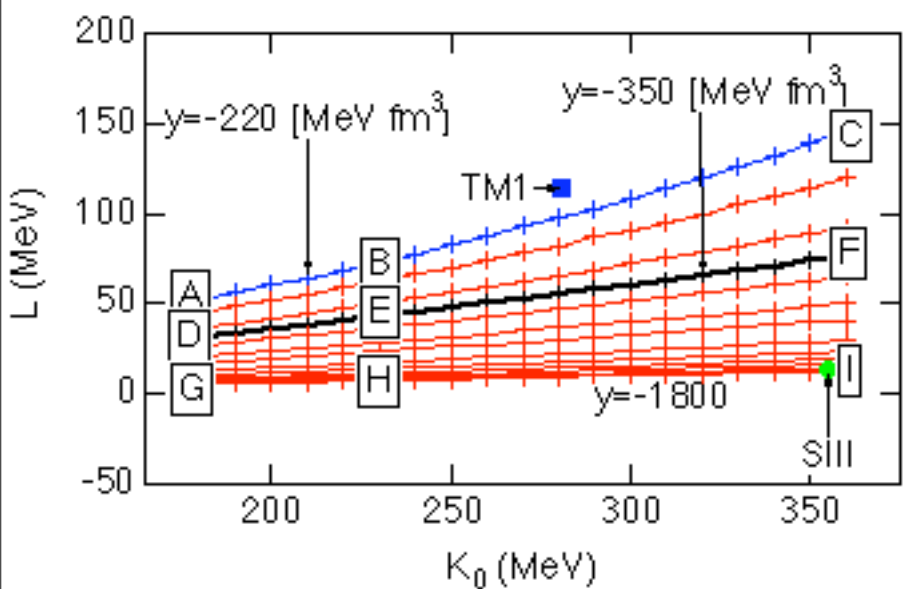
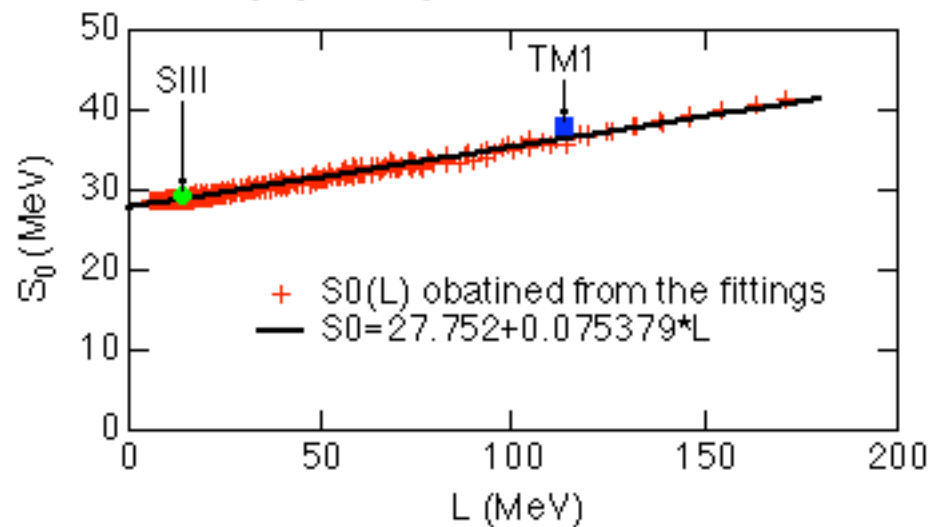
=> about 200 sets of empirical EOS+ $F_0$



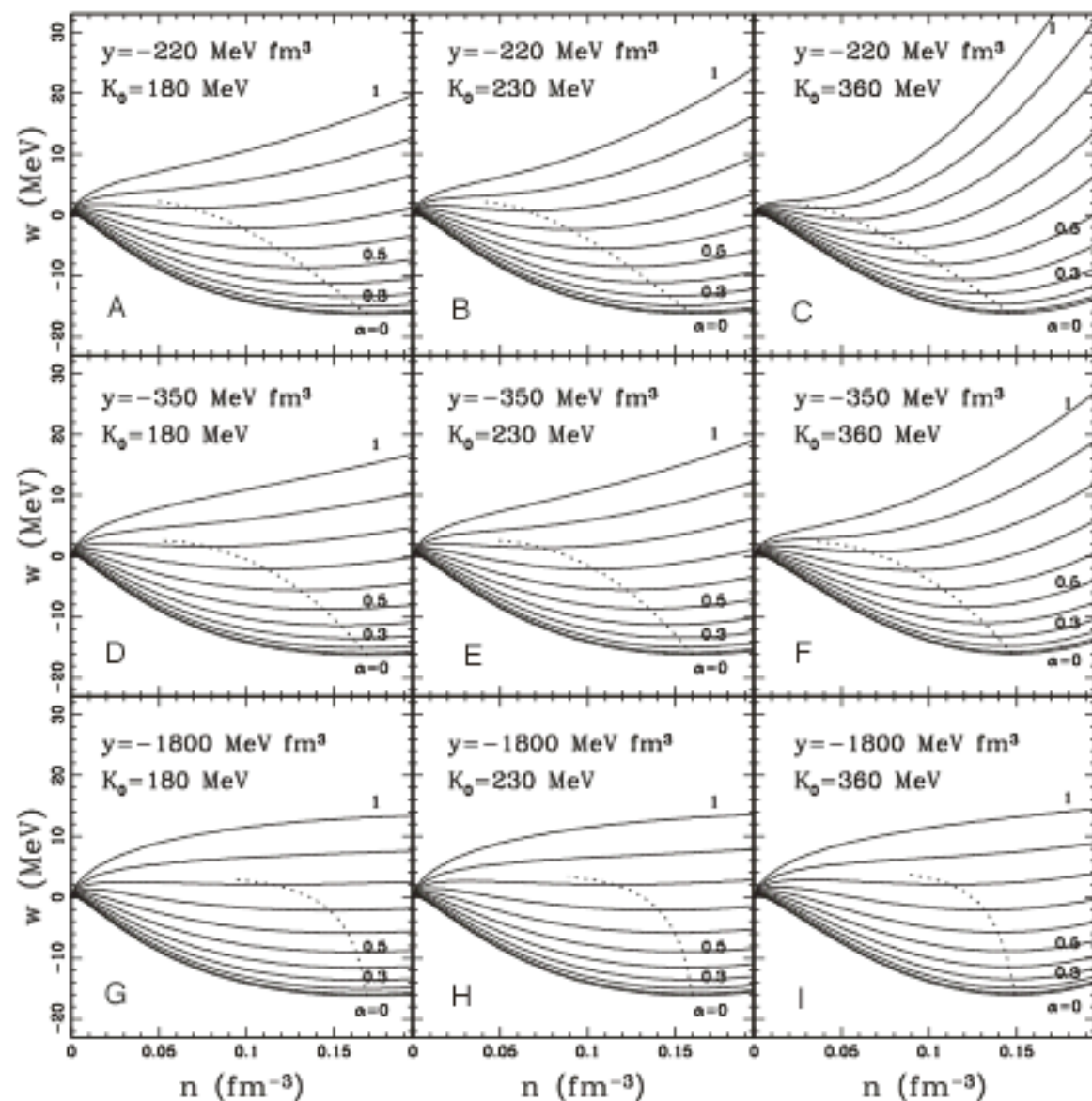
EOS parameter values obtained from stable nuclei

$S_0$ : symmetry energy

$L$ : density symmetry coefficient



9 representative EOS A-I

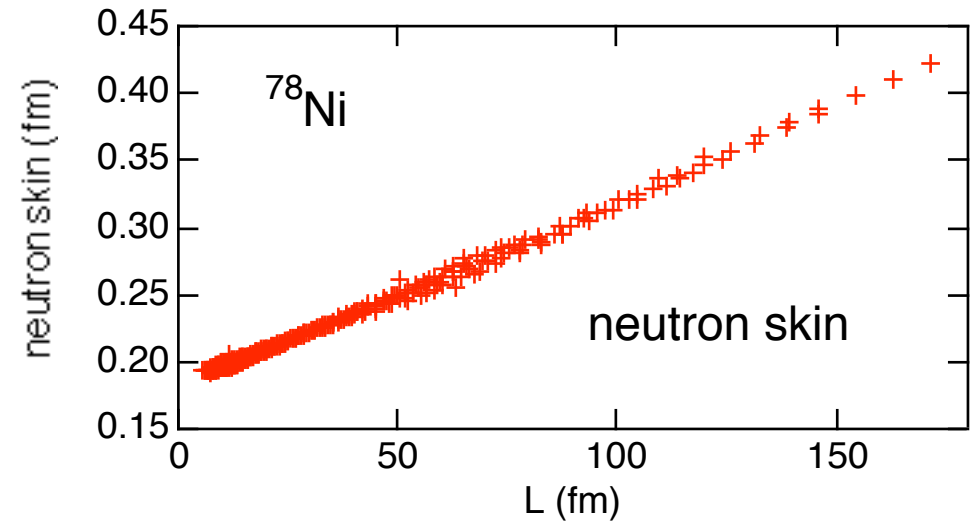
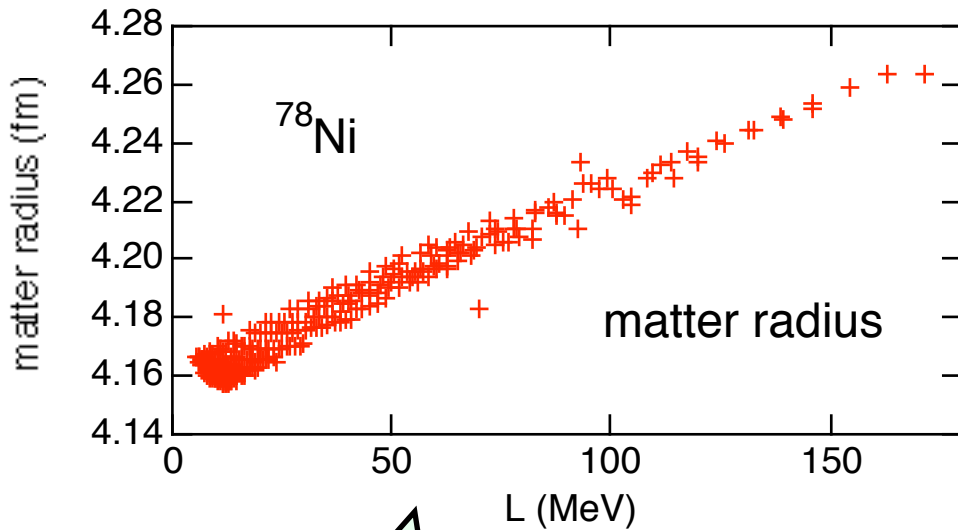


## Step 2

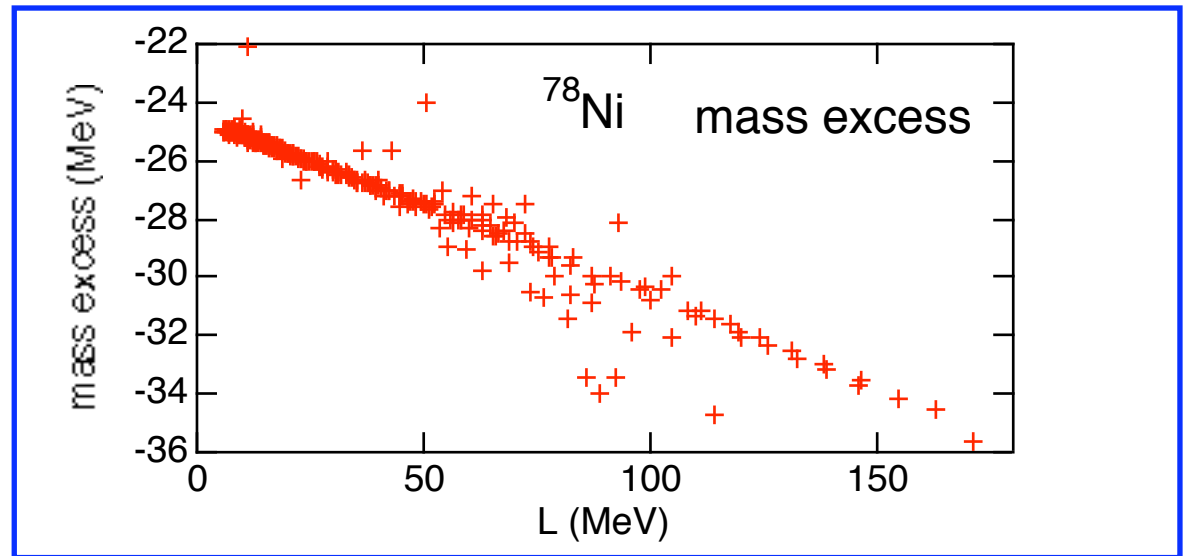
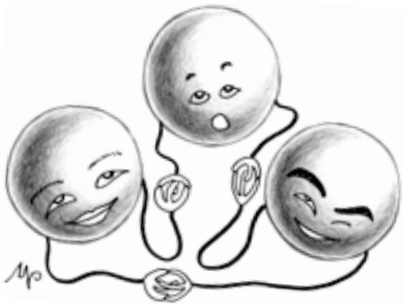
Calculate neutron-rich nuclei in labs  
with the 200 EOS's

K. Oyamatsu and K. Iida, Prog.Theor. Phys. 109, 631 (2003).

The mass, radius and neutron skin are dependent on  $L$  but not on  $K_0$ .

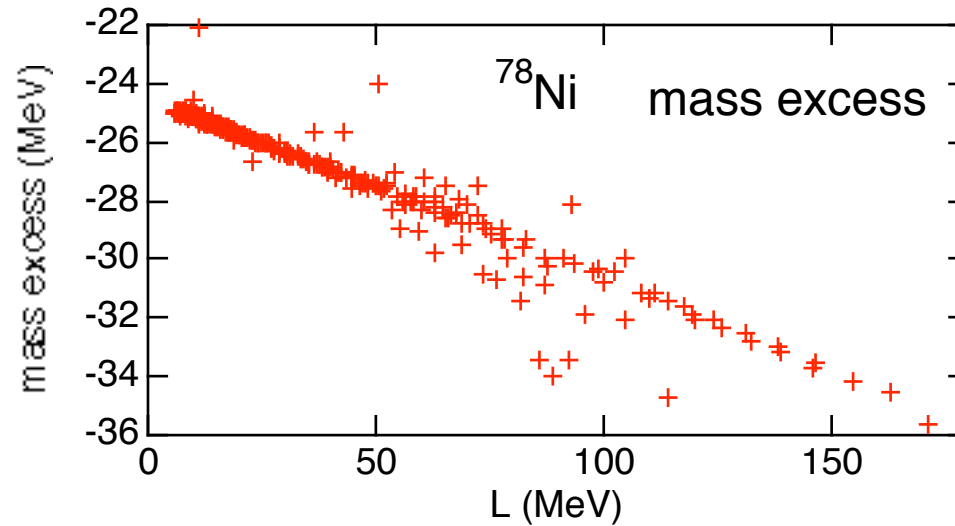


To be studied  
by Kurotama



Let's examine the  $L$  dependence of mass.

# L dependence comes from surface symmetry energy



Larger L => smaller mass

(>\_<) volume symmetry energy

Larger L => larger volume symmetry energy  $S_0$  => larger mass

(^\_^) surface symmetry energy

Oyamatsu and Iida, PRC81, 054302, 2010.

# Surface energy comes from ...

in the cases of beta-stable nuclei  
in neutron-star crusts and in laboratories

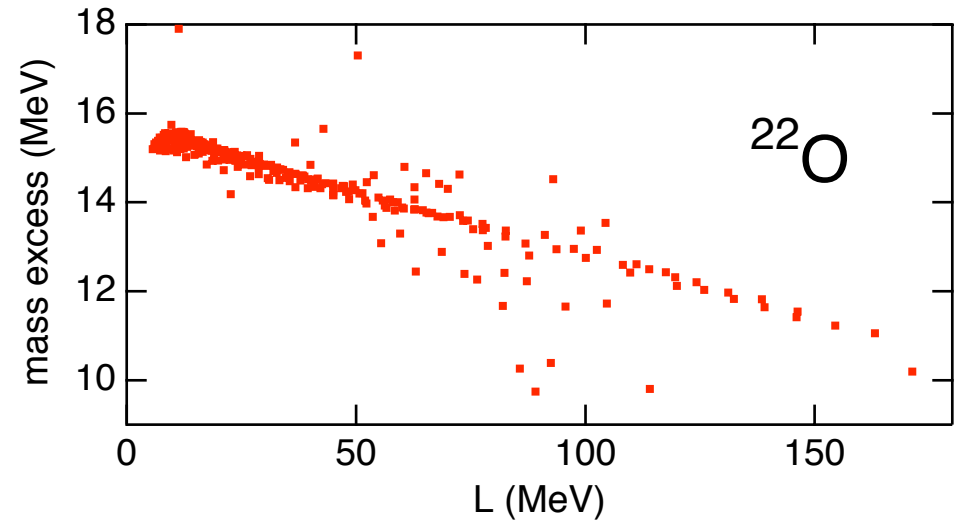
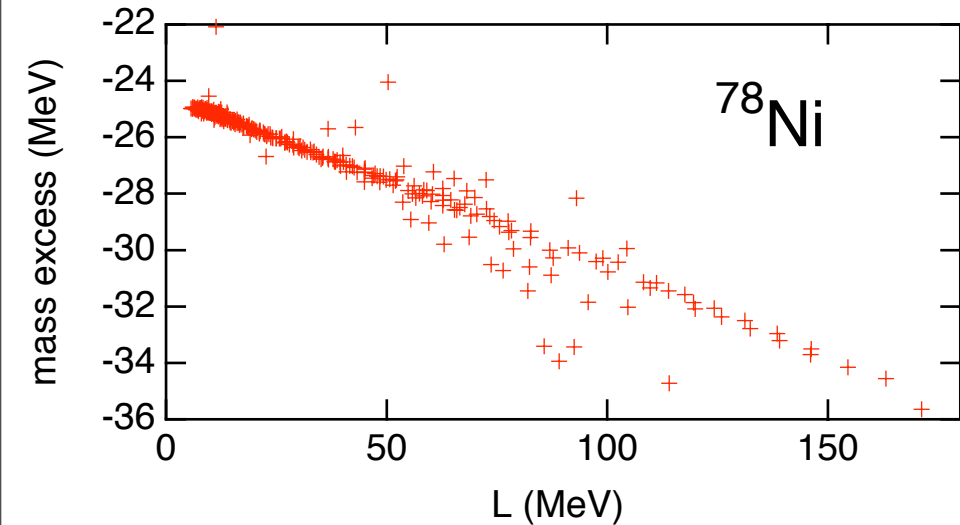
1/2 from  $F_0 \int dr |n(r)|^2$

the remaining  
1/2 mainly from  $\int dr \varepsilon(n_n(r), n_p(r))$  (EOS)

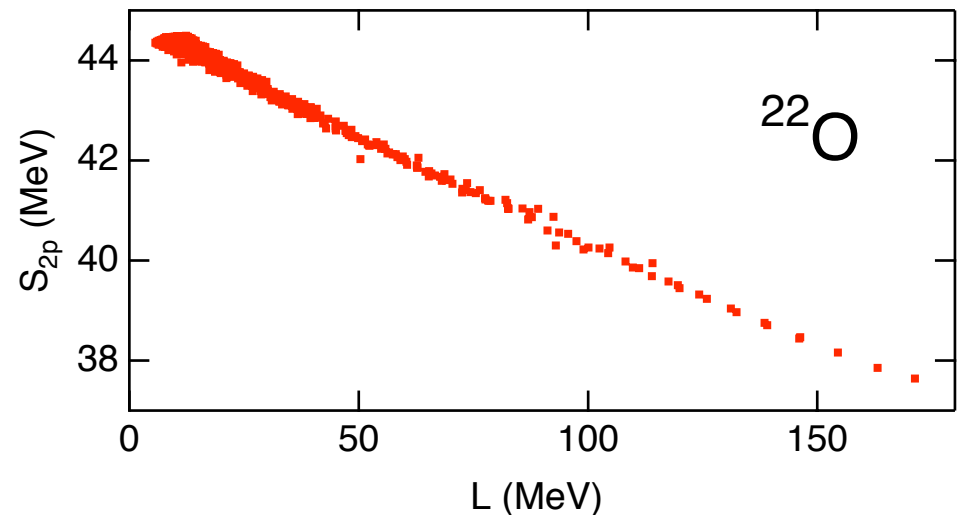
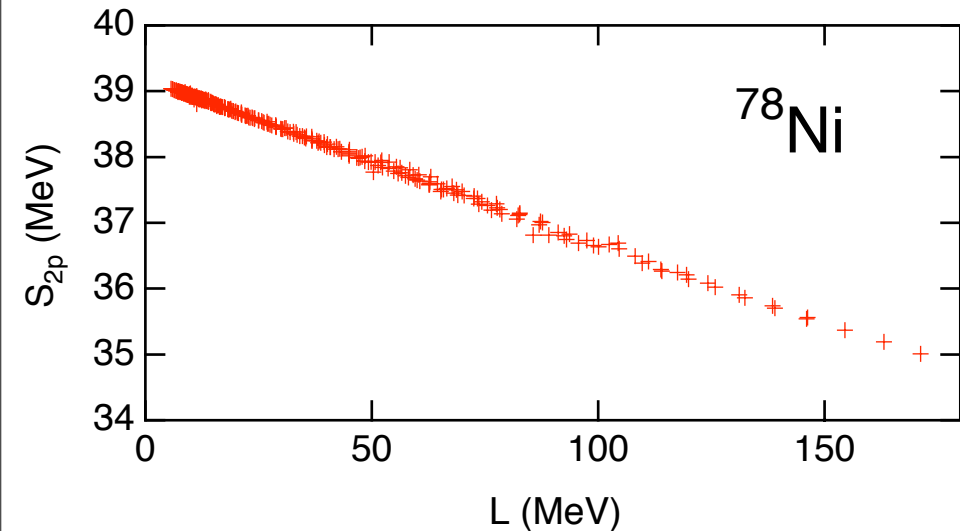
Anyway, L dependence emerge through density distribution.

Oyamatsu and Iida, PTP109, 631-650, 2003.

# $S_{2p}$ , $S_{2n}$ : clear L dependence better than mass



scatterings due to numerical errors in optimizing  $n_0$ ,  $w_0$ , and  $K_0$



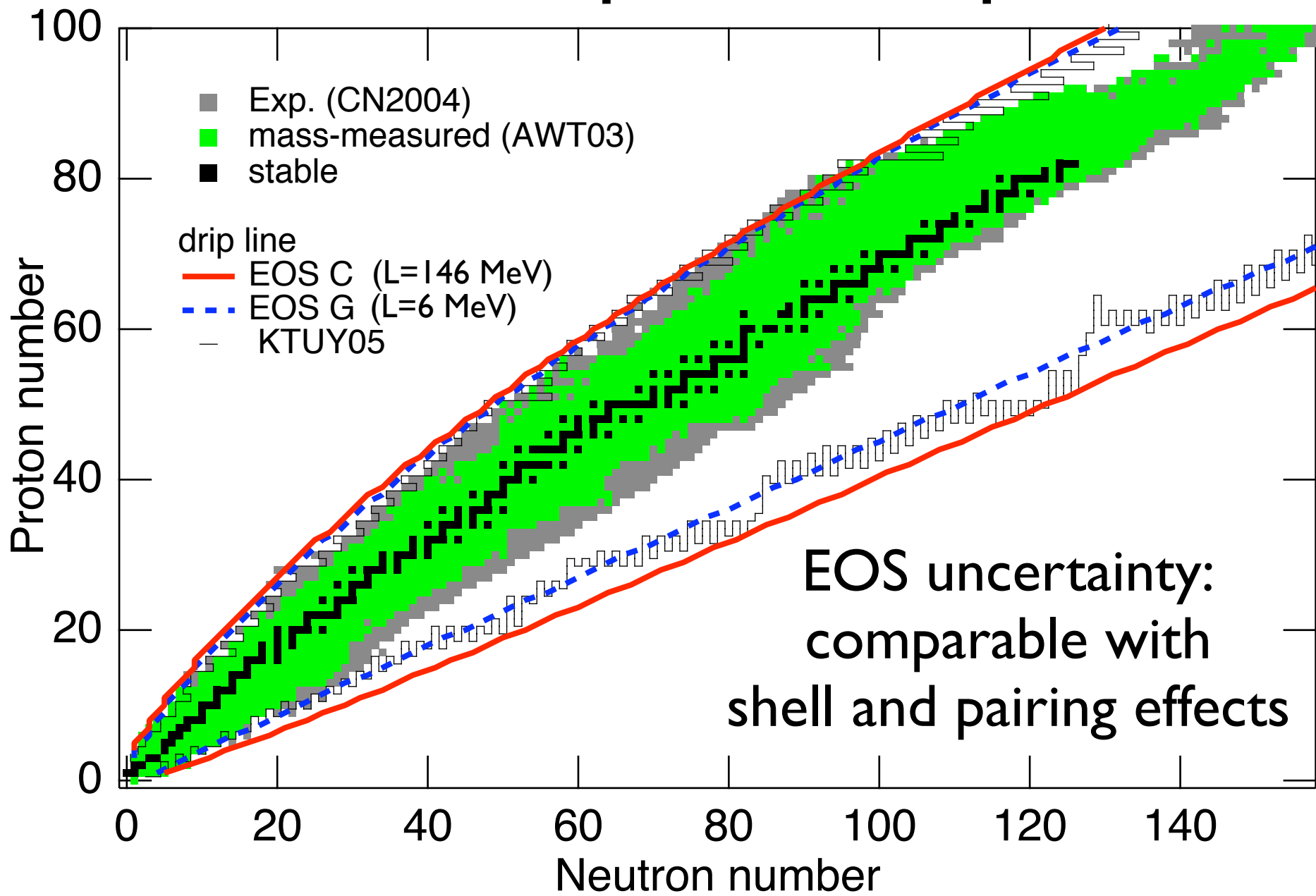
Oyamatsu and Iida, PRC81, 054302, 2010.

**Question :**

**How the drip line is affected  
by the EOS uncertainties?**

Oyamatsu, Iida, Koura, arXiv:1005.3183 (PRC (2010), in press)

# neutron and proton drip lines

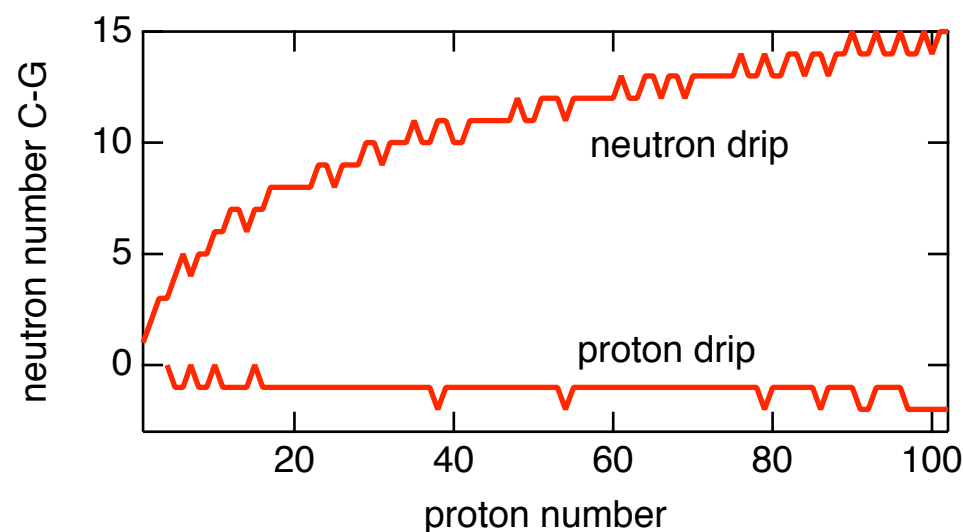
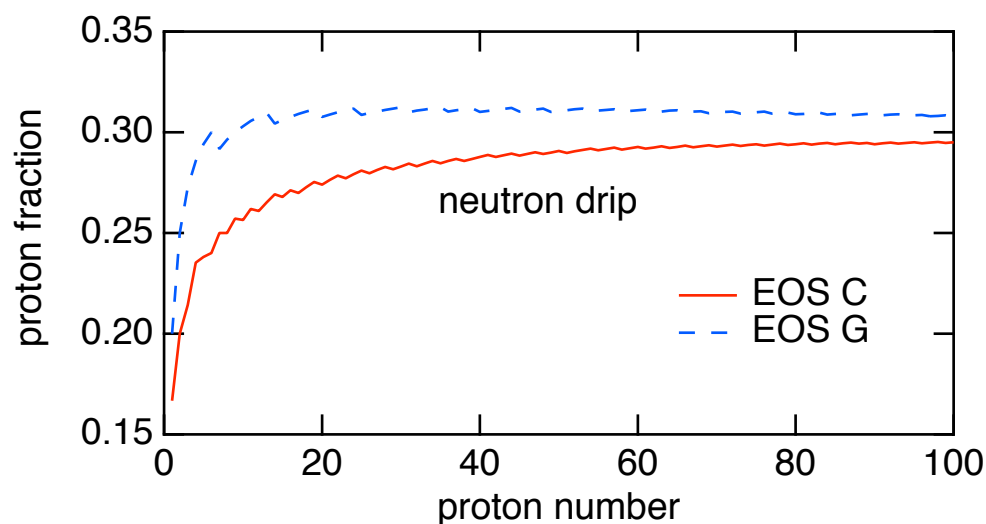


Oyamatsu, Iida, Koura, arXiv:1005.3183 (PRC (2010), to be published)



# Difference between C and G

L=147 MeV    L=6 MeV



Large difference in light nuclei  
while neutron number difference is large in heavy nuclei.

Oyamatsu, Iida, Koura, arXiv:1005.3183 (PRC (2010), to be published)

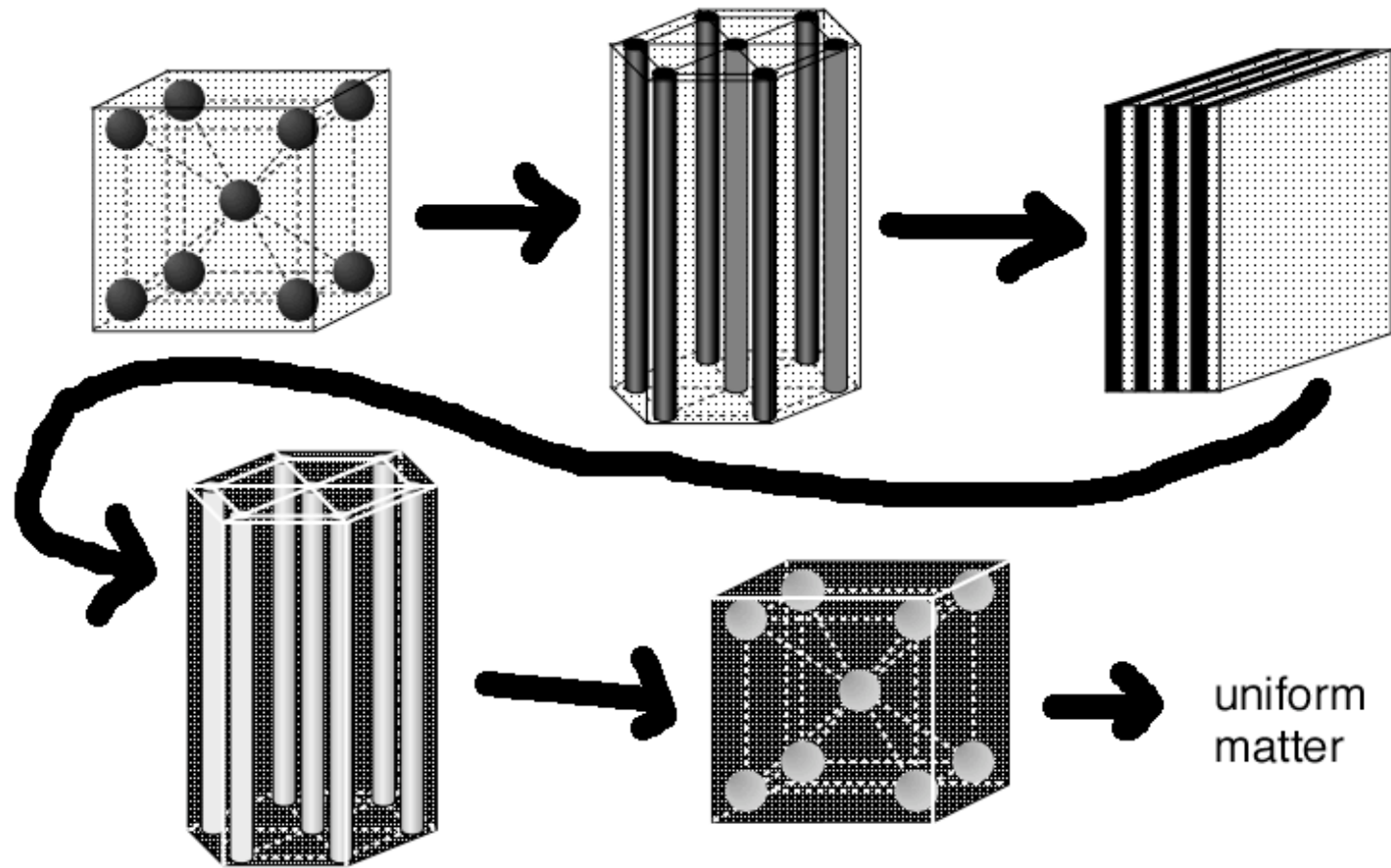
## Step 3

Calculate nuclei in neutron star crusts  
with 200 EOS's

Proton number and fraction  
Density region of pasta nuclei

K. Oyamatsu and K. Iida, Phys. Rev. **C75** (2007) 015801.

## spherical nuclei and pasta nuclei



K.Oyamatsu, NPA561, 431 (1993)

Dark domains means nuclei (proton clusters).

At low densities in neutron-star crusts, we have nuclei which are more or less spherical.

In the core we have uniform matter. Pasta nuclei could exist in between.

## Existence of pasta nuclei depends on the EOS.

# Estimate of density region of pasta nuclei

C.J. Pethick and D.G. Ravenhall, Annu. Rev. Nucl. Part. Sci. **45**, 429 (1995).

## lower boundary

stability against fission of spherical nuclei

In the liquid drop model, (Coulomb self energy) = 2 \* (surface energy)

$$\Rightarrow (\text{volume fraction of nucleus}) = 1/8$$

## upper boundary (core-crust boundary)

instability against forming proton clusters

$$v(Q) = v_0 + 2(4\pi e^2 \beta)^{1/2} - \beta k_{\text{TF}}^2 > 0$$

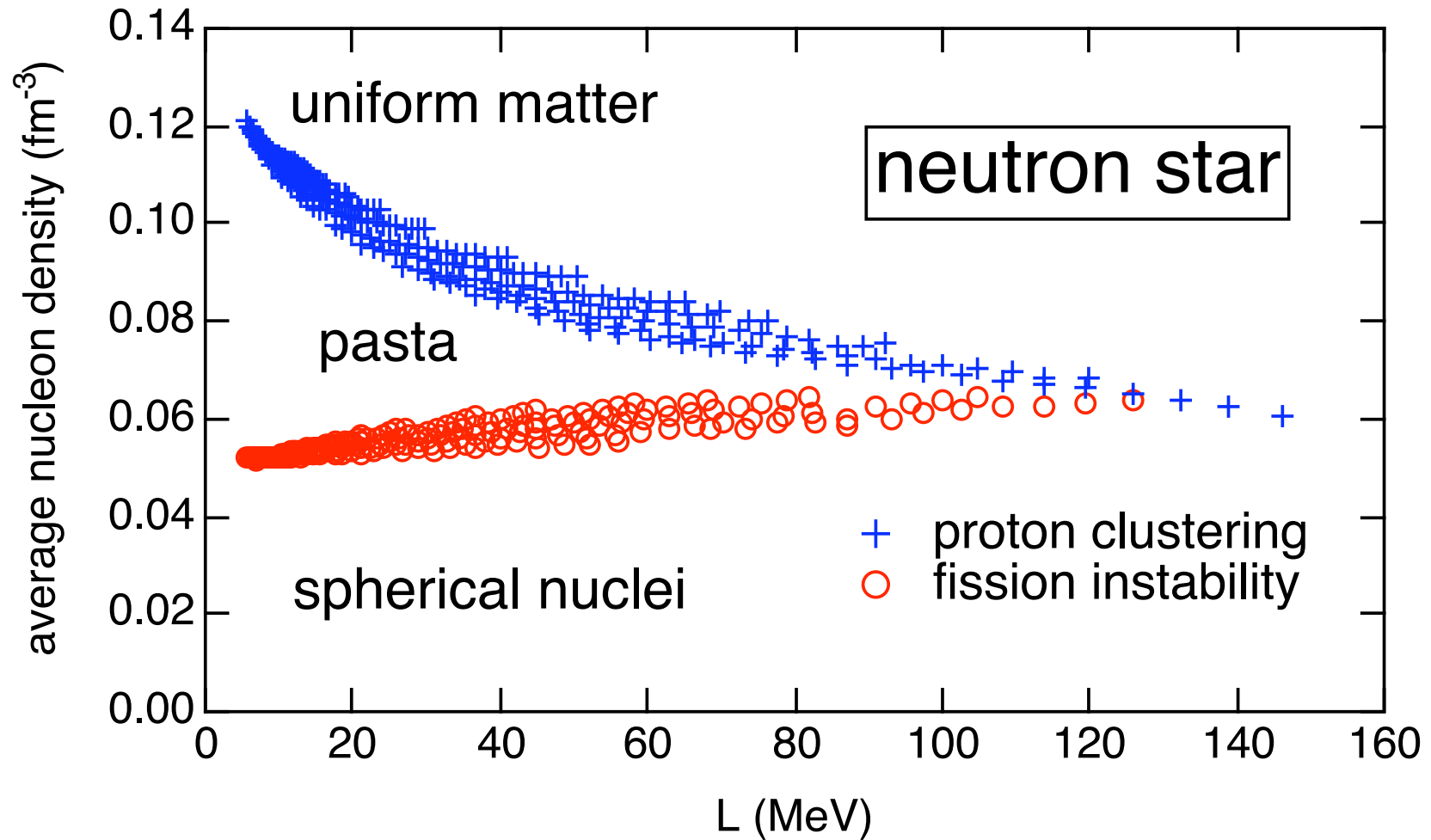
$$Q^2 = \left( \frac{4\pi e^2}{\beta} \right)^{1/2} - k_{\text{TF}}^2$$

$$v(Q) \approx v_0 = \frac{\partial \mu_p}{\partial n_p} - \frac{(\partial \mu_p / \partial n_n)^2}{\partial \mu_n / \partial n_n} = \left( \frac{\partial \mu_p}{\partial n_p} \right)_{\mu_n, \mu_e}$$

$$\beta = D_{pp} + 2D_{np}\zeta + D_{nn}\zeta^2, \quad \zeta = -\frac{\partial \mu_p / \partial n_n}{\partial \mu_n / \partial n_n}$$

$$k_{\text{TF}}^2 = \frac{4\pi e^2}{\partial \mu_e / \partial n_e} = \frac{4\alpha}{\pi} (3\pi^2 n_e)^{1/3}$$

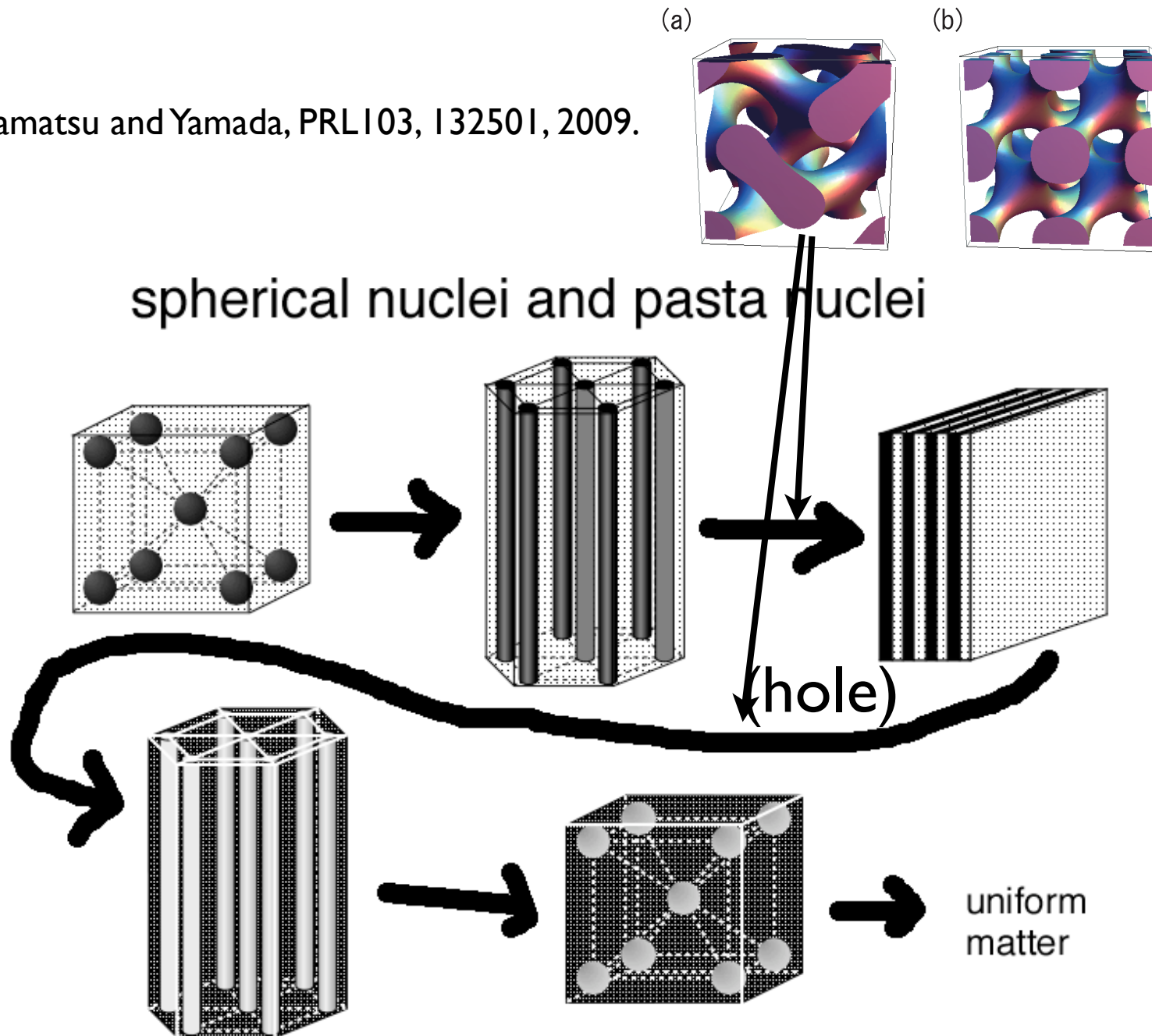
The upper bound (core-crust boundary density) is clearly dependent on  $L$  while the lower is almost constant.



Oyamatsu and Iida, PRC75, 015801, 2007.

If  $L < 100$  MeV,  
gyroid could appear at finite temperature.

Nakazato, Oyamatsu and Yamada, PRL103, 132501, 2009.



# Summary

- The values of  $L$  and  $K_0$  cannot be determined from masses and radii of stable nuclei.
- Radii and masses of unstable nuclei have sensitivity to  $L$ .
- The neutron drip lines shows an appreciable  $L$  dependence. The uncertainty is comparable with the shell and pairing effects.
- The core-crust boundary density of neutron star is dependent on  $L$ . The existence of the pasta phase is dominated by  $L$ . The pasta phase exists if  $L < 100$  MeV.
- Systematic experimental study of nuclear mass and size of unstable nuclei in laboratories will help determine the  $L$  value, the neutron drip line and the existence of pasta nuclei in neutron stars.