# The asymmetry term in the nuclear matter incompressibility deduced from the isoscalar giant monopole resonance in the Sn and Cd isotopes.

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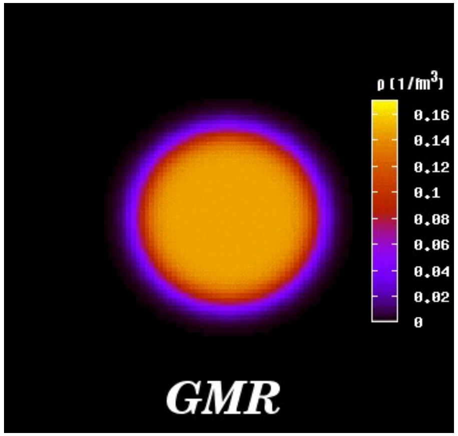
# Compression-modes of collective motions

- Isoscalar giant monopole resonance (ISGMR)

$$O=\Sigma r_i^2$$

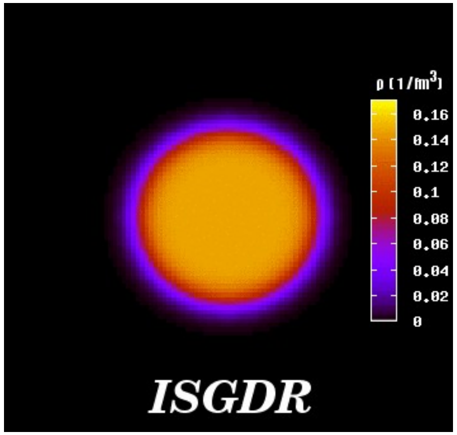
  - Breathing mode



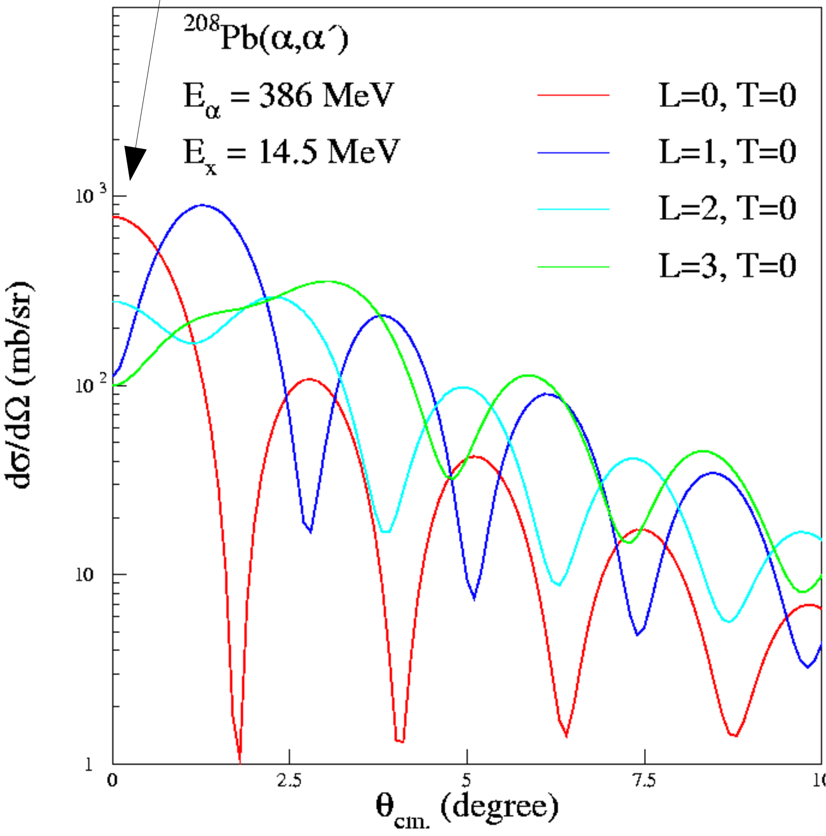
- Isoscalar giant dipole resonance (ISGDR)

$$O=\Sigma r_i^3 Y_1$$

  - Squeezing mode



Maximum at 0° (ISGMR)



# Incompressibility of the nuclear matter

- The incompressibility of nuclear matter $K_\infty$

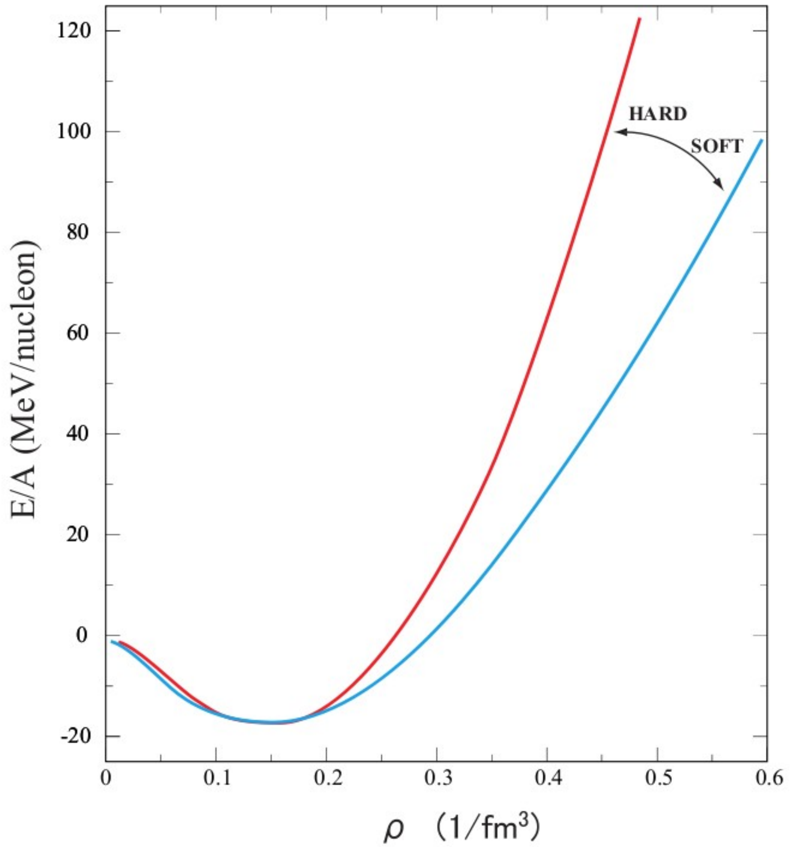
$$K_\infty = 9\rho_0^2 \frac{d^2}{d\rho^2}\left(\frac{E}{A}\right)_{\rho=\rho_0}$$

  - Curvature of the EOS

- The incompressibility of finite nuclei ($K_A$)

$$E_{ISGMR} = \hbar\sqrt{\frac{K_A}{m\langle r^2\rangle}}$$

$$E_{ISGDR} = \hbar\sqrt{\frac{7}{3}\frac{K_A + \frac{27}{25}\epsilon_F}{m\langle r^2\rangle}}$$



# Determination of the nuclear matter incompressibility

- Early attempt to fit the ISGMR data with the empirical expression of the incompressibility could not provide good constraints on the value of $K_\infty$.

S.Shlomo and D.H. Youngblood, Phys.Rev.C 47, 529(1993)

$$K_A \sim K_{vol}(1+cA^{-1/3})+K_\tau\left(\frac{N-Z}{A}\right)^2+K_{Coul}Z^2A^{-4/3}$$

$$K_\infty \sim K_{A\to\infty}$$

- The relation between $K_\infty$ and $K_A$ was obtained from the microscopic calculation with various interaction parameters

$$K_A \approx -3.5+0.64\,K_\infty$$

J.P.Blaizot et al, Nucl.Phys.A 591, 435(1995)

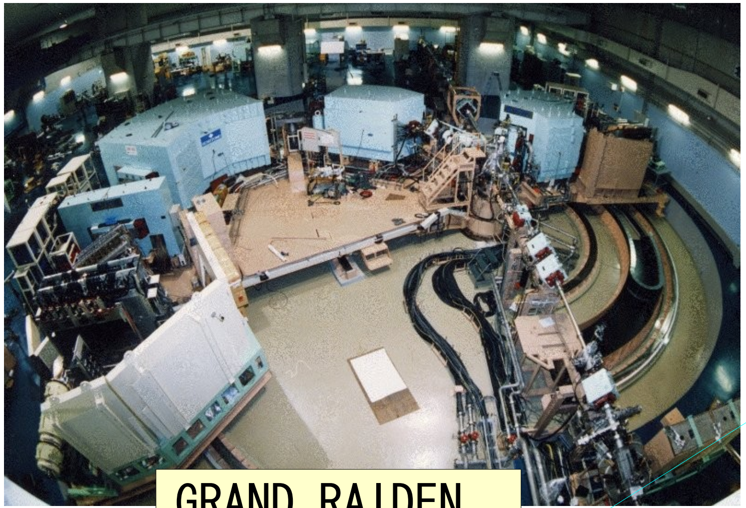
# Experimental setup in RCNP

Reaction:
(α, α')

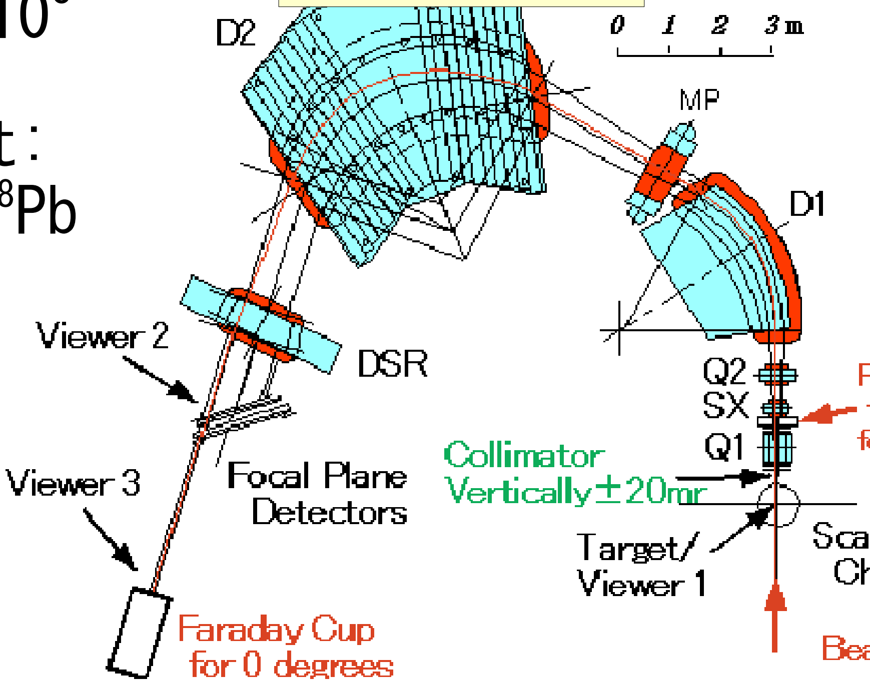
Beam:
400MeV

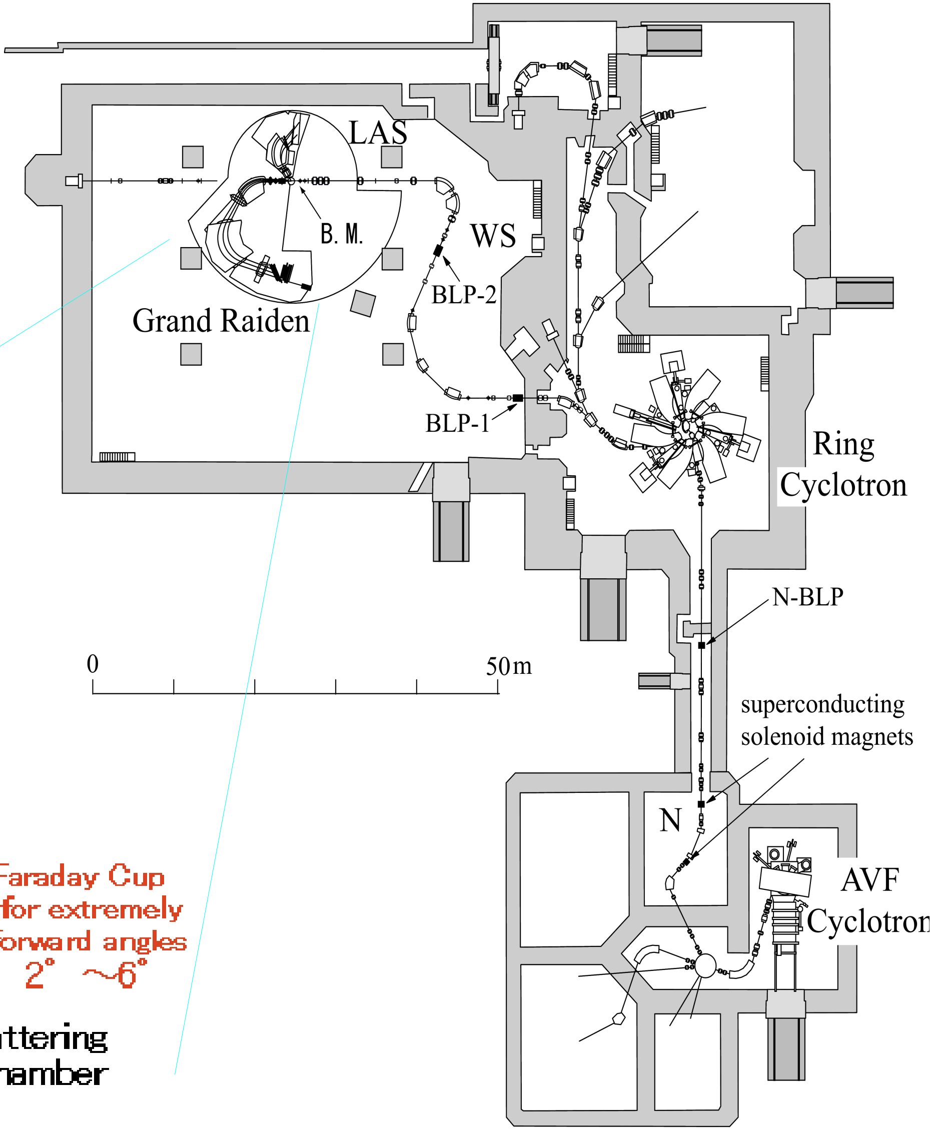
Angle:
0° ～10°

Target:
$^{12}$C～$^{208}$Pb



GRAND RAIDEN SPECTROMETER

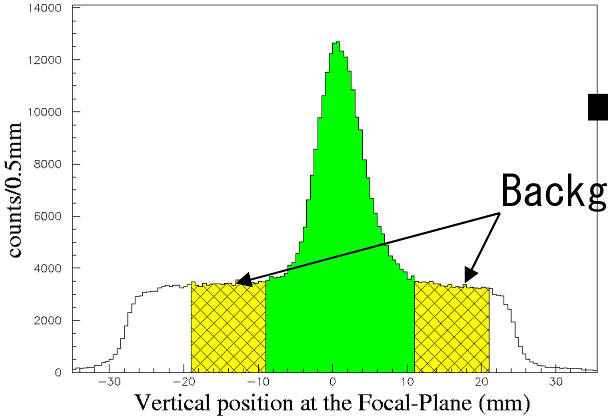




# Data Reduction

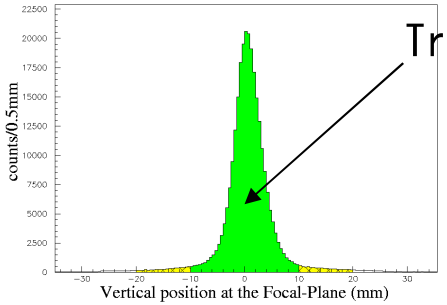
Subtract instrumental background from the energy spectra using the ion-optics of Grand Raiden

$\theta_{GR}$= 0°

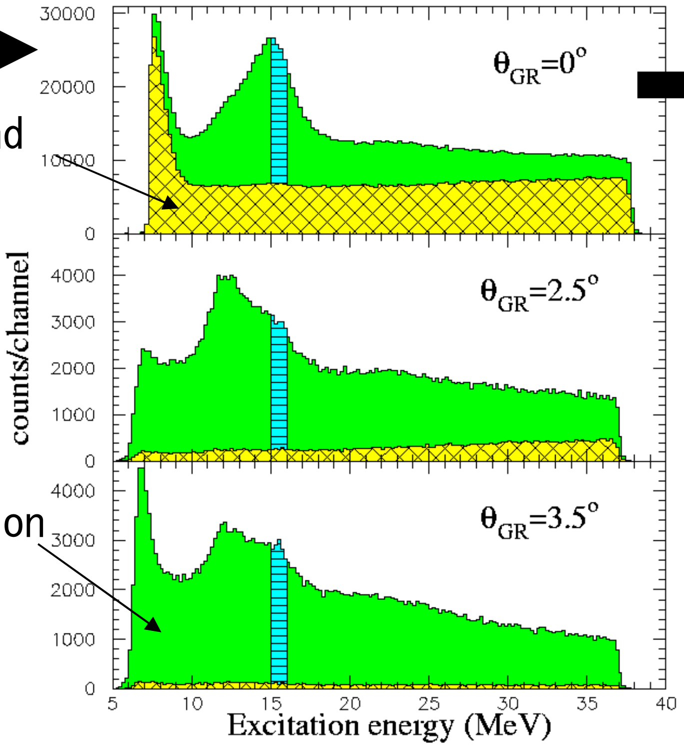


Background

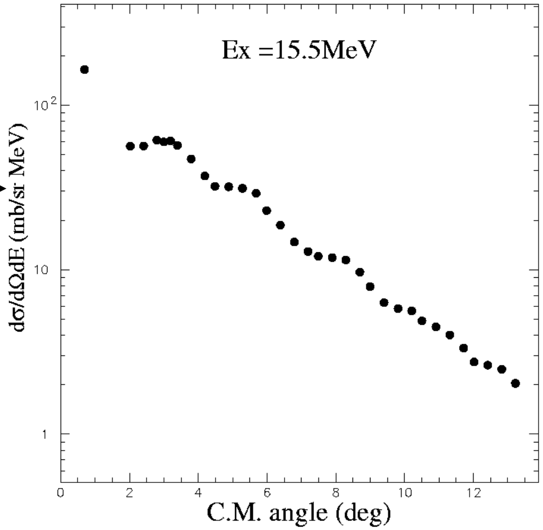
$\theta_{GR}$= 3.5°



True region



$^{144}$Sm



The angular distribution for each 1MeV bin

# The multipole decomposition analysis ( MDA )

- Multipole Decomposition

$$\frac{d^2\sigma}{d\Omega\, dE^{\exp}}(E,\theta)=\sum_L a_L(E)\frac{d\sigma}{d\Omega_L}(E,\theta)$$

- Folding model DWBA calculation
  - Optical potential

$$U(r)=V_F(r)+iW/\{1+\exp[(r-R_I)/A_I]\}$$

  - Density-dependent N-$\alpha$ interaction

$$V_{DDG}=-v[1-\beta\rho(r')^{2/3}]\exp(-|r-r'|^2/t^2)$$

in the analysis of Sn isotopes

- Interaction parameters were obtained by fitting elastic scattering.

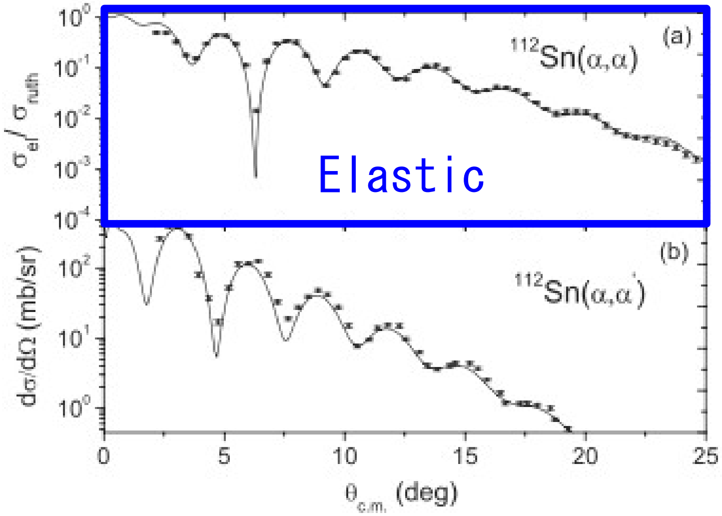
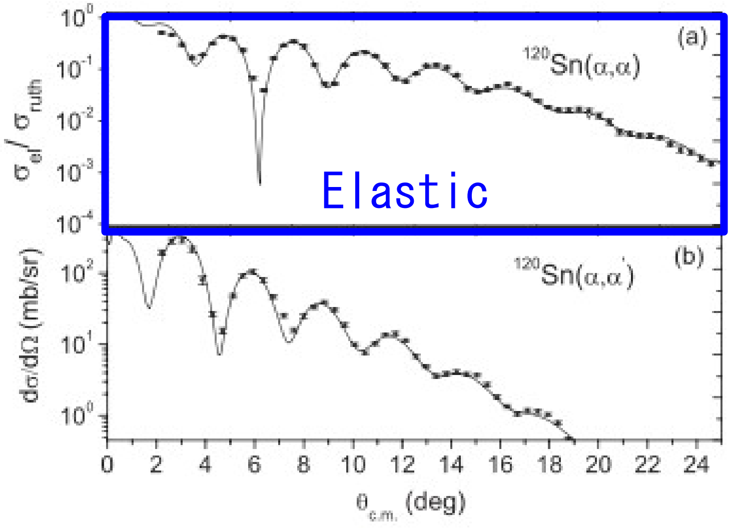


FIG. 4. (a) Ratio of the elastic $\alpha$-scattering cross sections to the Rutherford cross sections for $^{112}$Sn at 386 MeV. (b) Differential cross sections for excitation of the $2_1^+$ state in $^{112}$Sn. The solid lines are the results of the folding-model calculations.



T.Li et al, Phys.Rev.C 81, 034309(2010)

FIG. 5. Same as Fig. 4, except for $^{120}$Sn.

# The MDA 2

- Transition densities (T=0)

$$
\begin{aligned}
\delta\rho_{L=0} &= -\beta_0(E_x)(3\rho_0(r) + r\frac{d\rho_0(r)}{dr}), \\
\delta\rho_{L=1,T=0} &= -\frac{\beta_1(E_x)}{R\sqrt{3}}\Big[3r^2\frac{d}{dr} + 10r - \frac{5}{3}\langle r^2\rangle\frac{d}{dr} \\
&\quad +\epsilon(r\frac{d^2}{dr^2} + 4\frac{d}{dr})\Big]\rho_0(r), \\
\delta\rho_{L\geqq 2} &= -\delta_L(E_x)\frac{d\rho_0(r)}{dr},
\end{aligned}
$$

Ref. G.R. Satchler Nucl. Phys. A472(1987)215,
M.N. Harakeh, *et al.* Phys. Rev. C 23(1981)2329,
A. Kolomiets, *et al.* Phys. Rev. C 61(2000)034312.

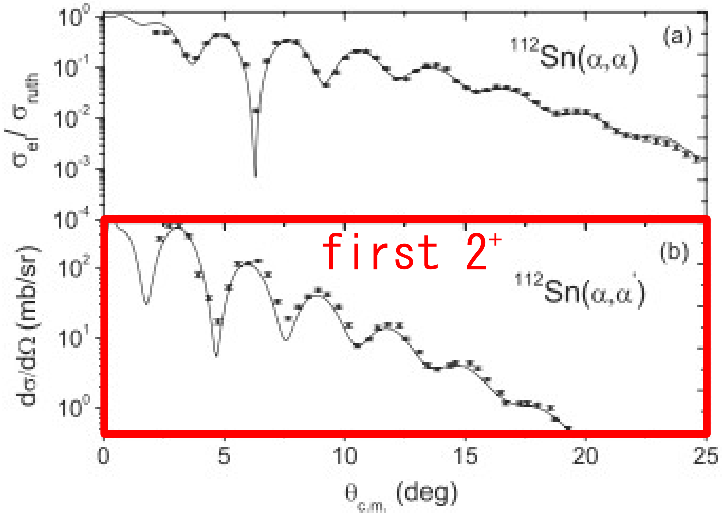
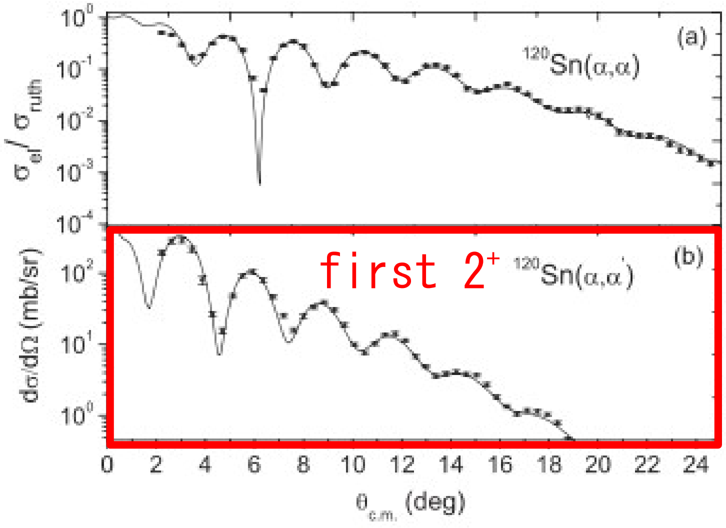


FIG. 4. (a) Ratio of the elastic α-scattering cross sections to the Rutherford cross sections for $^{112}$Sn at 386 MeV. (b) Differential cross sections for excitation of the $2_1^+$ state in $^{112}$Sn. The solid lines are the results of the folding-model calculations.



T.Li et al, Phys.Rev.C 81, 034309(2010)

FIG. 5. Same as Fig. 4, except for $^{120}$Sn.

# Results of $^{90}$Zr, $^{116}$Sn, and $^{208}$Pb

- Deduced $K_\infty$ from ISGMR and ISGDR energies of $^{208}$Pb by using equation of Blaizot was

  $K_\infty \sim 215$ MeV
  ($E_{GMR}$=13.5±0.2MeV)

  ($E_{GMR}$=13.96±0.2MeV
  D.H.Youngblood et al, PRC69, 034315(2004) )

M.Uchida et al,
Phys.Rev.C 69, 051301R(2004)

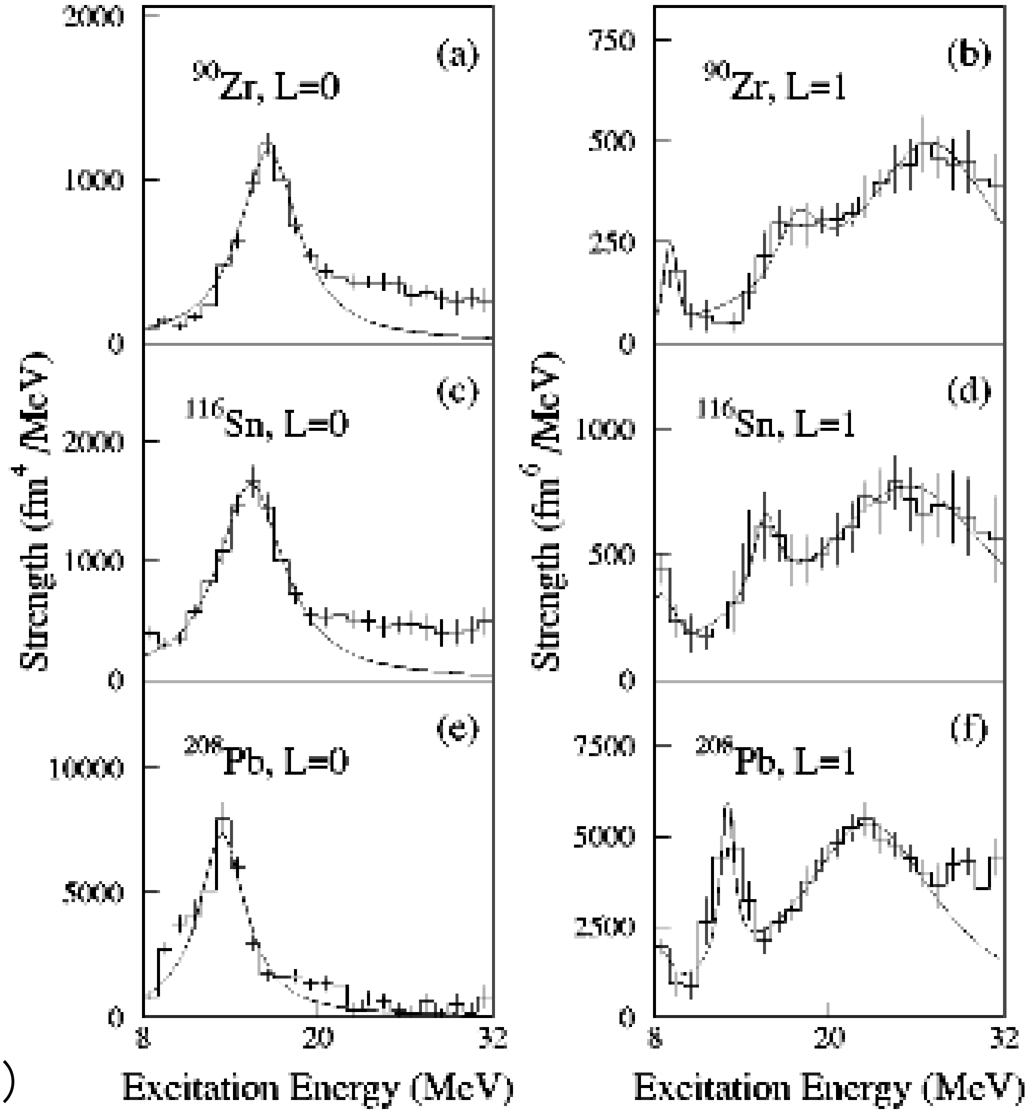


FIG. 3. Experimentally obtained strength distributions of the ISGMR and the ISGDR in $^{90}$Zr, $^{116}$Sn, and $^{208}$Pb. The error bars are

# Measurement of Sn isotopes

- Non-relativistic and relativistic calculations predicted different $K_\infty$ values.
- It was needed to provide a more constraint information, **the asymmetry term, $K_\tau$**.
- To extract the $K_\tau$ value, we adopted the empirical expression as $K_A \sim K_{vol}(1+cA^{-1/3})+K_\tau(\frac{N-Z}{A})^2+K_{Coul}Z^2A^{-4/3}$

  $K_A - K_{Coul}Z^2A^{-4/3}$ has a quadratic relation to $K_\tau$

  $K_{coul} \sim -5.2$MeV (H.Sagawa et al, Phys.Rev.C 76, 034327(2007))
- ISGMR centroid energies in wide range of asymmetry ratio, N-Z/A, have been measured. In the case of Sn isotopes, N-Z/A are from 0.107 for $^{112}$Sn to 0.194 for $^{124}$Sn.

# ISGMR strengths of Sn isotopes

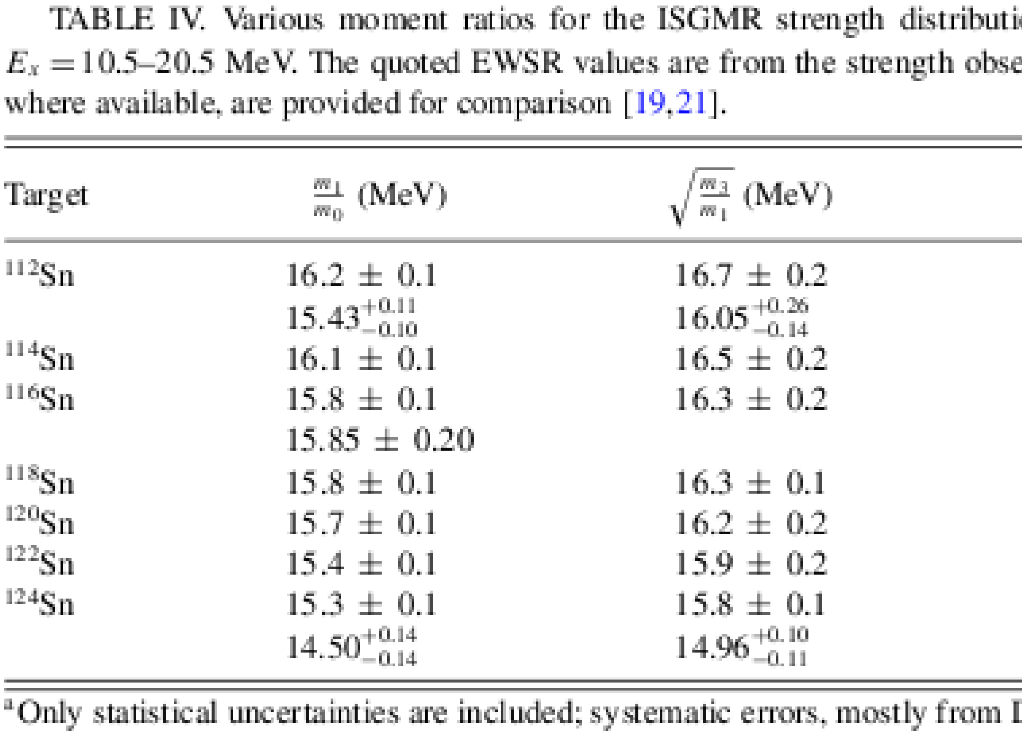


TABLE IV. Various moment ratios for the ISGMR strength distributi
$E_x = 10.5$–$20.5$ MeV. The quoted EWSR values are from the strength obse
where available, are provided for comparison [19,21].

| Target | $\frac{m_1}{m_0}$ (MeV) | $\sqrt{\frac{m_3}{m_1}}$ (MeV) |
|---|---|---|
| $^{112}$Sn | $16.2 \pm 0.1$ | $16.7 \pm 0.2$ |
| | $15.43^{+0.11}_{-0.10}$ | $16.05^{+0.26}_{-0.14}$ |
| $^{114}$Sn | $16.1 \pm 0.1$ | $16.5 \pm 0.2$ |
| $^{116}$Sn | $15.8 \pm 0.1$ | $16.3 \pm 0.2$ |
| | $15.85 \pm 0.20$ | |
| $^{118}$Sn | $15.8 \pm 0.1$ | $16.3 \pm 0.1$ |
| $^{120}$Sn | $15.7 \pm 0.1$ | $16.2 \pm 0.2$ |
| $^{122}$Sn | $15.4 \pm 0.1$ | $15.9 \pm 0.2$ |
| $^{124}$Sn | $15.3 \pm 0.1$ | $15.8 \pm 0.1$ |
| | $14.50^{+0.14}_{-0.14}$ | $14.96^{+0.10}_{-0.11}$ |

[a]Only statistical uncertainties are included; systematic errors, mostly from I

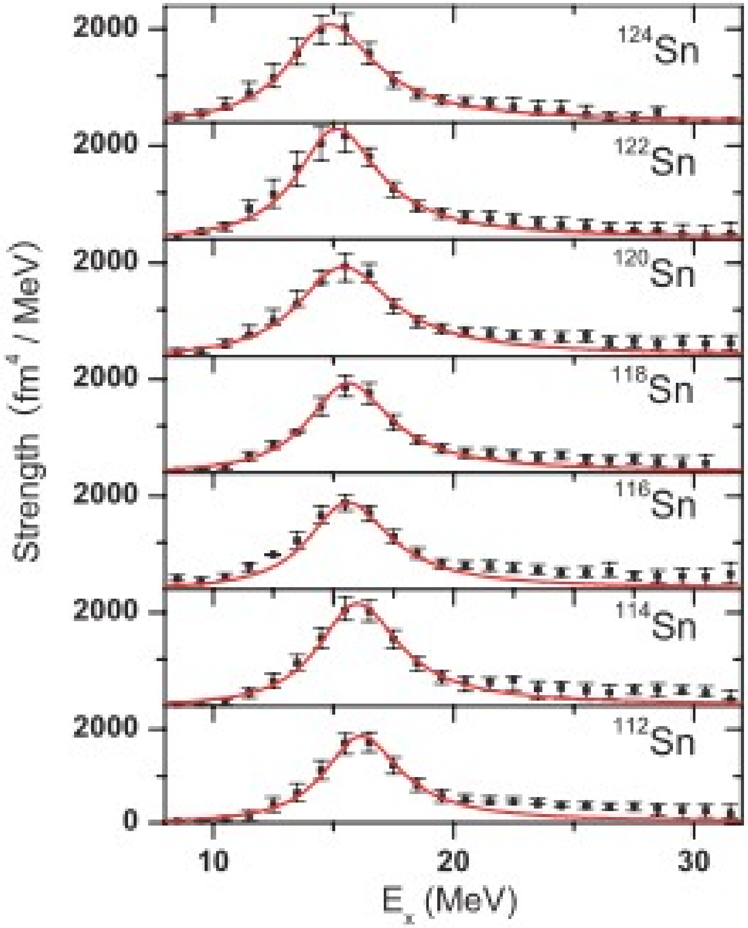


FIG. 9. (Color online) ISGMR strength distributions obtained for the Sn isotopes in the present experiment. Error bars represent the uncertainties from fitting the angular distributions in the MDA procedure. The solid lines show Lorentzian fits to the data.

T. Li et al, PRL99 162503(2007)

T. Li et al, Phys. Rev. C 81, 034309(2010)

# Dependence of ISGMR energies on the asymmetry ratio

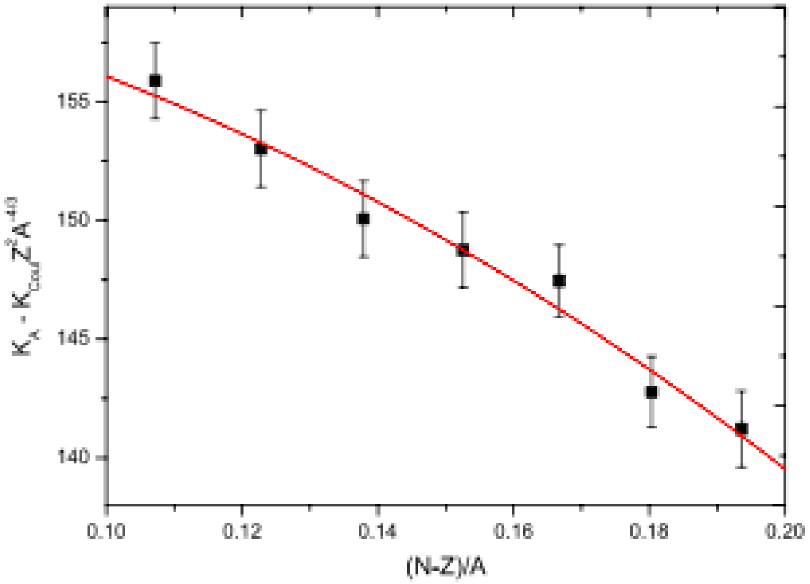


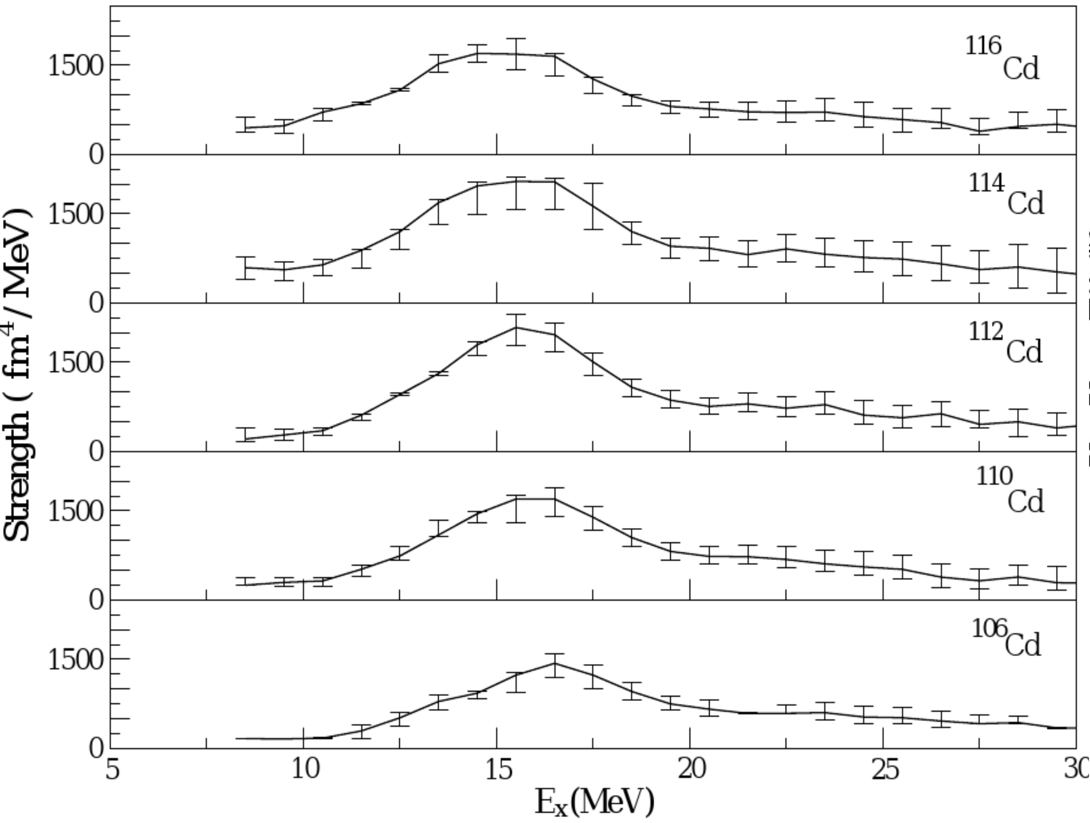
FIG. 4 (color online). Systematics of the difference $K_A - K_{\mathrm{Coul}}Z^2A^{-4/3}$ in the Sn isotopes as a function of the "asymmetry parameter" [$(N-Z)/A$]; $K_{\mathrm{Coul}} = -5.2$ MeV [33]. The solid line represents a least-squares quadratic fit to the data.

- **$K_\tau$ = −550 ± 100 MeV** T.Li et al, PRL99 162503(2007)
- This value was consistent with the result of the analysis of the HI experiment, −370±120 MeV.

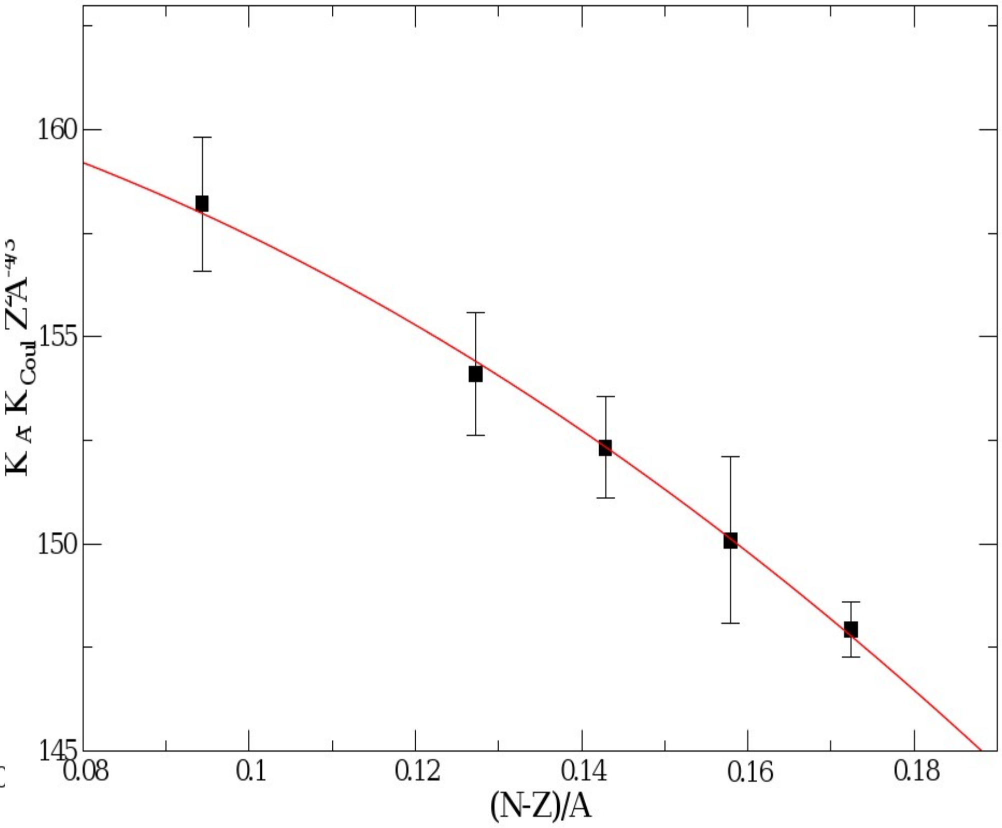
L.W.Chen et al, Phys.Rev.C 80, 014322(2009)

# Result of Cd isotopes

ISGMR strength



Preliminary result



$K_\tau$ = −480±100 MeV

analyzed by D.Patel

# Comparing with theories

- Moment ratio, m1/m0, of the ISGMR energy
- Non-relativistic and relativistic RPA calculations overestimated GMR energies of Sn isotopes

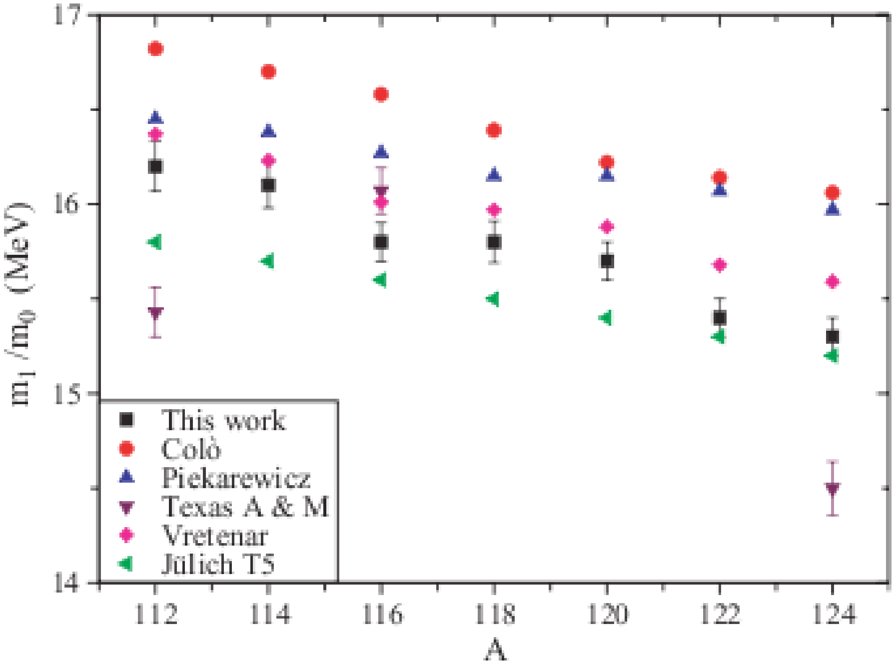


FIG. 13. (Color online) Systematics of the moment ratios $m_1/m_0$ for the ISGMR strength distributions in the Sn isotopes. The experimental results (filled squares) are compared with results from nonrelativistic RPA calculations (without pairing) by Colò *et al.* [65,66] (filled circles), relativistic calculations of Piekarewicz [67] (triangles), RMF calculations from Vretenar *et al.* [68] (diamonds), and QTBA calculations from the Jülich group [69] (sideways triangles). Results for $^{112}$Sn, $^{116}$Sn, and $^{124}$Sn reported by the TAMU group [19,21] are also shown (inverted triangles).

T.Li et al, Phys.Rev.C 81, 034309(2010)

# Is Tin soft?

- From the non-relativistic and relativistic calculation,
  - $K_\infty \backsim 240 \pm 20$ MeV

    It could consistently describe GMR energies of $^{90}$Zr and $^{208}$Pb.
- $K_\infty$ deduced from Sn isotopes was 10 % smaller

- Pairing effect ( "superfluidity" ) was critical in open shell nuclei.

  J.Li, G.Colo, and J.Meng, Phys.Rev.C 78,064304(2008)
- Hybrid model with FSUGold and NL3 reproduced ISGMR energies of Sn isotopes well.

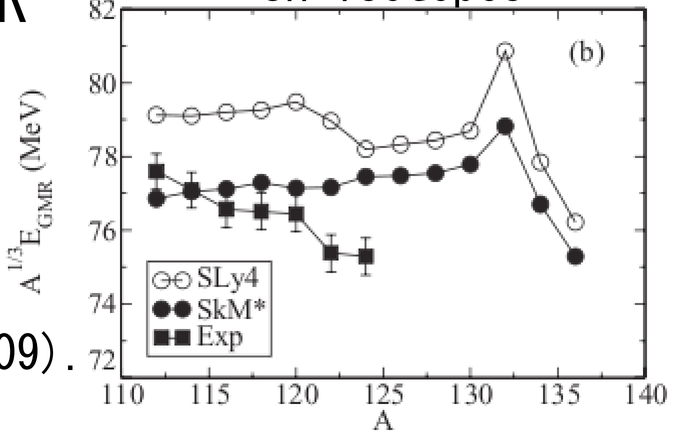
  J.Piekarewicz and M.Centelles, Phys.Rev.C 79, 054311(2009)

# MEM effect

- The calculation underestimated the ISGMR energy of $^{208}$Pb.
- Sn was not soft but $^{208}$Pb was stiff?

E. Khan, Phys. Rev. C 80, 011307(R) (2009).

Sn isotopes



- In the constrained Hartree-Fock method extended to the full Bogoliubov pairing treatment (CHFB), ISGMR centroid energies of double magic nuclei such as $^{208}$Pb were so stiff compared to other isotopes.

- It might be related to the difficulty to describe masses of double magic nuclei, "Mutual enhanced magicity (MEM) effct" ?

Pb isotopes

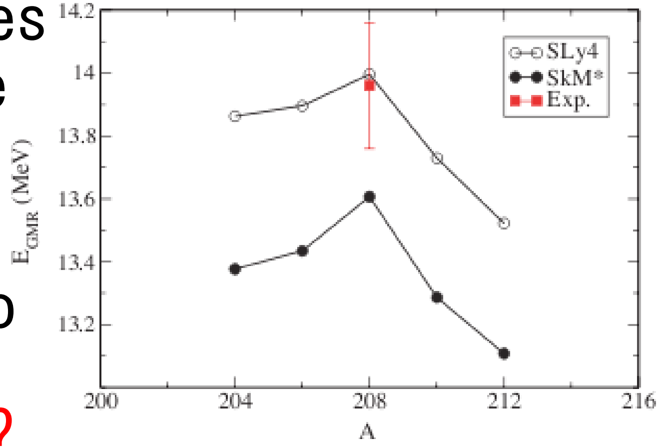


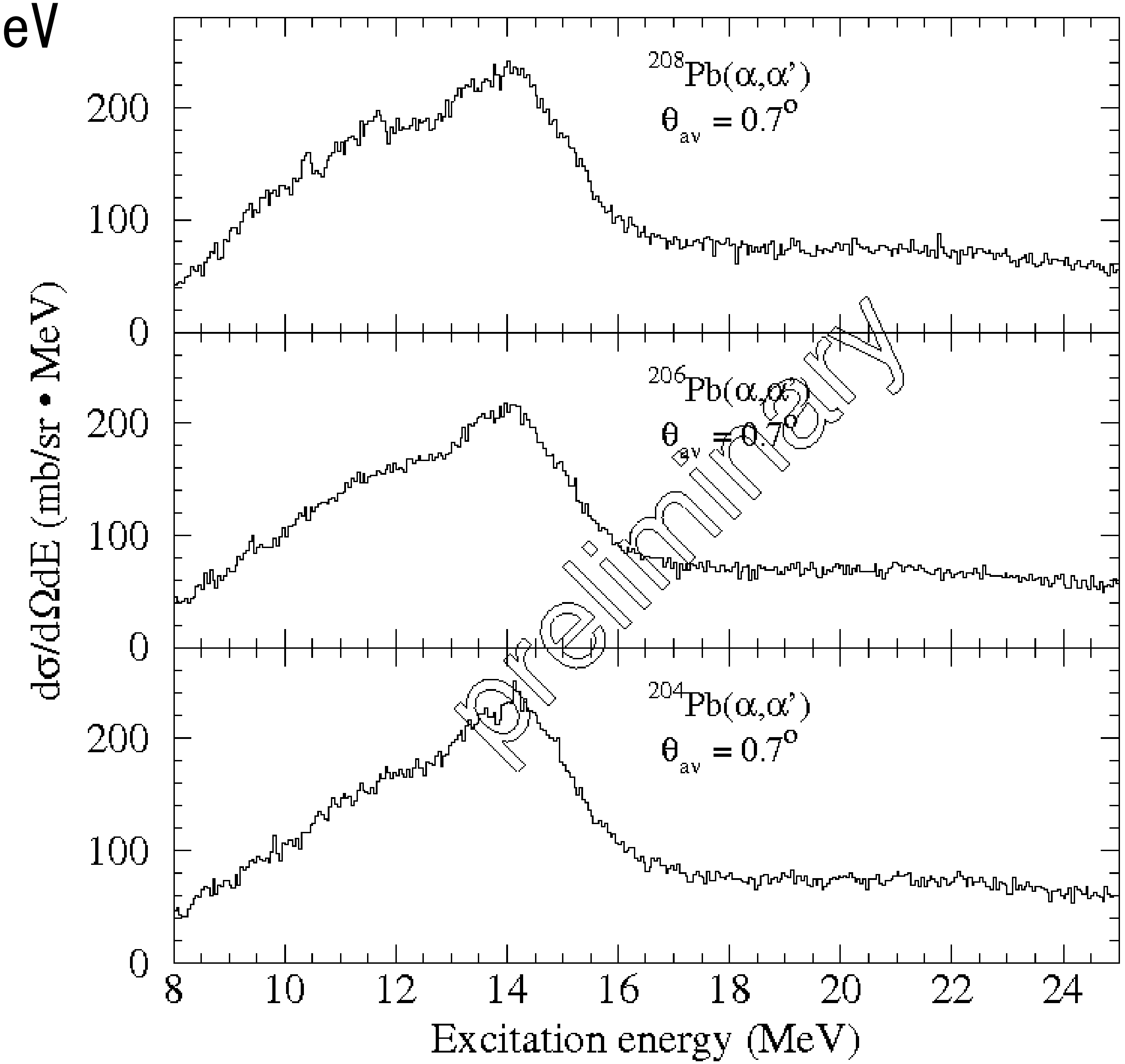
FIG. 1. (Color online) Excitation energies of GMR in $^{204-212}$Pb isotopes calculated with the CHFB method and the SLy4 and SkM* interactions. The experimental data are taken from Ref. [19].

E. Khan, Phys. Rev. C 80, 057302 (2009).

# Preliminary result of Pb isotopes

$E_\alpha$ ～ 400 MeV

Pb(α, α') at 0°



Experiment in May 2010

# Summary

- From compressional-mode giant resonances, we have an "experimental" value for **$K_\infty$= 240 ± 20 MeV.**
- From ISGMR of Sn isotopes, we get the experimental value for $K_\tau$ = −550±100 MeV. The result of Cd isotopes supported this value.
- The combination of these two values constrains the standard interaction in EOS and nuclear structure calculations.
- MEM effect need to be investigated.

# Collaborators

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Thank you for your attentions!