

# ***Probing low-density pairing properties via neutron-rich nuclei***

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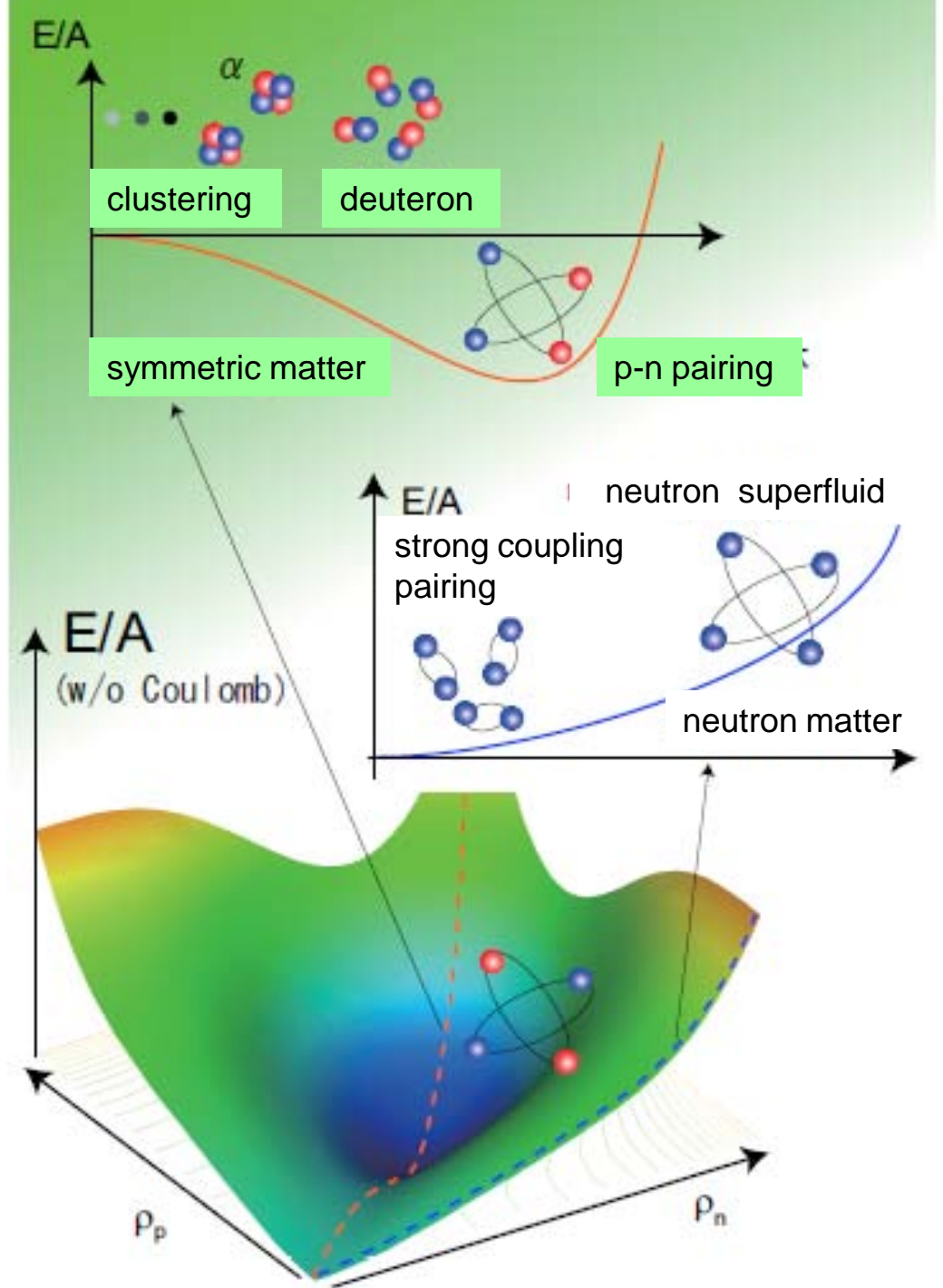
## 1. Dilute neutron matter

strong-coupling pairing, BCS-BEC crossover, large scattering length

## 2. Are there features of strong coupling pairing in finite nuclei ?

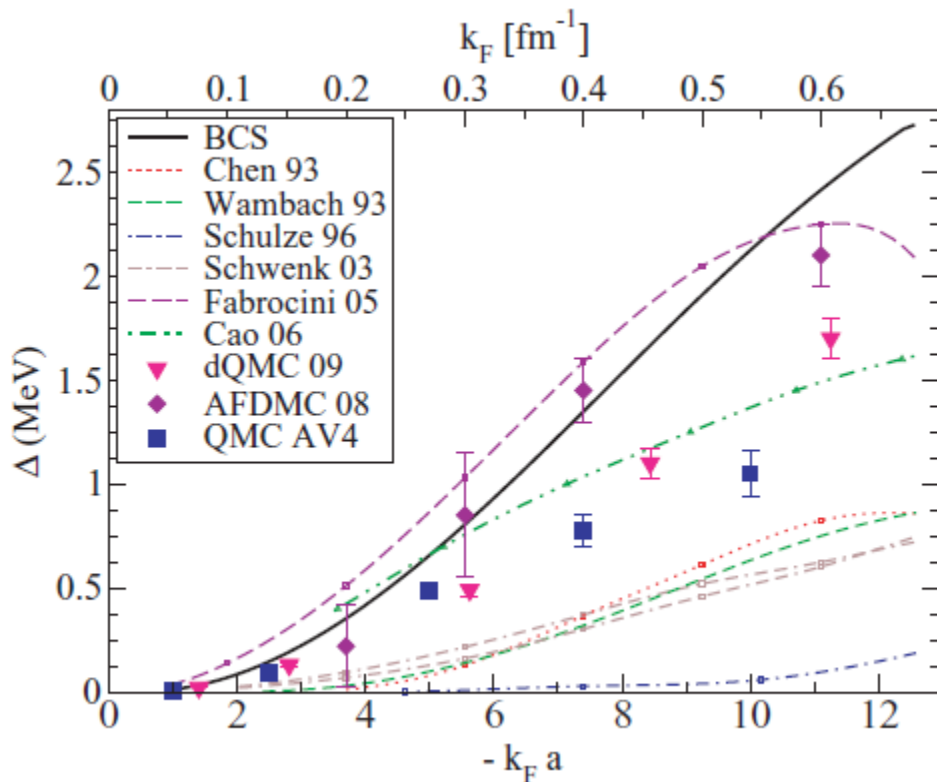
## 3. How do we probe?

# Correlations in dilute matter

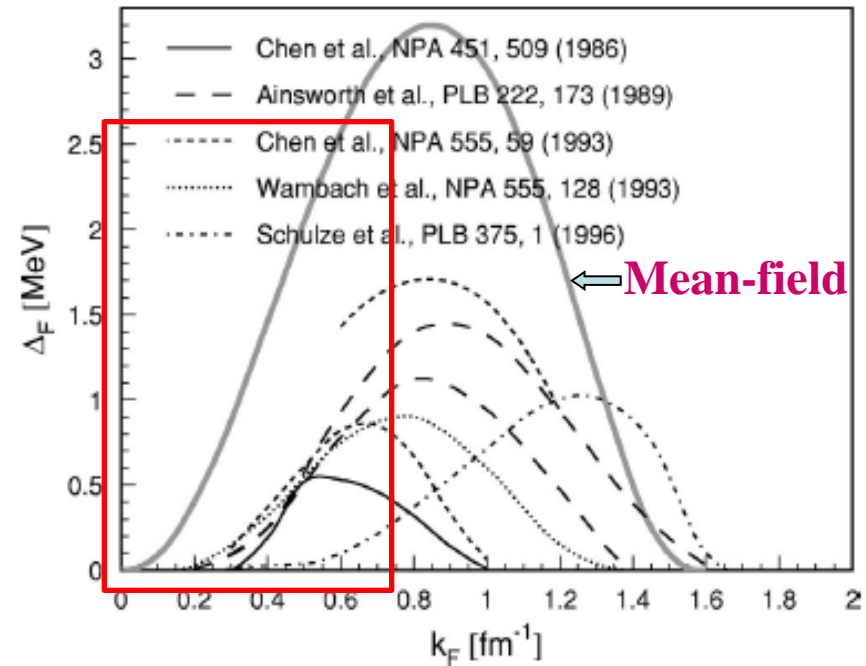


# pairing gap in dilute neutron matter

$\Delta = (1 \sim 0.5) \Delta_{\text{mean-field}}$  in recent calculations



Gezerlis & Carlson, PRC81 (2010)



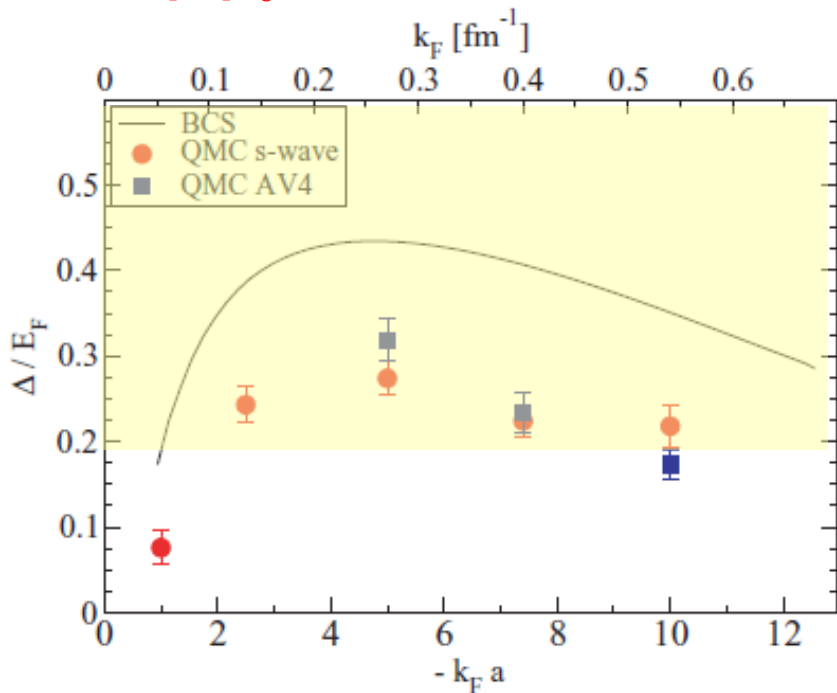
Lombardo & Schulze 2001

# Strong coupling pairing & BCS-BEC crossover

“Large” pair gap vs. Fermi energy  $\Delta/e_F > 0.2$  at low-densities

Monte-Carlo calculation

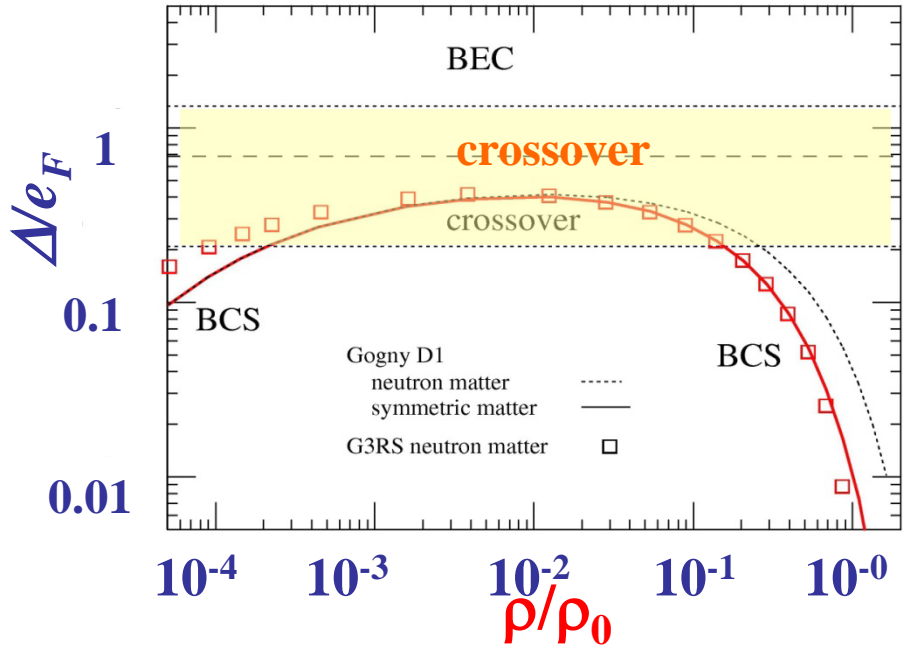
$\rho/\rho_0 = 10^{-3} \sim 0.5 \times 10^{-1}$



Gezerlis & Carlson, PRC81 (2010)

Mean-field calculation (BCS approx.)

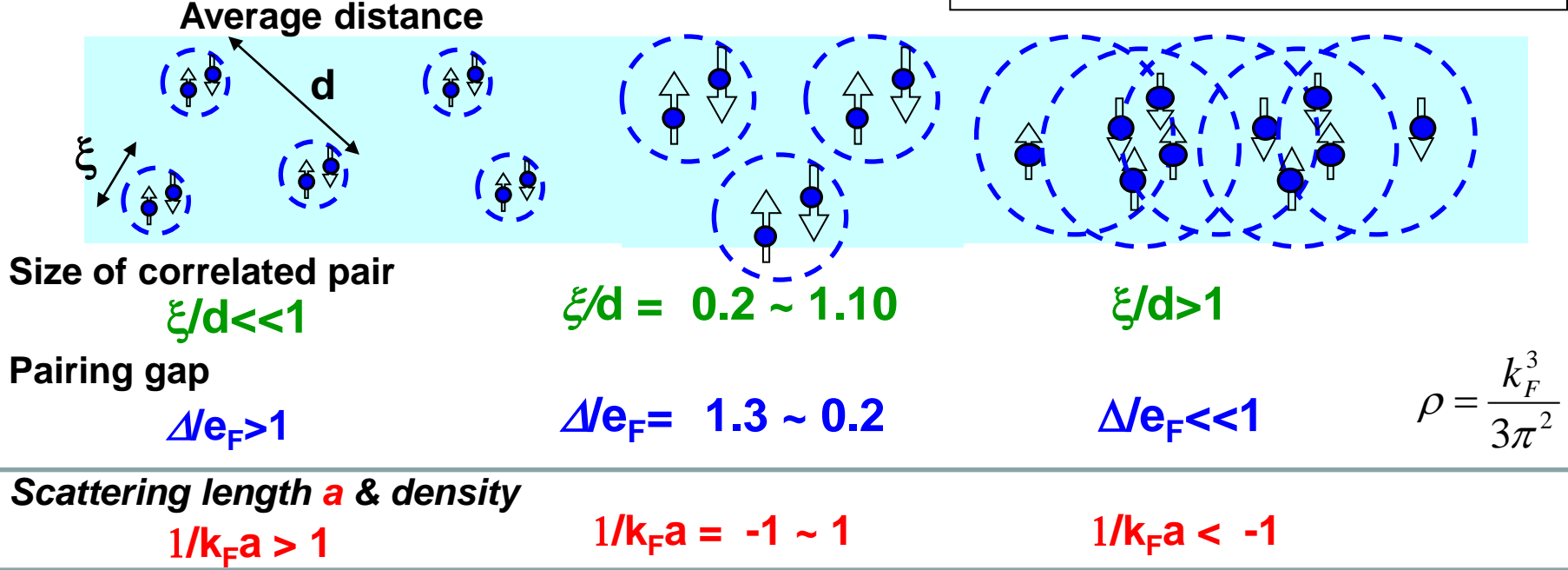
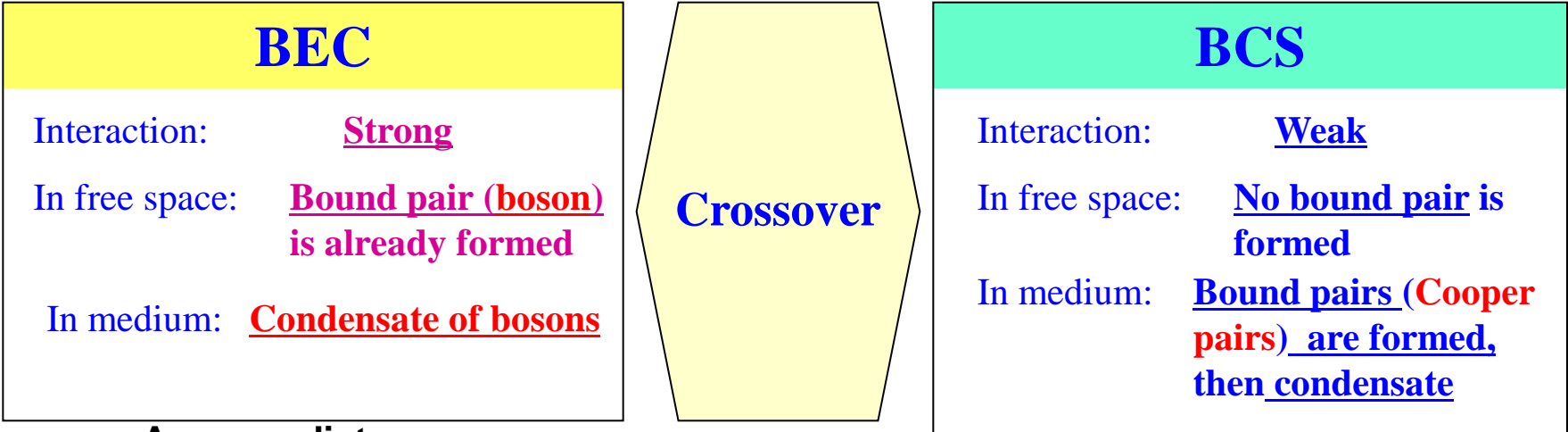
$\rho/\rho_0 = 10^{-4} \sim 2 \times 10^{-1}$



MM, PRC73,044309(2006)

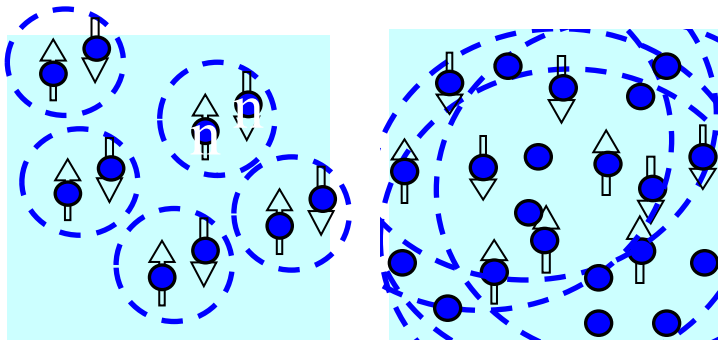
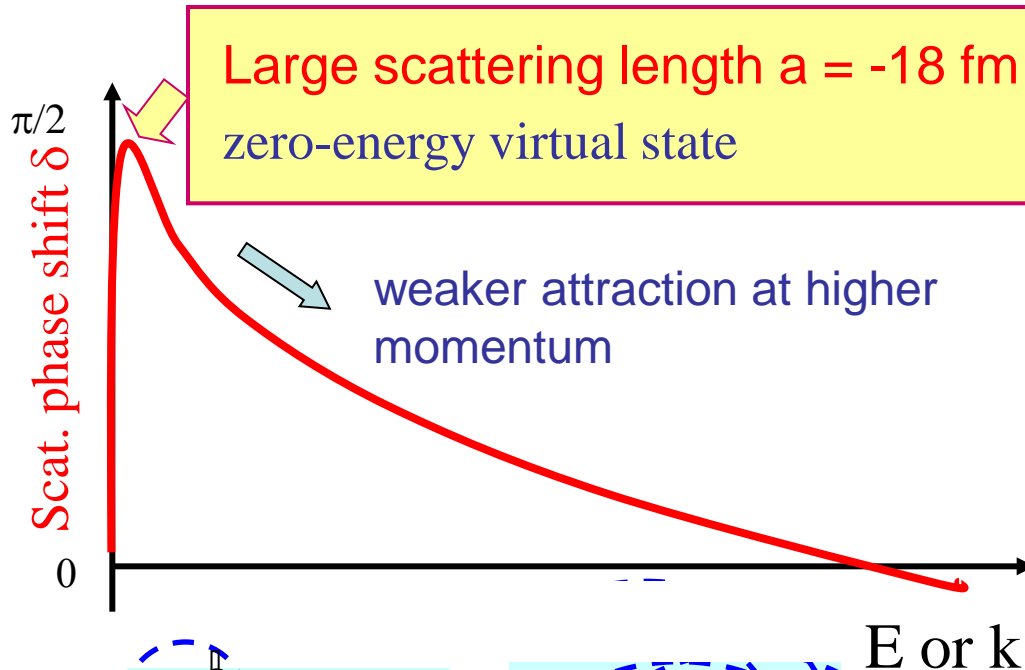
# Bose condensation and strong coupling pairing: BCS-BEC crossover 5

Leggett 1980, Nozieres & Schmitt-Rink 1985



# nn interaction $^1S$ has a large scattering length at low momentum (low density)

$^1S$  phase shift



Strong coupling

$$k_F a > -1$$

may be realized for

$$k_F > 0.05 \text{ fm}^{-1}$$

$$\rho/\rho_0 > 10^{-4}$$

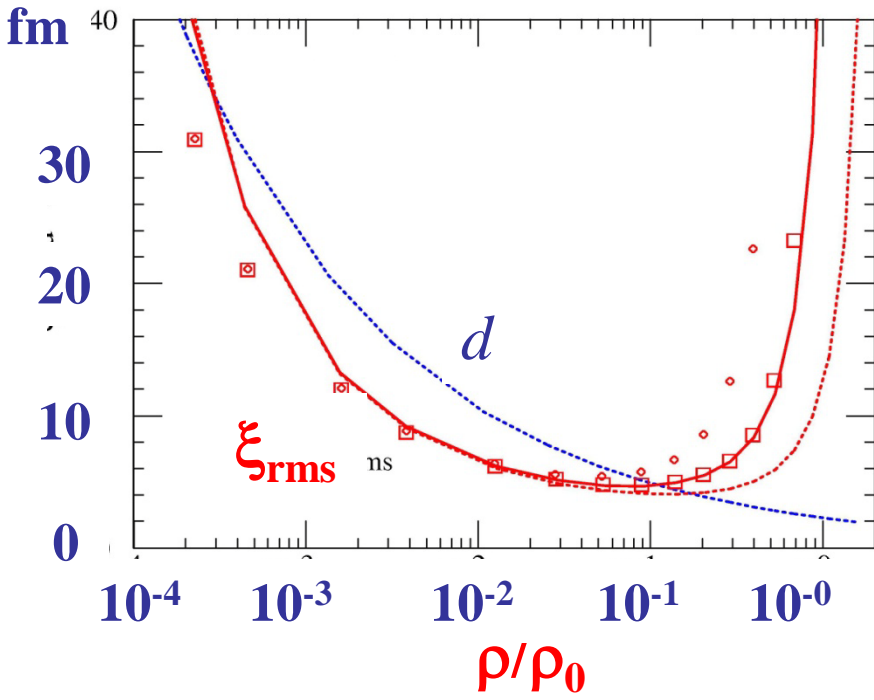
but not for

$$\rho/\rho_0 \sim 1$$

# Neutron Cooper pair in dilute matter

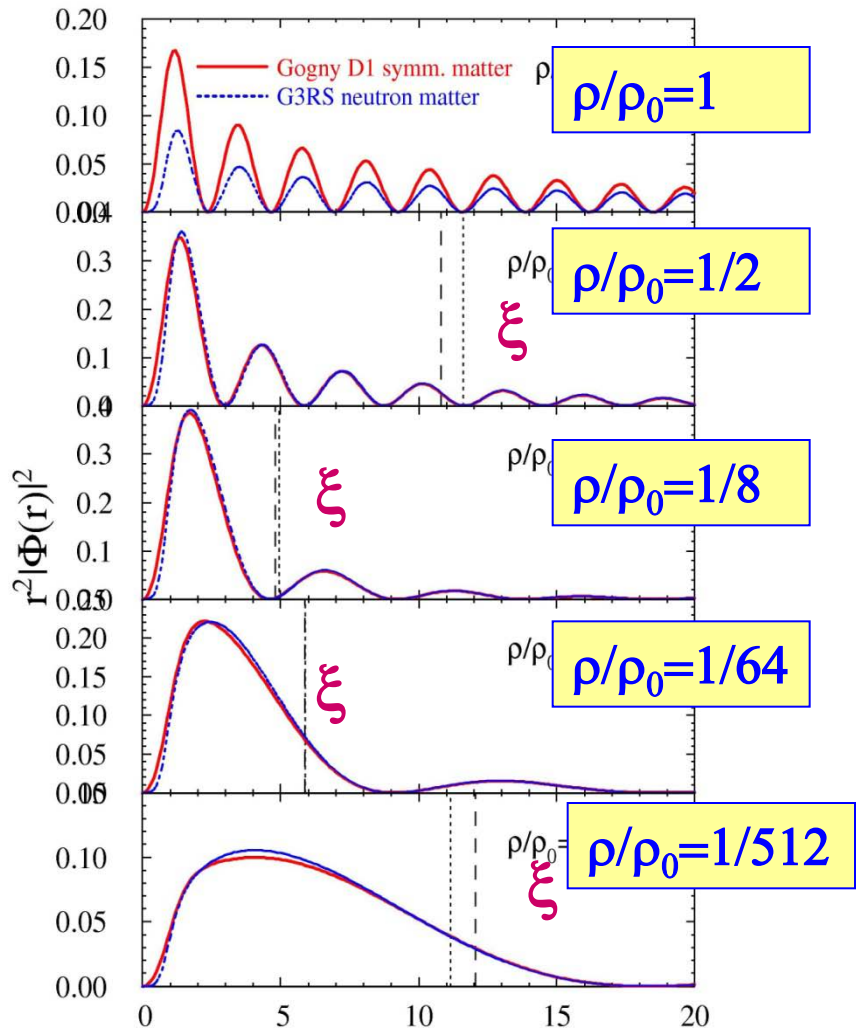
BCS calculation using  $\xi$  A bare force (G3RS)  
 $\xi$  Gogny force (D1)

## Neutron pairing gap



Pair wave function has large amplitude at short relative distances  $r \sim 2-3$  fm

## Cooper pair wave function



Relative distance  $r$  (fm) between paired neutrons

MM, PRC73,044309(2006)

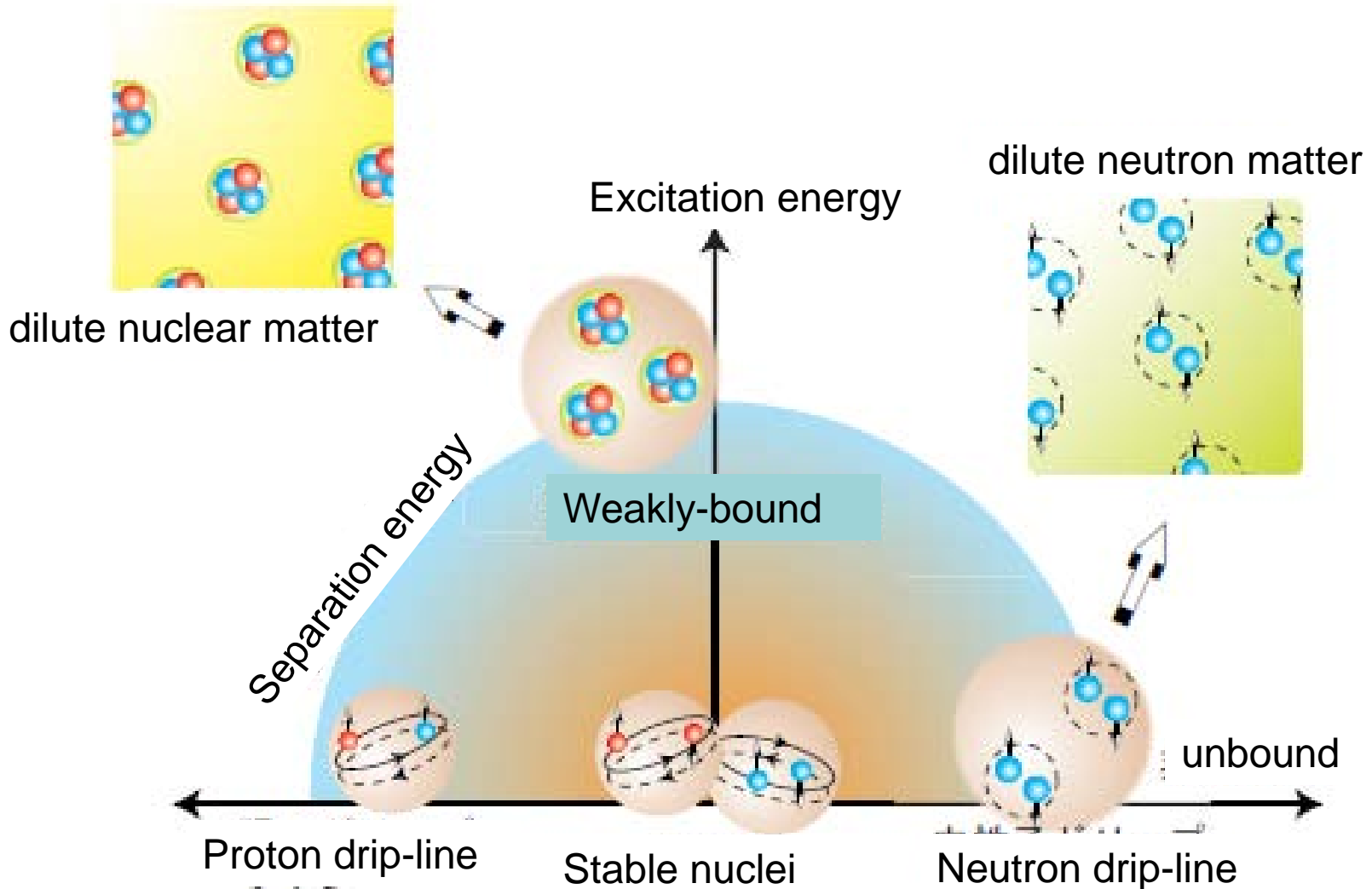
Margueron et al, PRC77,054309(2008)

# Question 1

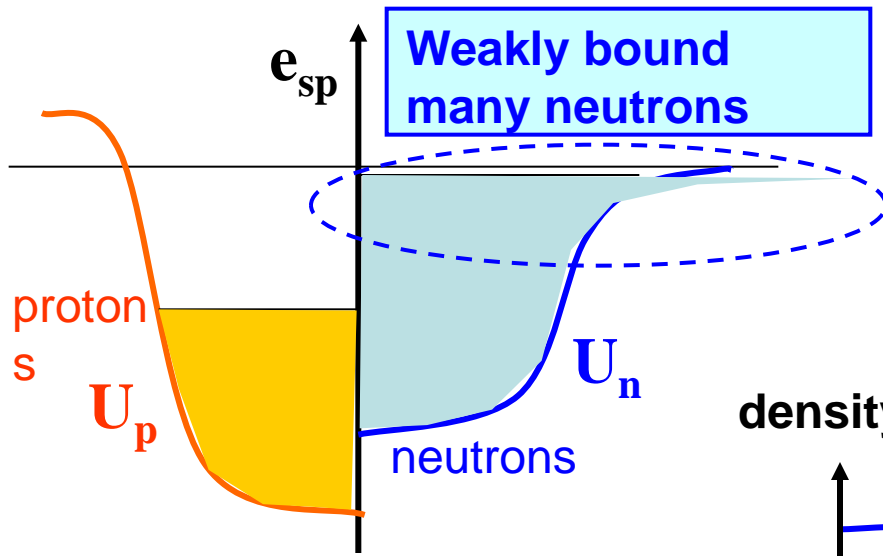
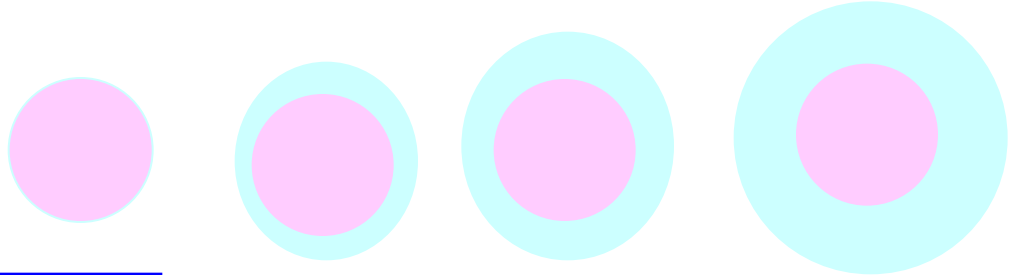
***Do nuclei, in particular, neutron-rich nuclei exhibit features of the strong-coupling, e.g., the spatial correlation in Cooper pairs?***



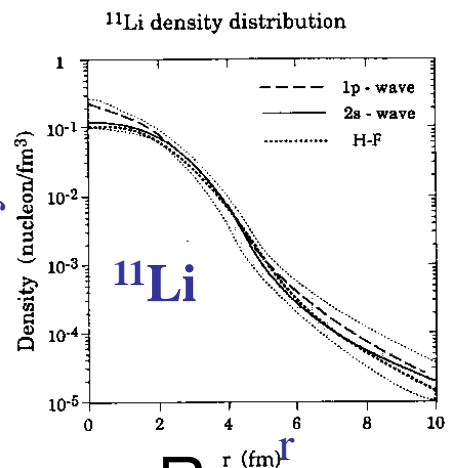
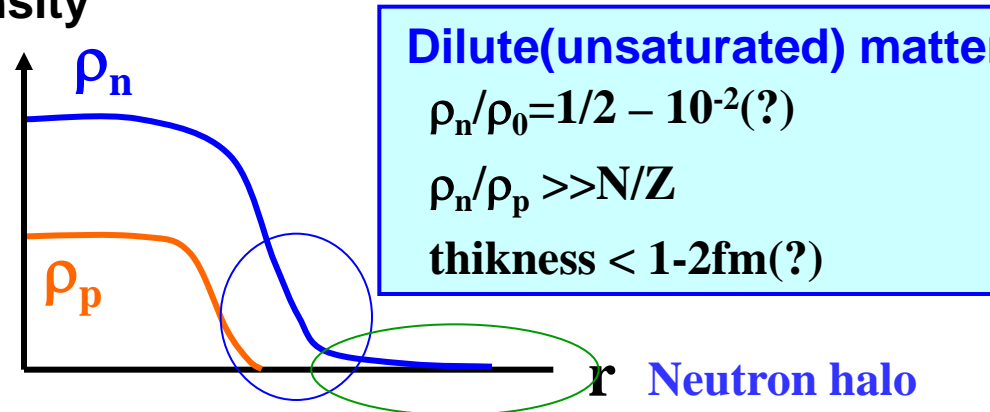
# Dilute matter vs. Weakly-bound nuclei / excited states near separation energies



# “Dilute matter” in n-rich nuclei



**Neutron skin**



Tanihata et al. PLB287 (1992)

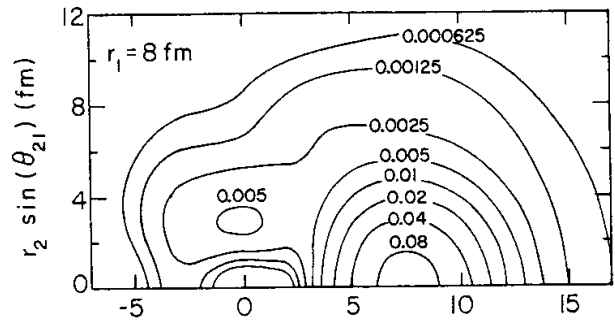
# Spatial pair correlation & surface-enhanced pairing in n-rich nuclei

neutron skin nuclei in medium mass region

**Spatially compact pair in nuclei**

2n-halo nucleus  $^{11}\text{Li}$

G.F.Bertsch, H.Esbensen, Ann. Phys. 209(1991) 327

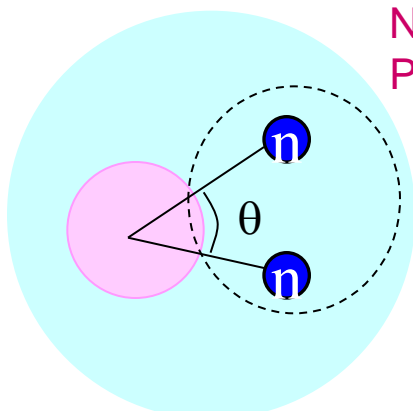


**Recent Coulomb break-up exp.**

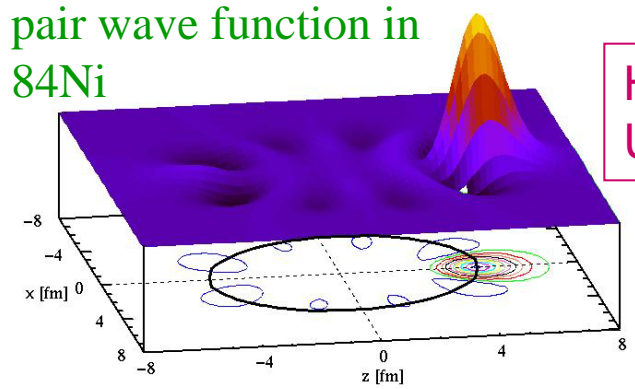
Nakamura et al.  
PRL30,252502 (2006)

$$\theta_{nn} = 48^{+14}_{-18} \text{ deg}$$

$$R_{c,2n} = 5.01 \pm 0.32 \text{ fm}$$



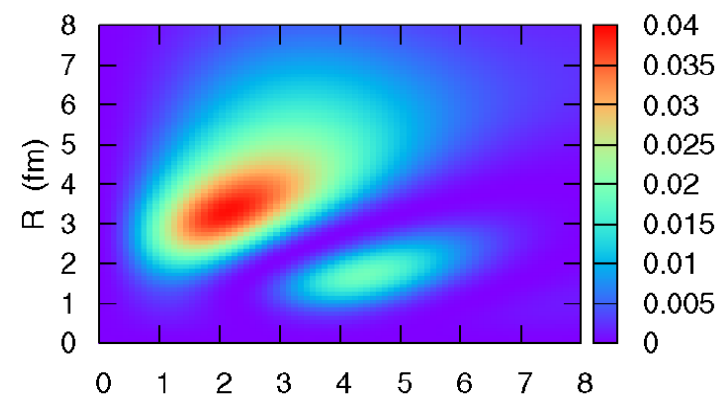
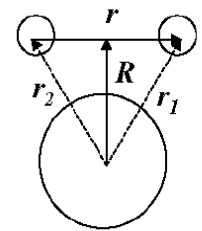
pair wave function in  $^{84}\text{Ni}$



HFB calc.  
Using DDDI

Matsuo et al PRC71,064326(2005)  
Pillet, Sandulescu, Schuck, PRC76, 024310 (2007)

K.Hagino et al., Phys.Rev. Lett.99, 022506(2007)



# Strong coupling pairing & Spatial correlation in Cooper pair

Closed-core +  $2N$  e.g.  $^{210}\text{Pb}$

Ibarra et al. NPA288, (1977)

Janouch & Liotta PRC27 (1983)

etc

Slab

Kanada-En'yo et al. PRC79, (2009)

Semi-infinite matter

Pankratov et al. PRC79, (2009)

pn pairing in symmetric matter

Alm et al. NPA551, (1993)

Baldo et al. PRC52, (1995)

etc

# Density functional theories: a link between matter and nuclei

The Skyrme functional
Pair correlation energy functional

$$E = E_{Skyrme}[\rho, \vec{\nabla}\rho, \Delta\rho, \tau, \vec{j}, \vec{s}, \vec{J}] + E_{pair}[\rho, \tilde{\rho}, \tilde{\rho}^*]$$

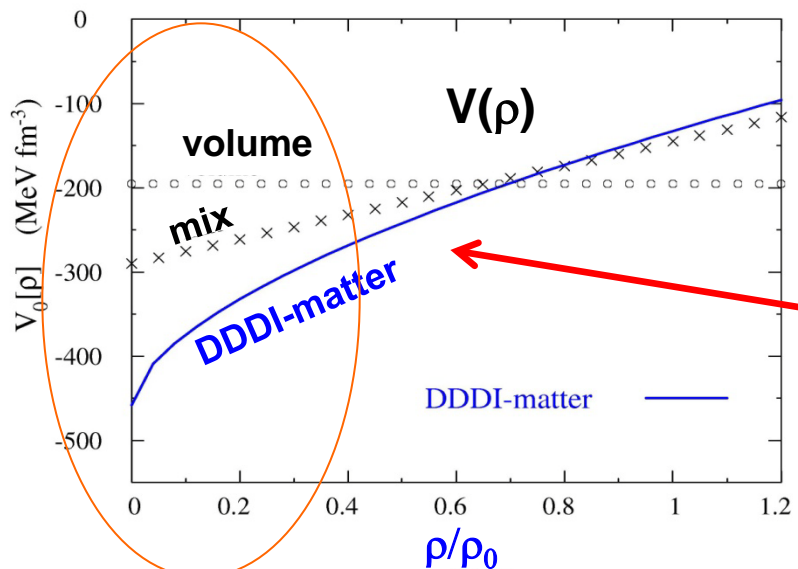
**A simple functional DDDI (density-dependent delta interaction)** Esbensen-Bertsch 1999, Garrido et al 1999, Matsuo 2006

$$E_{pair}[\rho, \tilde{\rho}, \tilde{\rho}^*] = \int d\vec{r} V_q[\rho_n, \rho_p] \tilde{\rho}(\vec{r}) \tilde{\rho}^*(\vec{r})$$

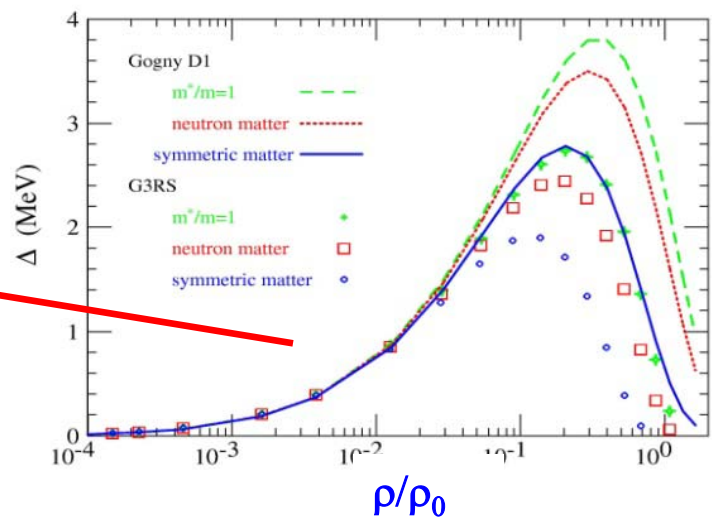
Pair density

$$\tilde{\rho}(\vec{r}) = \langle HFB | \psi^+(\vec{r} \uparrow) \psi^+(\vec{r} \downarrow) | HFB \rangle$$

Density dependence is flexibly incorporated as a simple functional



Gap in neutron matter



# Pairing functional at work

## DDDI-matter, reproducing matter pair gap $\Delta_{\text{matter}}$ (BCS)

DDDI a-18

$$V_n[\rho_n] = v_0 \left( 1 - \frac{0.845(\rho_n / 0.08)^{0.59}}{0.71} \right)$$

DDDI-mix

Linear weak dependence on density

$$V_n[\rho_n, \rho_p] = v_0 \left( 1 - 0.5(\rho_n + \rho_p / 0.16)^{0.71} \right)$$

DIDI-volume

No density dependence

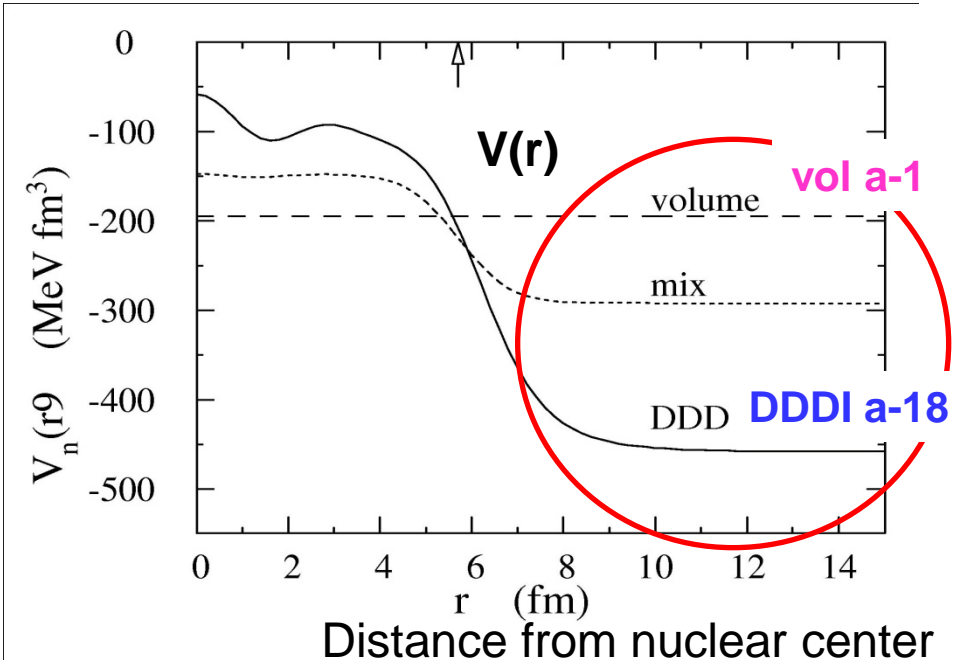
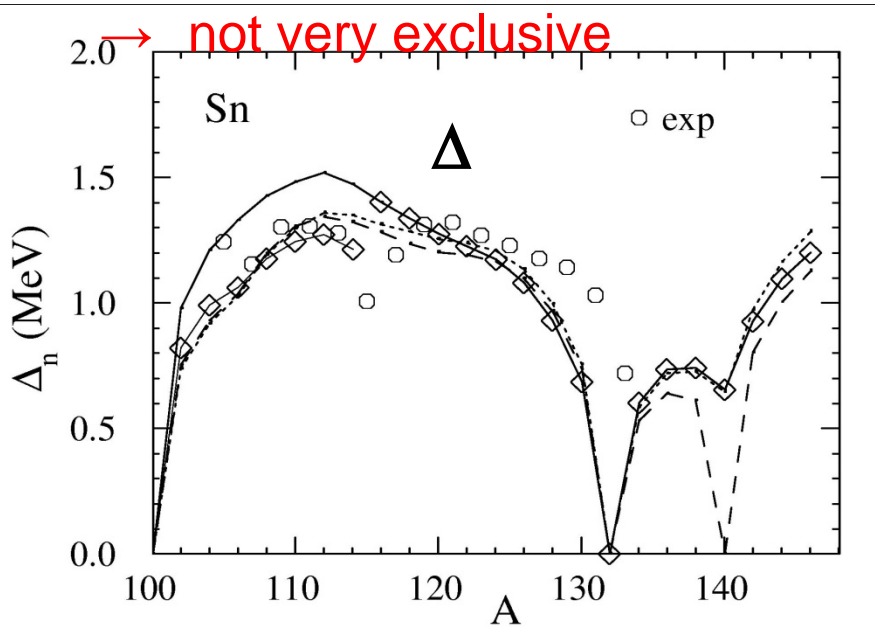
vol a-1

$$V_n[\rho_n, \rho_p] = v_0$$

$v_0 = -458.4 \leftarrow a_{nn} = -18.5 \text{ fm}$   
 $v_0 = -292 \leftarrow a_{nn} = -1.4 \text{ fm}$   
 $v_0 = -195 \leftarrow a_{nn} = -0.63 \text{ fm}$

Different low-density limit

## average pair gap in Sn istopes



# Spatial correlation at surface of n-rich nuclei

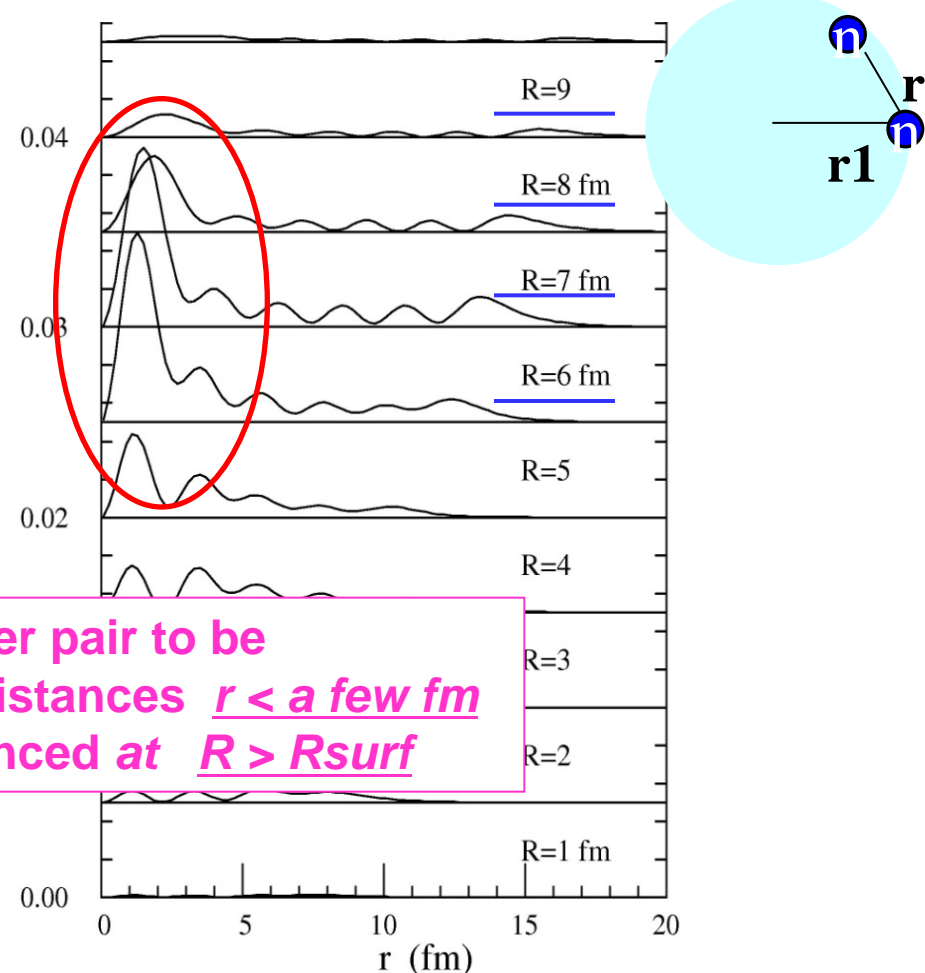
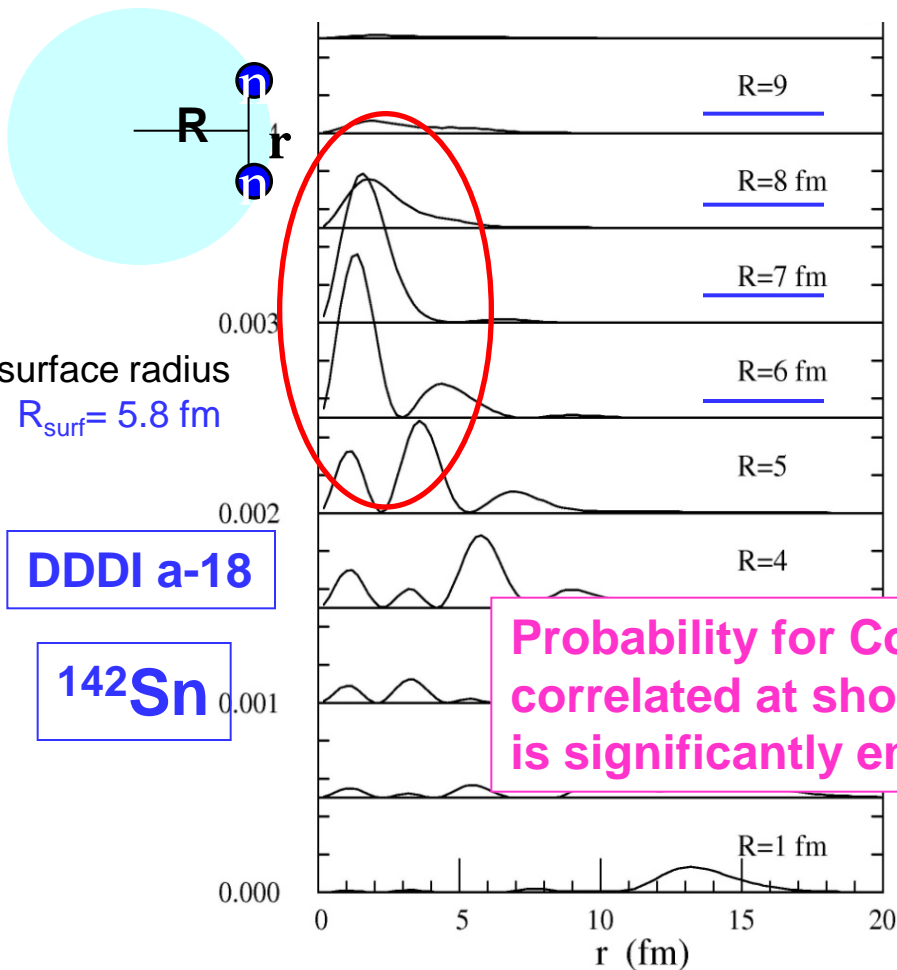
Cooper pair wave function  $\Psi_{pair}(\vec{r}_1, \vec{r}_2) = \langle HFB | \psi^+(\vec{r}_1 \uparrow) \psi^+(\vec{r}_2 \downarrow) | HFB \rangle$

Probability distribution 1

$$P_c(R, r) = R^2 r^2 \int d\Omega |\Psi_{pair}(\vec{R} - \frac{\vec{r}}{2}, \vec{R} + \frac{\vec{r}}{2})|^2$$

Probability distribution 2

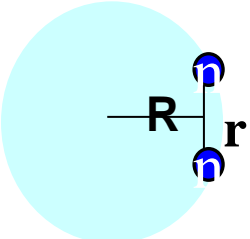
$$P_1(r_1, r) = r_1^2 r^2 \int d\Omega |\Psi_{pair}(\vec{r}_1, \vec{r}_1 + \vec{r})|^2$$



# Sensitivity to the low-density pairing

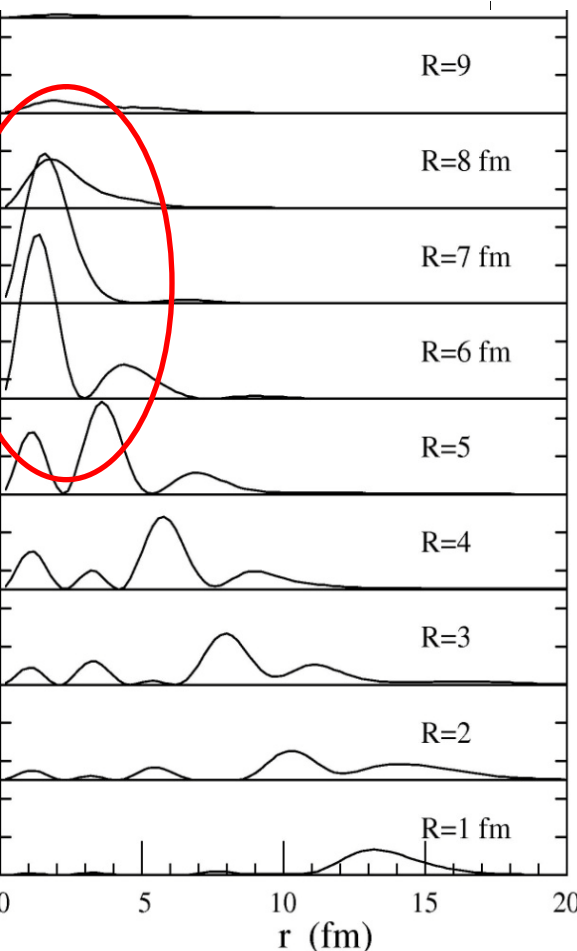
## Probability distribution 1

$$P_c(R, r) = R^2 r^2 \int d\Omega |\Psi_{pair}(\vec{R} - \frac{\vec{r}}{2}, \vec{R} + \frac{\vec{r}}{2})|^2$$

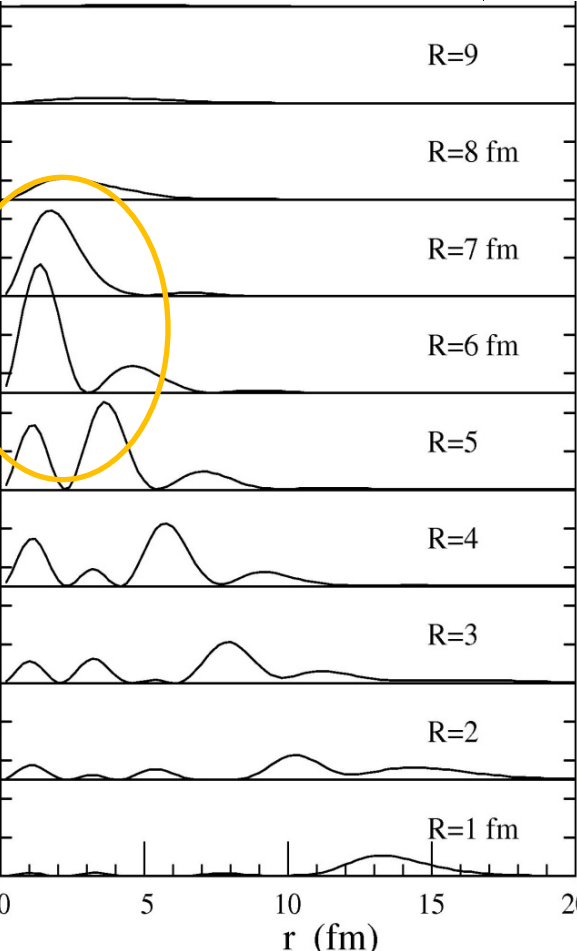


$^{142}\text{Sn}$

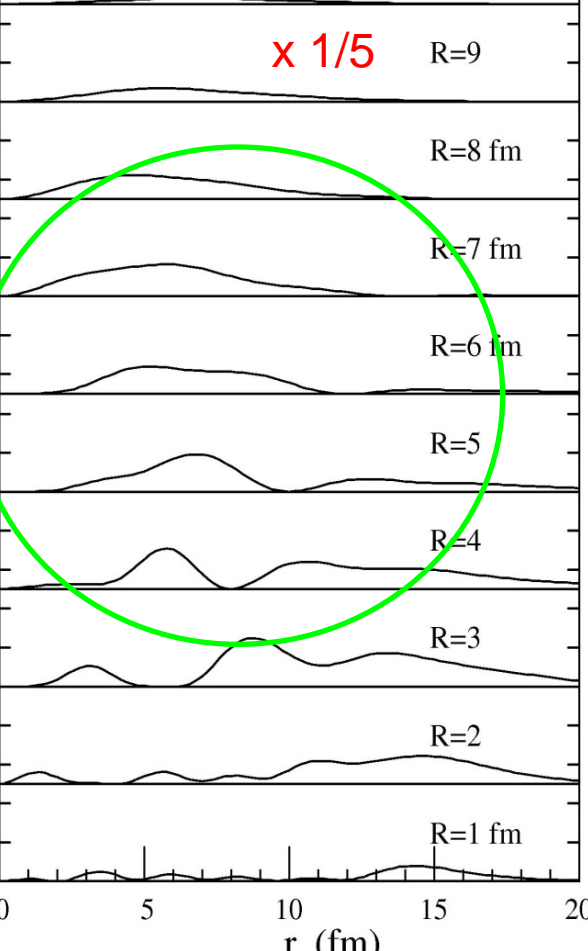
DDDI a-18



volume a-1



Single-j J=0 pair  $(p_{3/2})^2$

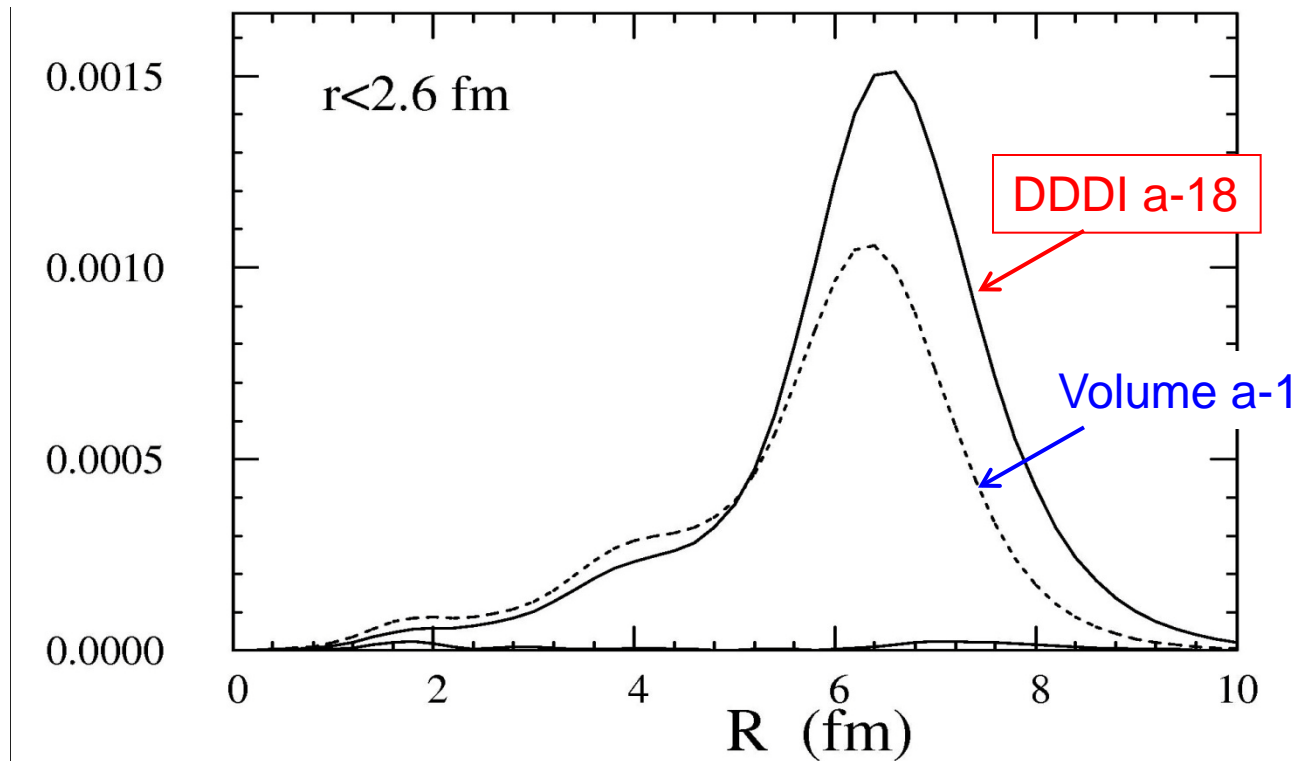
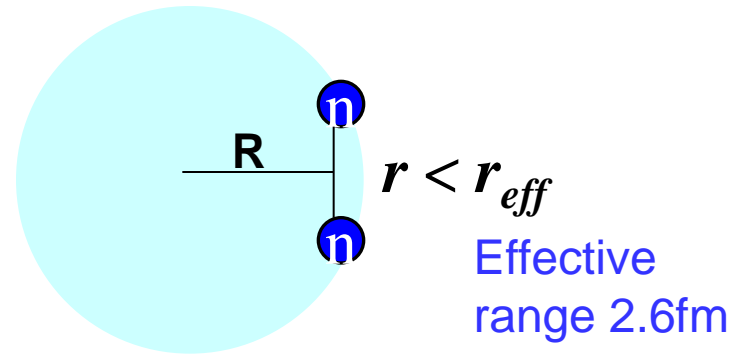




# Pair contact probability $r < 2.6$ fm

Probability of pair at short relative distances within the interaction range

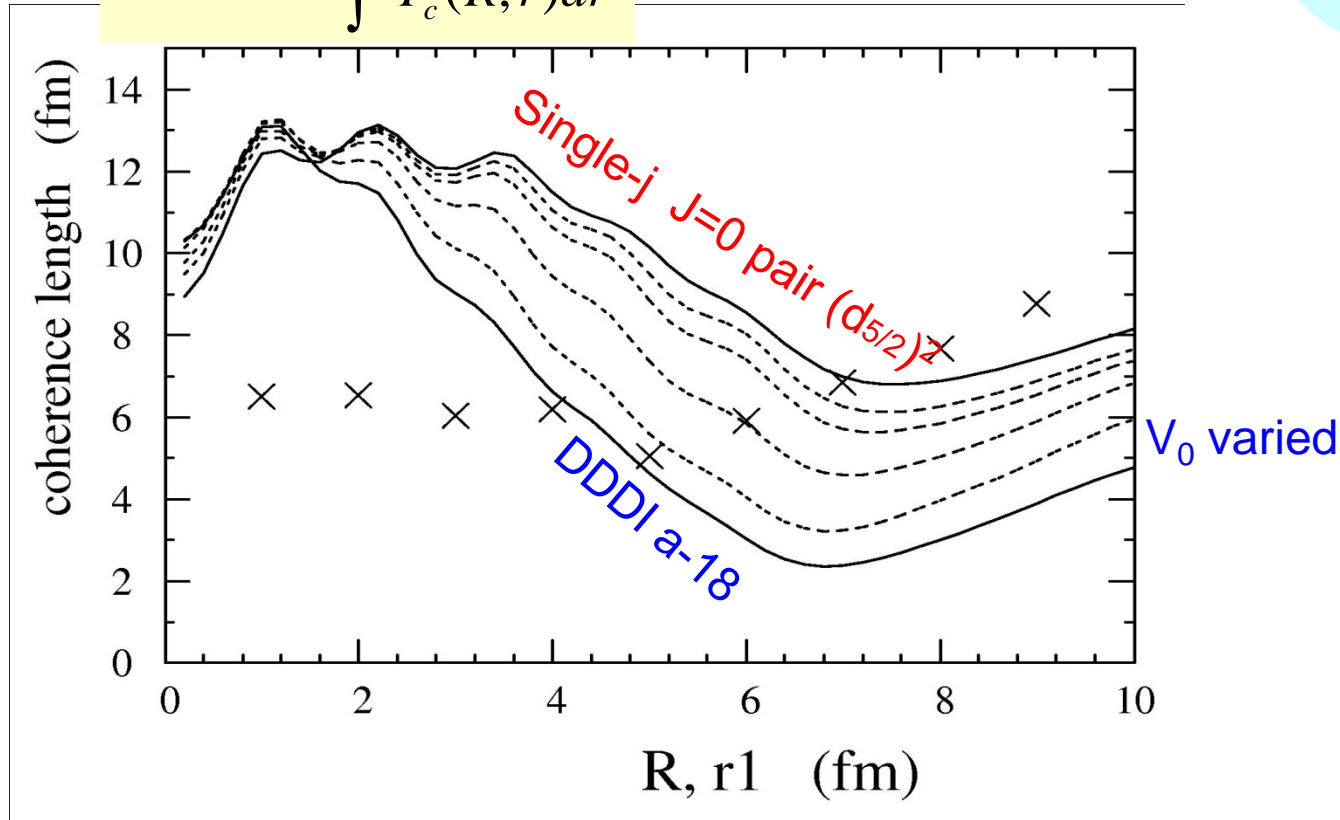
$$p(R) = \int_0^{r_{eff}} P_c(R, r) dr$$



# Comment on coherence length

Rms radius of 'Cooper pair' as a function of R

$$\xi = \langle r^2 \rangle = \frac{\int r^2 P_c(R, r) dr}{\int P_c(R, r) dr}$$



1. It is influenced by the finite size effect

2. And also depends on choices of coordinate.

3. But still it reflects the spatial correlation to some extent.

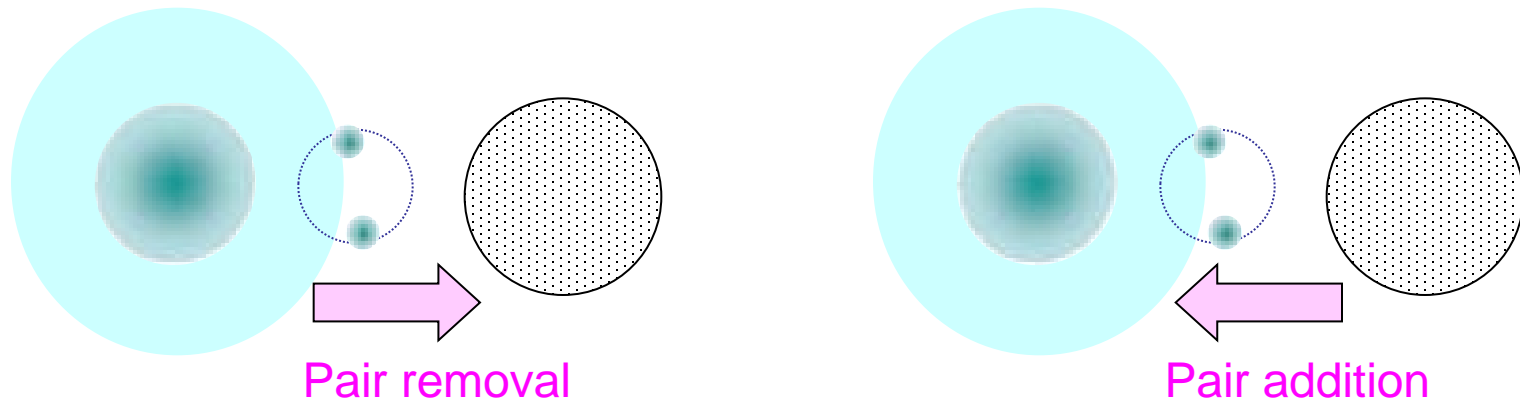
[1] Pillet, Sandulescu, Schuck, Berger, PRC81 (2010)

[2] Hagino et al J Phys.G37 (2010)

# Question 2

***How can we probe the features of the strong coupling pairing in the surface? the strong spatial correlation in the surface?***

***Pair transfers in neutron-rich nuclei***



Pair transfer process, for Sn isotopes,  **$^{132}\text{Sn}$**

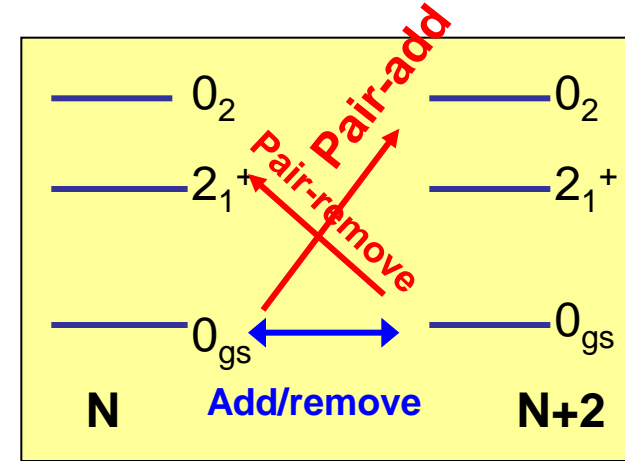
***Anomalous  $0^+$  state, a new kind of pair vibration***

# Pair correlation and pair transfer in heavier n-rich<sub>20</sub> nuclei: Skyrme HFB + QRPA approach

Khan, Grasso, Margueron, PRC80, 044328 (2009)

Matsuo, Serizawa, arXiv:1007.1705

Shimoyama, Matsuo, in preparation



## 1. ground-ground pair transfer

$0_{gs}$ - $0_{gs}$  pair-add/remove transition density

Hartree-Fock-Bogoliubov mean-field calc.

$$\langle 0_{gs}, N \pm 2 | \psi^+(\vec{r} \uparrow) \psi^+(\vec{r} \downarrow) | 0_{gs}, N \rangle \approx \langle 0_{gs} | \psi^+(\vec{r} \uparrow) \psi^+(\vec{r} \downarrow) | 0_{gs} \rangle = \tilde{\rho}(\vec{r})$$

Pair density in the ground state

## 2. Pair transfer to **excited $0^+, 2^+$ states**

$0_{gs}$ - $2^+$  pair-add transition density

$$\langle 2^+, N + 2 | Y_{2M} \psi^+(\vec{r}) \psi^+(\vec{r}) | 0_{gs}, N \rangle$$

$0_{gs}$ - $2^+$  pair-remove transition density

$$\langle 2^+, N - 2 | Y_{2M} \psi(\vec{r}) \psi(\vec{r}) | 0_{gs}, N \rangle$$

QRPA calc. of excitation modes

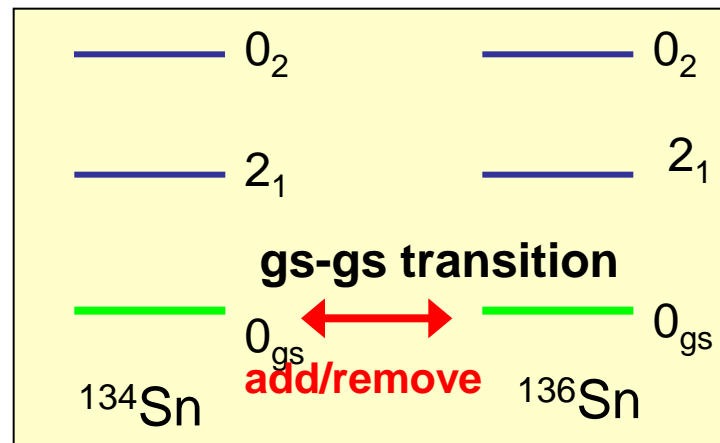
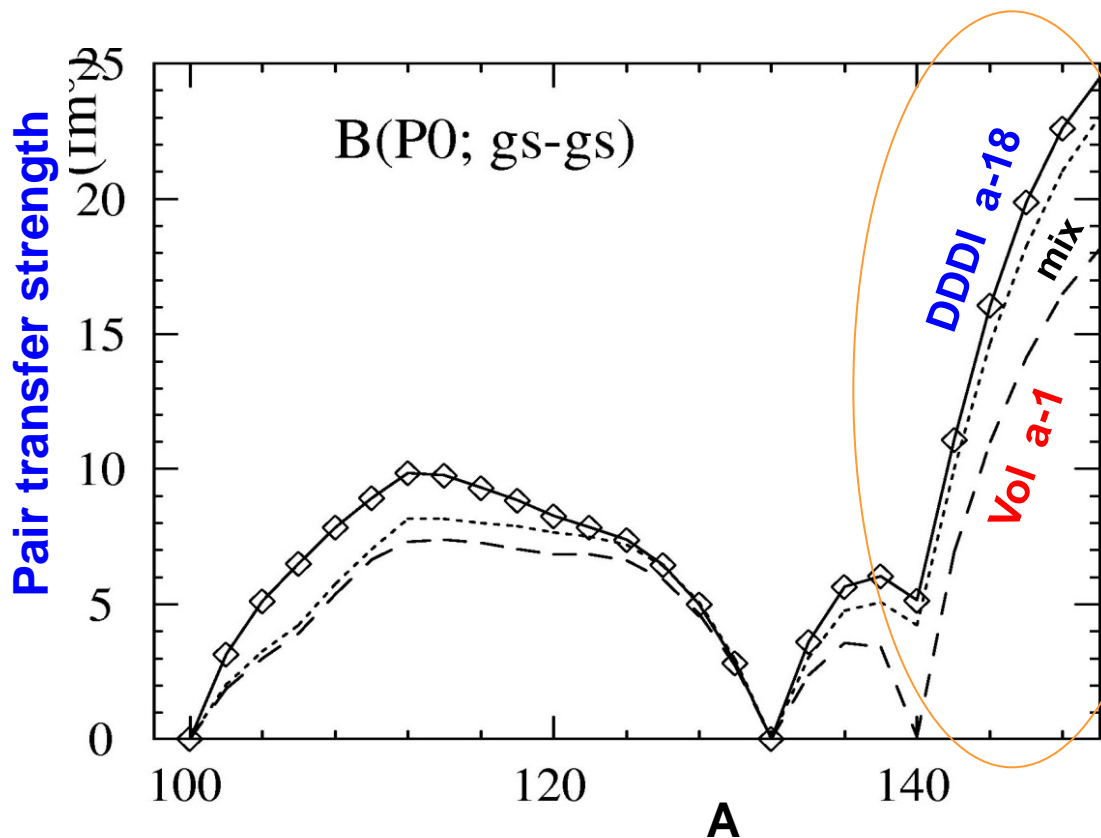
The same Skyrme, but Landau-Migdal approximation to residual ph int.

# $0_{gs}-0_{gs}$ pair transfer strength in $>^{132}\text{Sn}$

2n-add/removal transfer strength

$$B(P0; 0_{gs} N \rightarrow 0_{gs} N \pm 2) = M_{gg}^2$$

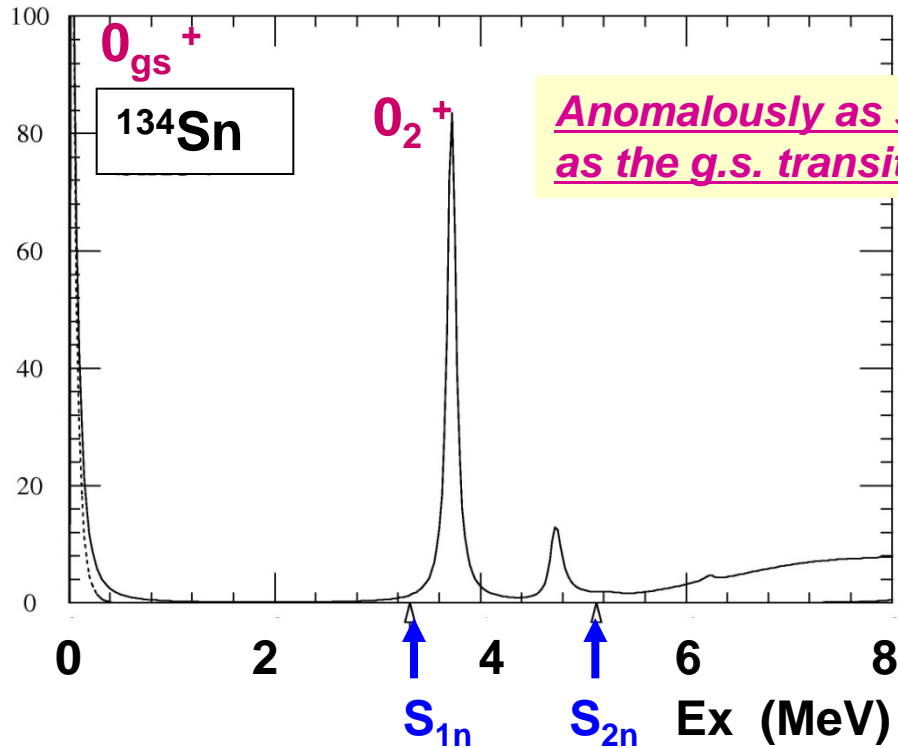
$$M_{gg} = \langle 0_{gs} | \int Y_{00} \psi^+(\vec{r}) \psi^+(\vec{r}) d\vec{r} | 0_{gs} \rangle = 4\pi \int \tilde{\rho}(\vec{r}) r^2 dr$$



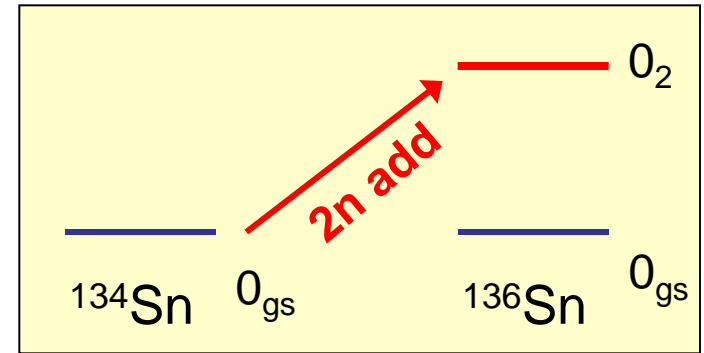
- Ground-ground 2n transfer is significantly increased in very n-rich isotopes ( $>^{140}\text{Sn}$ )**
- Sensitivity to the correlation at  $R > R_{surf}$**

# Anomalous $0_2^+$ pair transfer in $^{132-140}\text{Sn}$

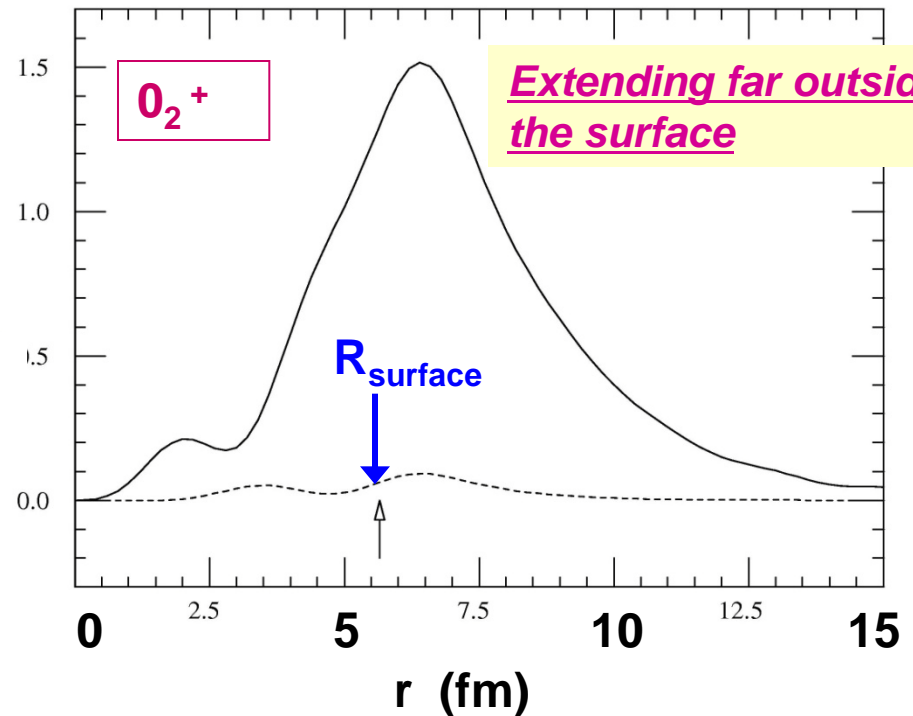
## Pair-addition strength function



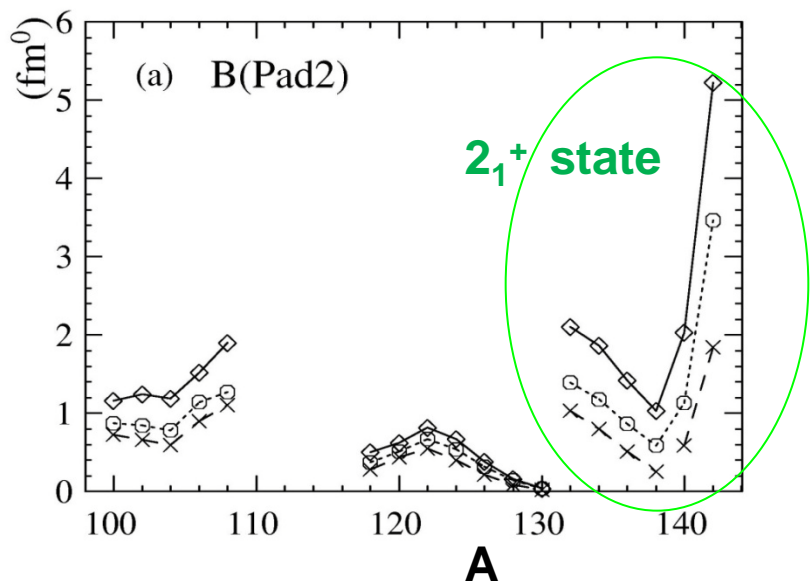
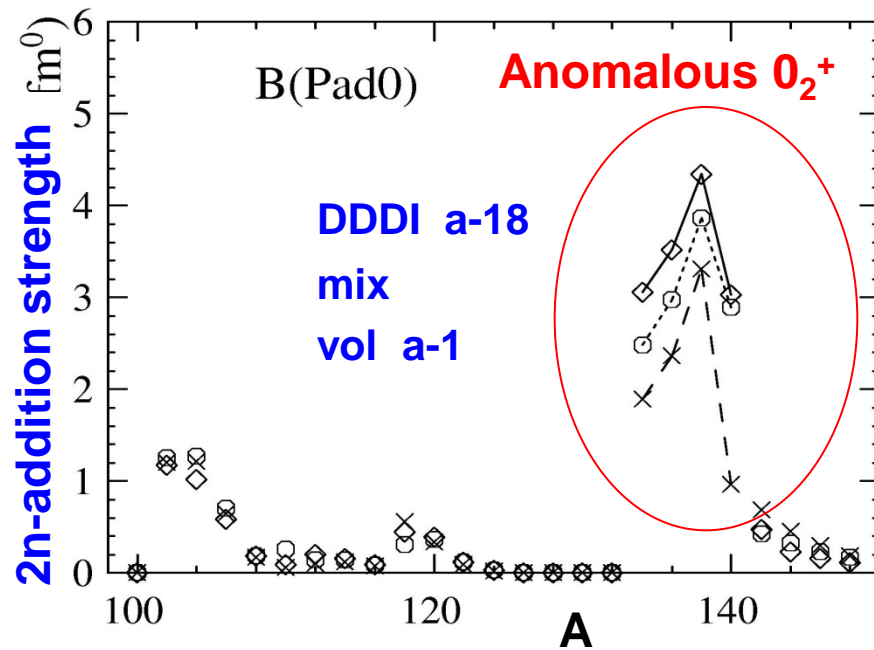
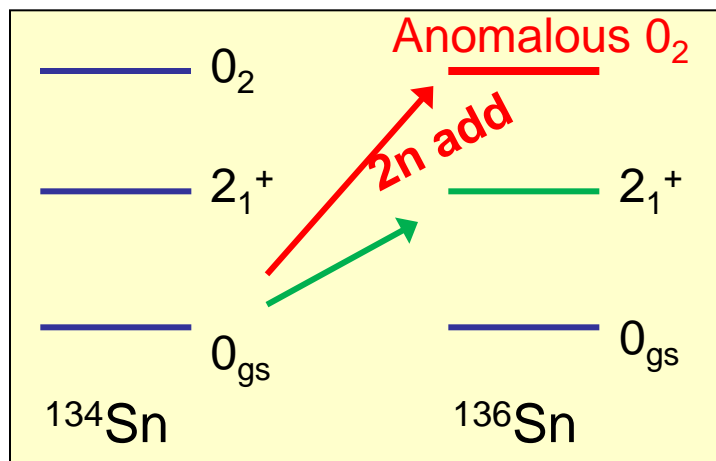
*These features are very different from those of the pair vibration in stable nuclei*



## Pair-addition transition density



# Pair-addition transfers to $0_2$ and $2_1$ states in $^{132-140}\text{Sn}$ may provide probes



*2n-add transfer to the anomalous  $0_+$  and  $2_+$  provides is sensitive to the surface correlation*

# Another probe?

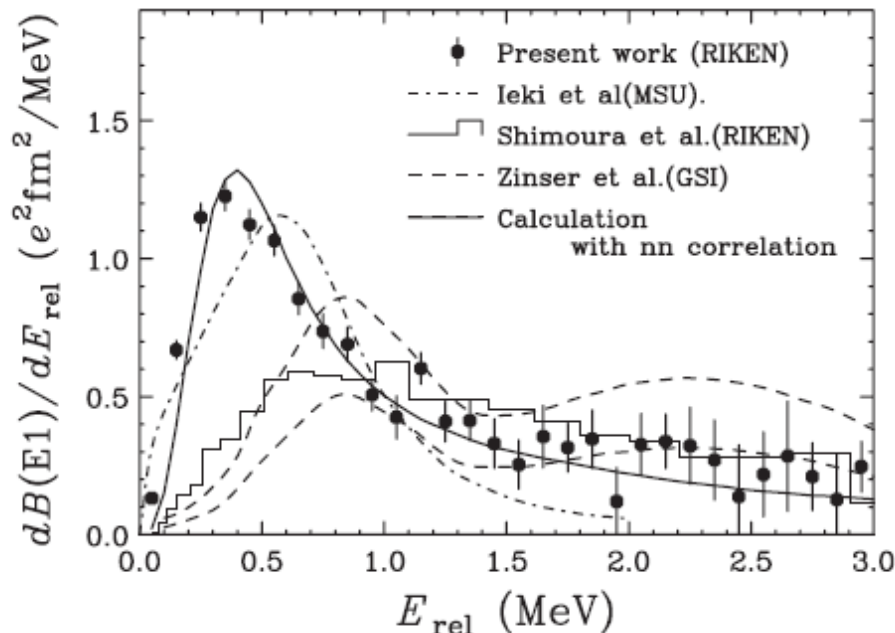
## Soft neutron modes above $S_{2n}$

RIKEN exp. Nakamura et al.  
PRL30,252502 (2006)

2n break-up through soft dipole excitation in nuclei near n-drip line

2n-halo nuclei

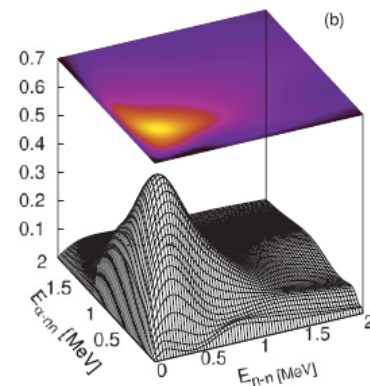
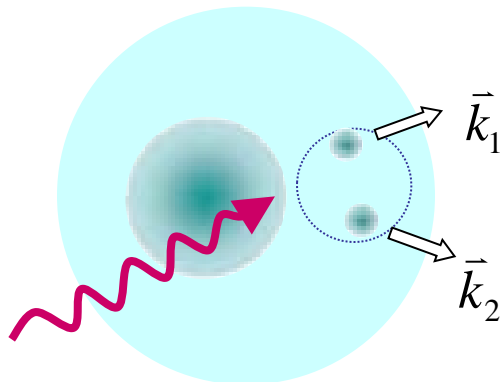
$^{11}\text{Li}$ ,  $^6\text{He}$ , etc.



## 2n correlation ??

Hagino et al., PRC80, 031301 (2009)

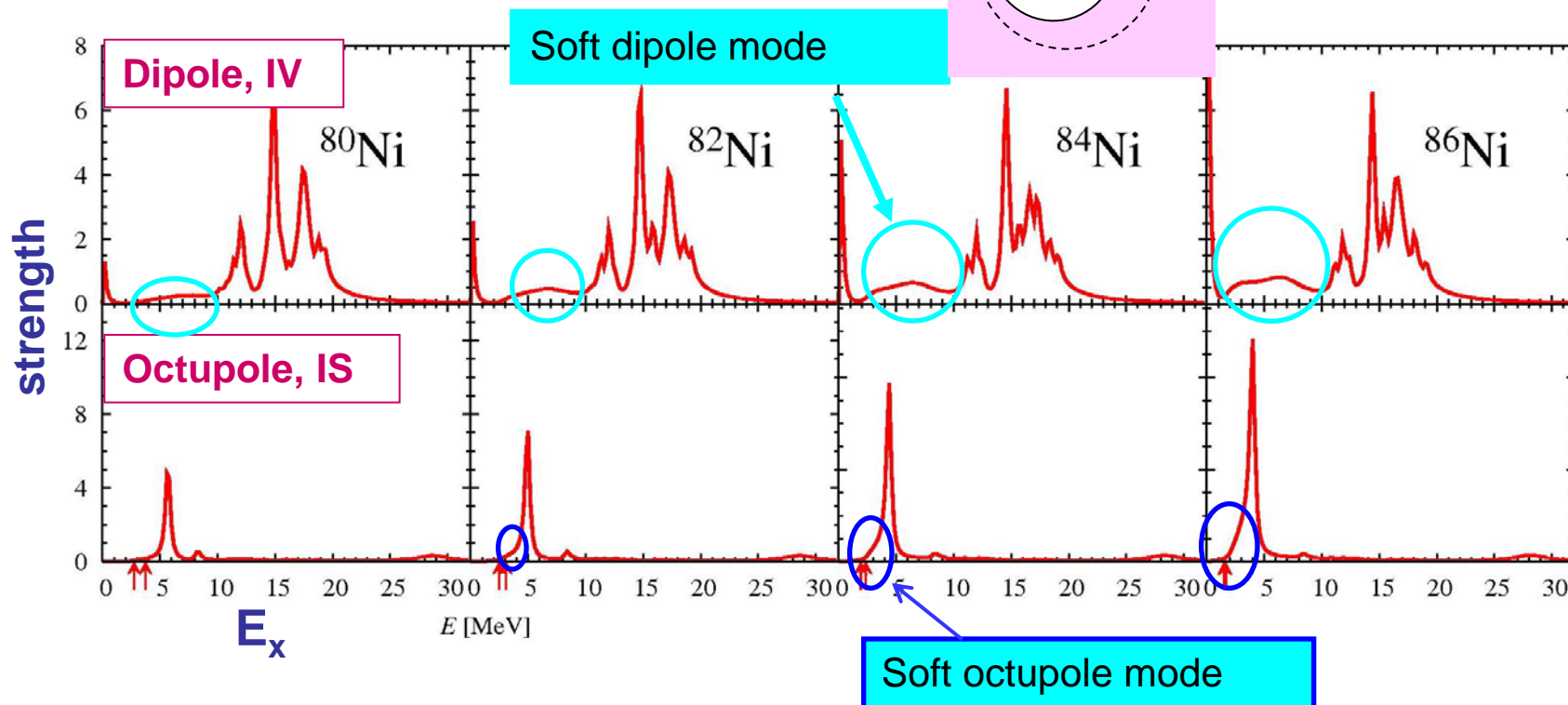
Kikuchi et al., PRC81, 044308 (2010)





# Soft neutron modes above $S_{2n}$ in skin nuclei

$S_{1n} \sim S_{2n} \sim < 2-3 \text{ MeV}$



Continuum QRPA with SLy4 & mix pairing

Serizawa, Matsuo, PTP121 (2009)

# Conclusions

- ***Pairing properties of dilute neutron matter*** are interesting.
- Neutron rich-nuclei appear to share those properties to some extent, especially in ***neutron correlations in the surface region***.

Spatial correlation of Cooper pair at short distances,  
e.g. the pair contact probability.

- There exist a few ideas to probe them via neutron-rich nuclei.
  - ***Two neutron transfers*** in neutron-rich  $>^{132}\text{Sn}$  isotopes.
    - ◆ ***Anomalous  $0_2^+$  states*** in  $^{134--140}\text{Sn}$ .
    - ◆  ***$2_1^+$  states*** in  $^{134--140}\text{Sn}$ .
    - ◆ ***Enhanced gs-gs transfer*** in  $^{132--140}\text{Sn}$ .
  - 2n correlation in 2n break-up via soft modes → on the way
- ***We await experimental data***