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Nuclear Symmetry Energy and its Effect on Neutron Stars

S. Nishizaki

Faculty of Humanities and Social Sciences,
Iwate University, Morioka 020-8550, Japan

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4. Summary

1. Introduction

- The nuclear symmetry energy a_{sym} plays an important role to determine the composition of nuclear matter (NM).

$$E(N,Z)/A = a_{\text{vol}} + a_{\text{sym}}(N-Z)^2/A^2 \\ + a_{\text{surf}}A^{-1/3} + a_{\text{Coul}}Z^2/A^{4/3} + \dots$$

a_{vol} : negative

a_{sym} , a_{surf} , a_{Coul} : positive

1) $a_{\text{sym}} = 0$

$Z = 0$ i.e. pure neutron matter

2) $a_{\text{sym}} = \text{finite}$, neglecting a_{Coul}

$Z = N$ i.e. symmetric nuclear matter

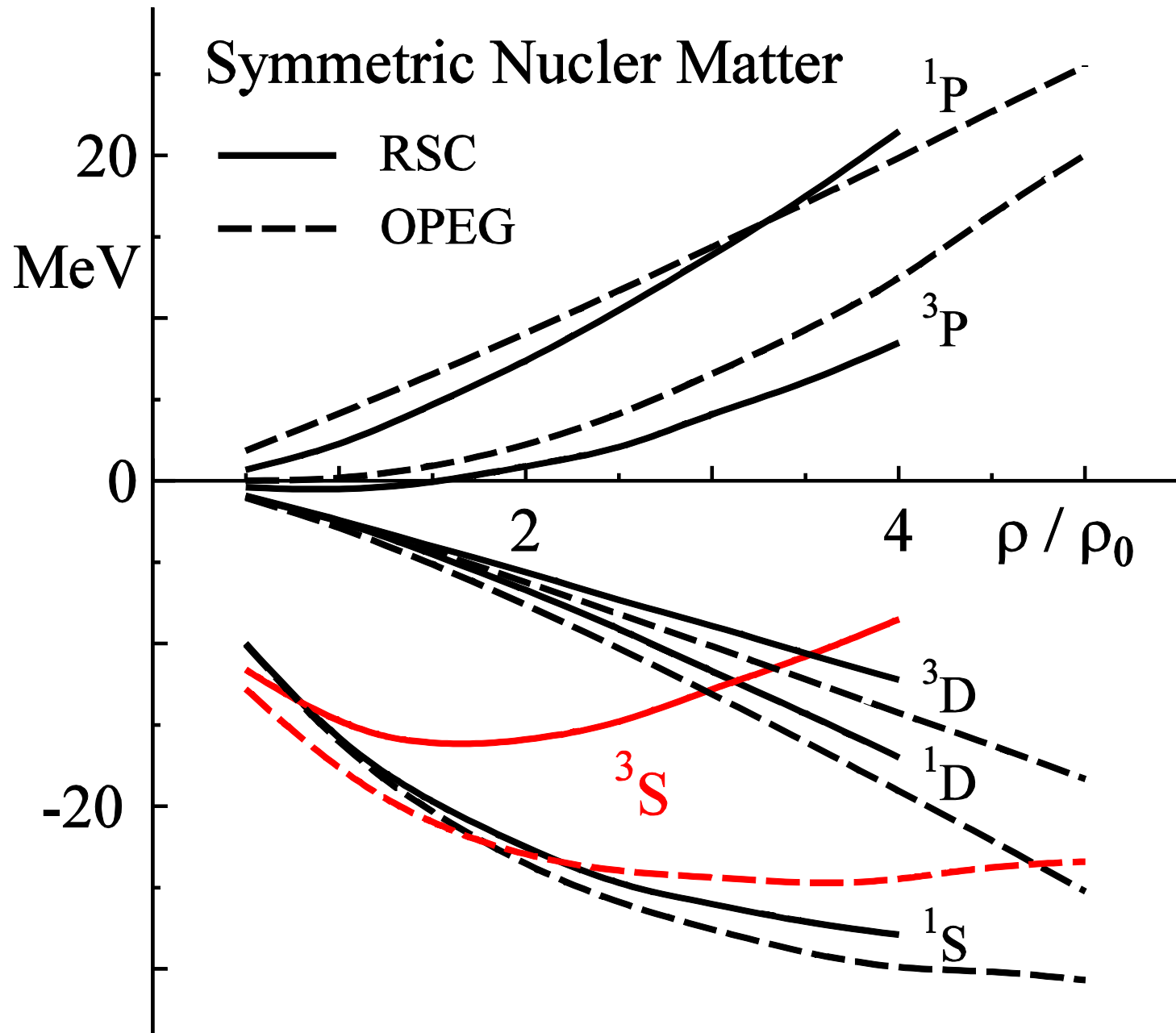
3) composition of neutron stars (nucleons,
hyperons, leptons)

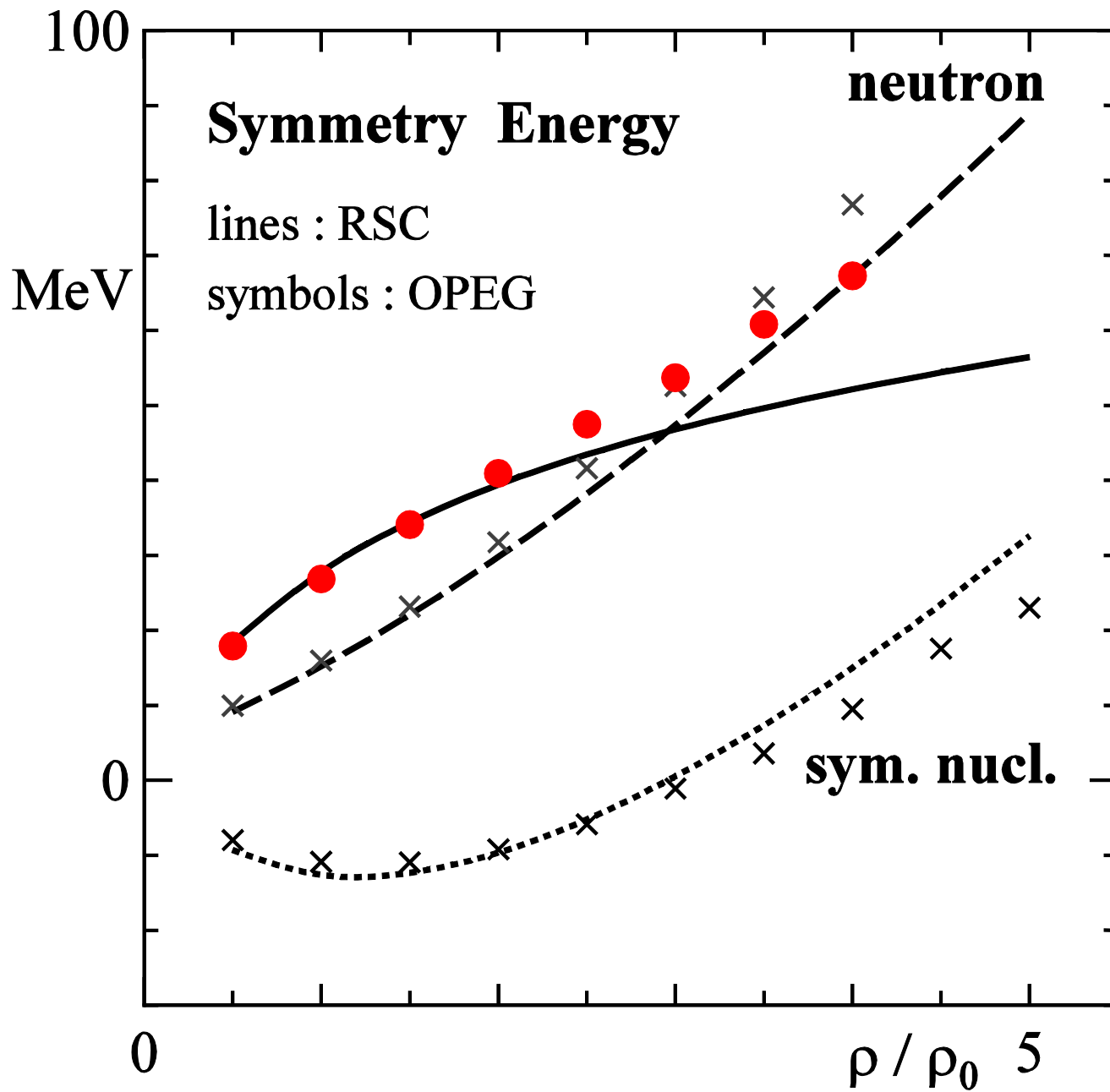
equation of state (EOS)

→ properties of neutron stars

2. Nuclear Symmetry Energy a_{sym}

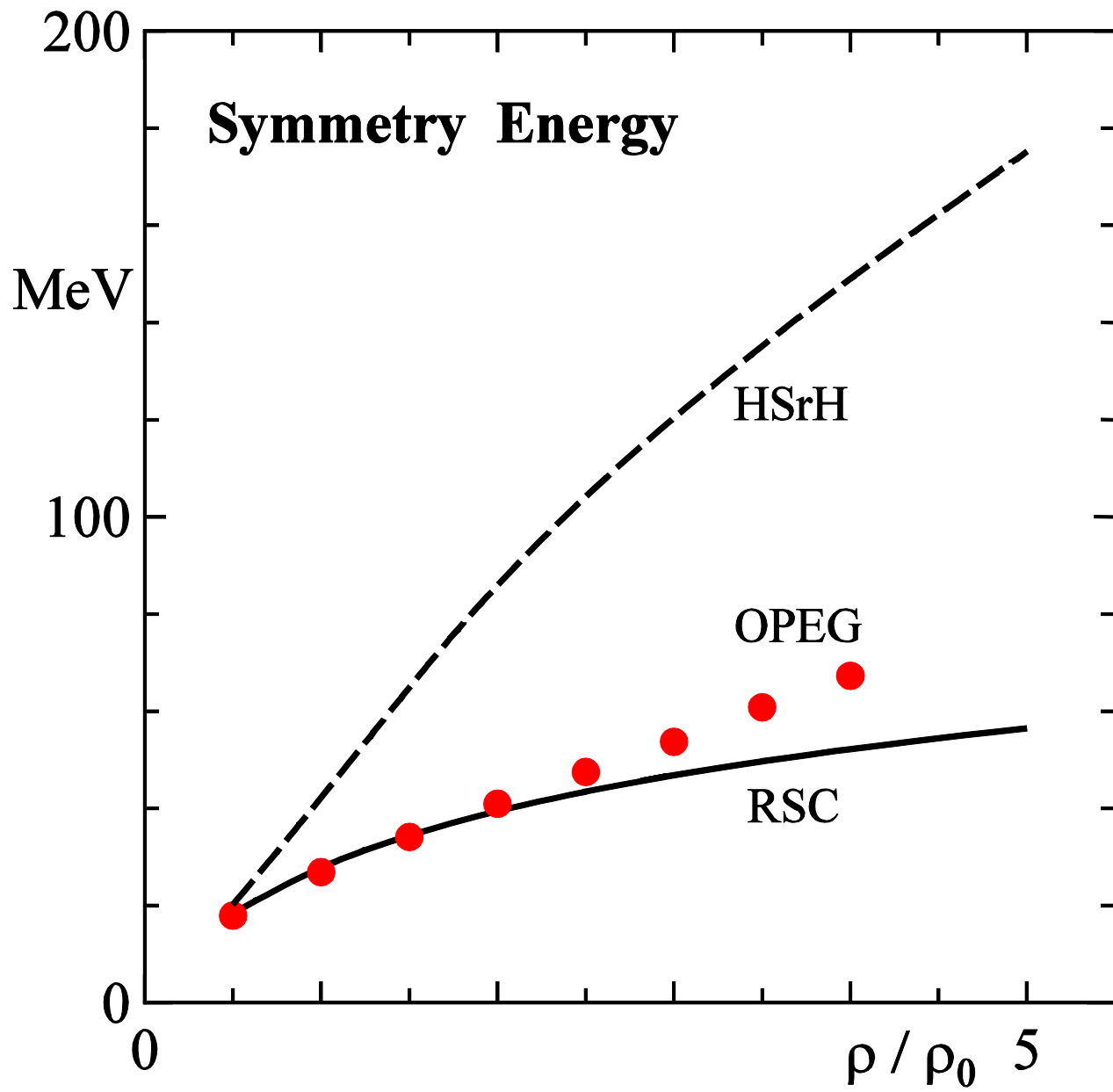
- a_{sym} depends on two-nucleon potentials
G-matrix calculation with
 - RSC** : R.V.Reid, Ann. of Phys. 50(1968), 411.
 - OPEG** : R.Tamagaki, Prog. Theor. Phys. 39(1968), 91.
- In case of **stronger tensor force (RSC)**, the attractive contribution of **$^3S(-^3D)$** is reduced at higher density compared with the case of **weaker tensor force (OPEG)**.





Relativistic Nuclear Model

- C.J.Horowitz and B.D.Serot, Nucl. Phys. A368(1981), 503.
 - relativistic Hartree model with σ , ω , ρ mesons(HSrH)
- a_{sym} is larger than those from G-matrix and steeply depends on the density.
 - kinetic energy term with the density-dependent m^*
 - ρ meson contribution linearly depending on the density



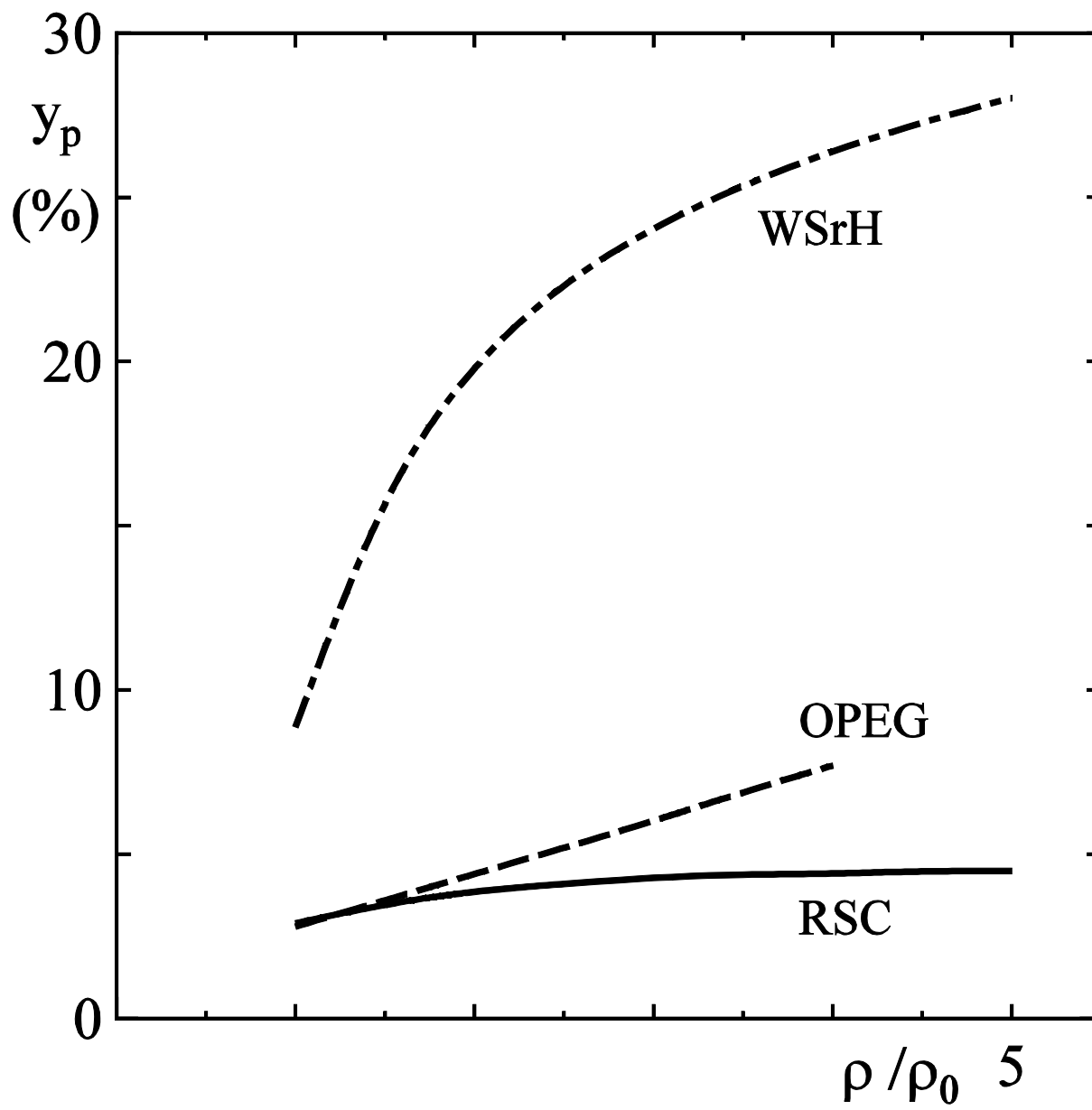
Proton Mixing in Neutron Star Matter

- neutrons, protons, electrons, muons
chemical equilibrium
charge neutrality
- Simple approximation for energy per nucleon

$$E(\rho, \alpha) = E_0(\rho) + E_2(\rho)\alpha^2$$

$$E_0(\rho) = E(\rho, 0), \quad E_2(\rho) = E(\rho, 1) - E(\rho, 0) = a_{\text{sym}}$$

$$\alpha = (\rho_n - \rho_p)/\rho, \quad \rho = \rho_n + \rho_p$$



3. Hyperons in Neutron Stars

3.1 Effective interaction approach

1) Effective interactions are derived from G-matrix calculation with realistic NN, YN, YY interactions.

NN from RSC: ρ - and α -dependent

S.Nishizaki et al., Prog. Theor. Phys. 86(1991), 853.

YN and YY from Nijmegen-D(NHC-D):

ρ - and y_Y -dependent

S.Nishizaki et al., Prog. Theor. Phys. **105** (1991), 607; **108**(2002), 703.

M.M.Nagels et al., Phys. Rev. **D12**(1975), 744; **D15**(1977), 2547; **D20**(1979), 1633.

realistic NN,YN,YYinteraction ← scattering data, hypernuclei

↓ G-matrix ··· short range correlation

Effective interactions V

density-dependent two-body interaction

(1) NN interaction

$V_{NN}(\text{RSC})$, PTP 86(1991), 853.

$$V(r;\rho,\alpha;\beta,\gamma) = \sum_{i=1}^5 c_i(r;\rho,\alpha;\beta,\gamma) \cdot e^{-(r/\lambda_i)^2}$$

$$c_i(r;\rho,\alpha;\beta,\gamma) = a_i(\alpha;\beta,\gamma) + b_i(\alpha;\beta,\gamma)(k_F)^{1/2}$$

$$\alpha = (\rho_n - \rho_p)/\rho, \quad \beta = {}^1E, {}^3O, {}^3E, {}^1O, \quad \gamma = nn, pp, np$$

D.W.L.Sprung and P.K.Banerjee, NP A168(1971), 273.

(2) ΛN , $\Lambda\Lambda$ interaction \leftarrow G-matrix in (n,Λ) -matter
 NHC-D, NHC-F, NHC-Dm

$$V_{\Lambda n}: \rho = \rho_n + \rho_\Lambda, \quad y_\Lambda = \rho_\Lambda / \rho$$

$$V_{\Lambda p}: \rho = \rho_p + \rho_\Lambda, \quad y_\Lambda = \rho_\Lambda / \rho$$

$$V_{\Lambda\Lambda}: \rho = \rho_n + \rho_p + \rho_\Lambda, \quad y_\Lambda = \rho_\Lambda / \rho$$

PTP 105(2001), 607.

(3) $\Sigma^- N$, $\Sigma^- \Sigma^-$ interaction \leftarrow G-matrix in (n,Σ^-) -matter
 NHC-D

$$V_{\Sigma^- n}: \rho = \rho_n + \rho_{\Sigma^-}, \quad y_{\Sigma^-} = \rho_{\Sigma^-} / \rho$$

$$V_{\Sigma^- p}: \rho = \rho_p + \rho_{\Sigma^-} \quad (I=3/2), \quad y_{\Sigma^-} = \rho_{\Sigma^-} / \rho \quad I=1/2?$$

$$V_{\Sigma^- \Sigma^-}: \rho = \rho_n + \rho_p + \rho_{\Sigma^-}, \quad y_{\Sigma^-} = \rho_{\Sigma^-} / \rho$$

PTP 108(2002), 703.

2) A ρ -dependent two-body interaction is used as the three-body force to reproduce the saturation properties of symmetric NM.

TNR at higher ρ , **TNA** at lower ρ

I.E. Lagaris and V.R. Pandharipande, Nucl. Phys. **A359**(1981), 349.

3) EOS is derived by interacting Fermi gas model.

4) Properties of neutron stars are calculated by TOV equations.

R.C. Tolman, Phys. Rev. 55 (1939), 364.

J.R. Oppenheimer and G.M. Volkoff, Phys. Rev. 55 (1939), 374.

Prob. $\rho_s = 1.19\rho_0$, $E_s = -12.2\text{MeV}/A$
for Nuclear Matter at $T = 0 \text{ MeV}$

3 body force, relativistic effects, etc

“unknown parts” \rightarrow phenomenological approach

TNI I.E.Largaris and V.R.Pandharipande,
NP A359(1981), 349.

(1) repulsive at high density (TNR)

(2) attractive at low density (TNA)

$$V_{\text{TNI}}(r;\rho) = V_{\text{TNR}}(r;\rho) + V_{\text{TNA}}(r;\rho)$$

$$V_{\text{TNR}}(r;\rho) = V_1 e^{-(r/\lambda_r)^2} \cdot (1 - e^{-\eta_1 \rho})$$

$$V_{\text{TNA}}(r;\rho) = V_2 e^{-(r/\lambda_a)^2} \cdot \rho \cdot e^{-\eta_2 \rho} (\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)^2$$

(3) TNI for hyperons

$$V_{\text{TNR}}(r;\rho_N) \rightarrow V_{\text{TNR}}(r;\rho_B), \rho_B = \rho_n + \rho_p + \rho_\Lambda + \rho_{\Sigma^-}$$

TOV equation

$$\begin{aligned} \frac{dP}{dr} = & - \left\{ \frac{GM(r)e(r)}{r^2} \right\} \\ & \times \left\{ 1 + \frac{4\pi r^3 P(r)}{M(r)} \right\} \cdot \left\{ 1 + \frac{P(r)}{e(r)} \right\} / \\ & \left\{ 1 - \frac{2GM(r)}{r} \right\} \\ M(r) = & \int^r 4\pi r^2 e(r) dr \end{aligned}$$

$e(r)$: energy density , $P(r)$: pressure

Hyperon Mixing in Neutron Star Matter

$(n, p, \Lambda, \Sigma^-, e^-, \mu^-)$ -system at zero Temperature

Baryon Number Conservation: $y_n + y_p + y_\Lambda + y_{\Sigma^-} = 1$

Charge Neutrality: $y_p = y_{e^-} + y_{\mu^-} + y_{\Sigma^-}$

Chemical Equilibrium: $\mu_n = \mu_p + \mu_{e^-}$

$$\mu_{\Sigma^-} = \mu_n + \mu_{e^-}$$

$$\mu_\Lambda = \mu_n$$

$$\mu_{\mu^-} = \mu_{e^-}$$

$y_a = \rho_a/\rho$: Fractions, μ_a : Chemical Potentials

NN : RSC

Λ N : NHC-Dm

$\Lambda\Lambda$: NHC-D

Σ^-n : NHC-D

$\Sigma^-\Sigma^-$: NHC-D

Σ^-p : same as Λn

$\Lambda\Sigma^-$: neglect

TNI3u($\kappa=300\text{MeV}$)

Threshold density

$$\rho_t(\Lambda) = 4.01\rho_0$$

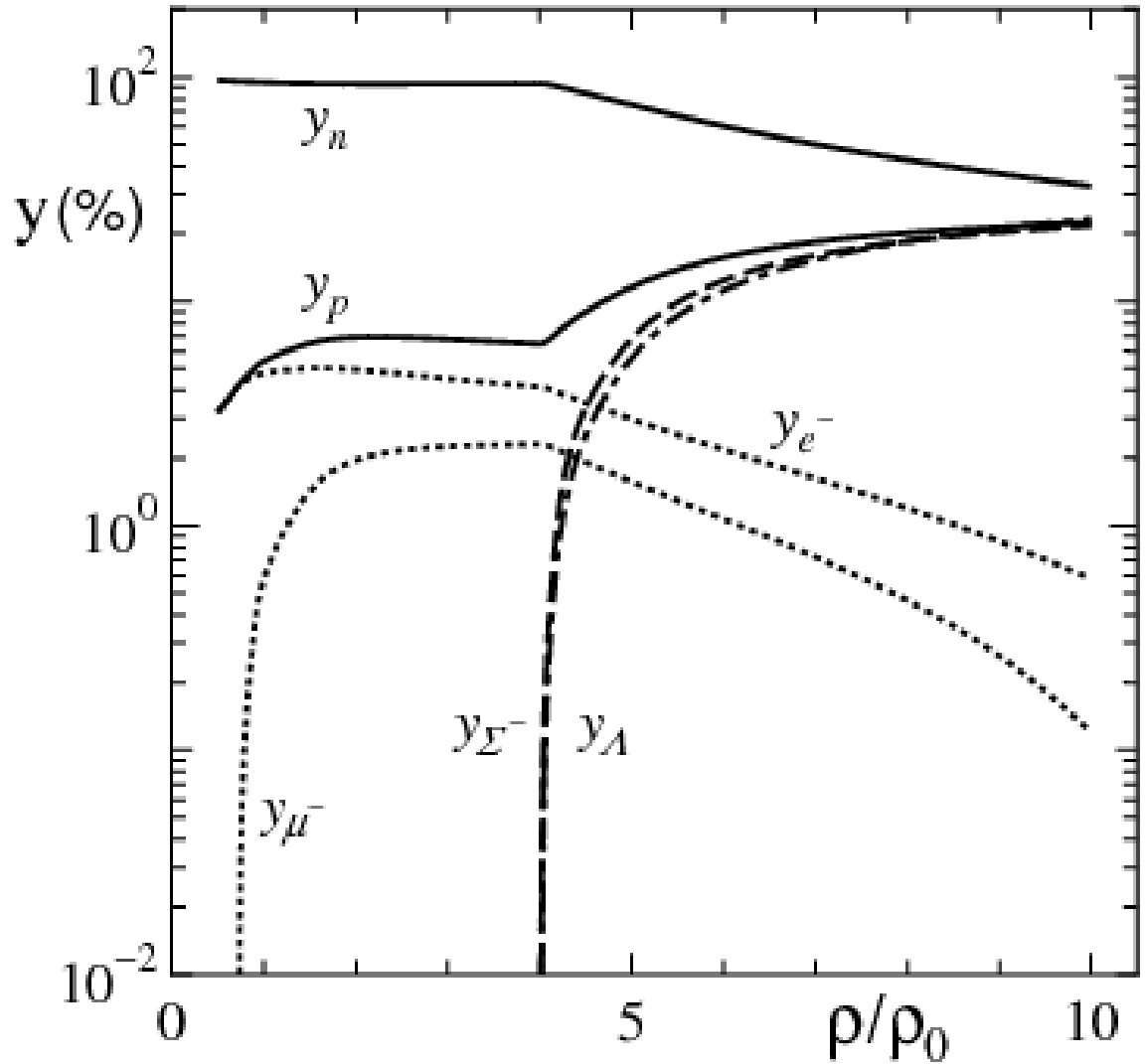
$$\rho_t(\Sigma^-) = 4.01\rho_0$$

Y-Mixing at $8\rho_0$

$$y_\Lambda, y_{\Sigma^-} : \sim 19\%$$

$$\text{cf. } y_n \sim 42\%$$

$$y_p \sim 20\%$$

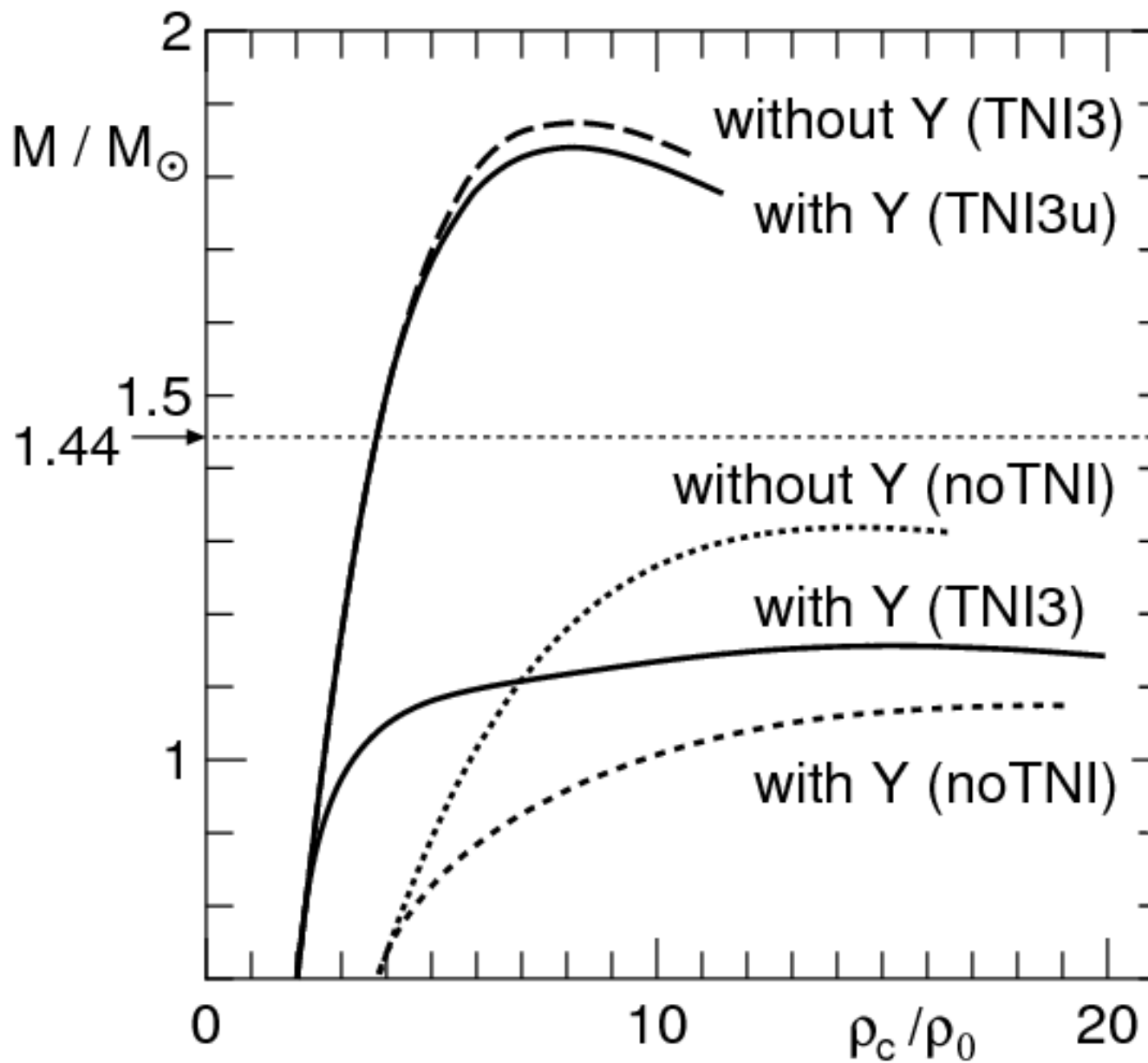


Neutron Star

$$M_{\max} = 1.83M_{\odot}$$

$$R = 9.55\text{km}$$

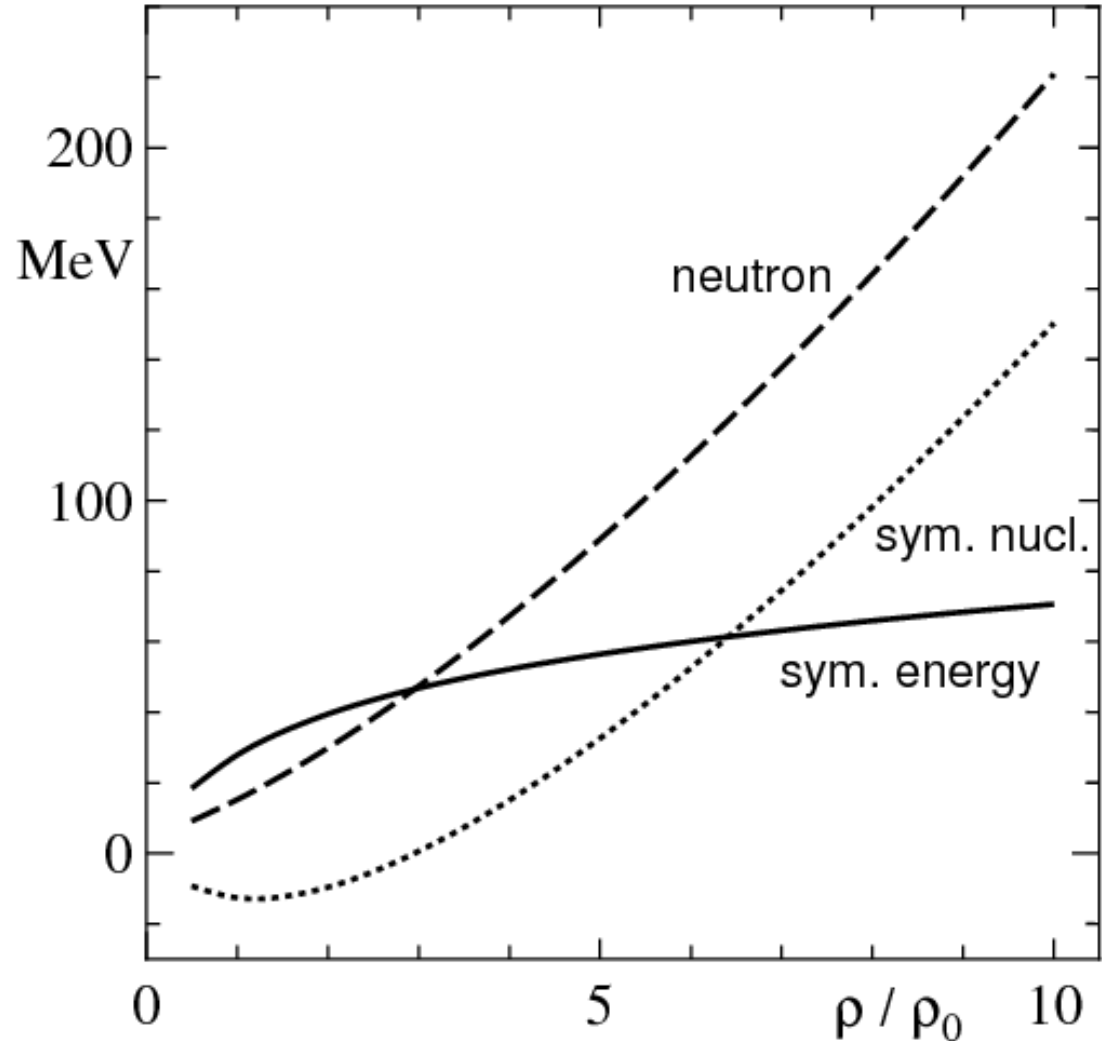
$$\rho_c = 8.26\rho_0$$



3.2 Symmetry energy effect

Present symmetry energy weakly depends on density.

How affects the ρ -dependence of a_{sym} on hyperon mixing in neutron stars?



○ NN interaction

$$V_{\text{TO}} = \alpha_{\text{TO}} V_{\text{TO}}(\text{RSC}) \quad \dots \quad \text{nn, pp, np}$$

$$V_{\text{TE}} = \alpha_{\text{TE}} V_{\text{TE}}(\text{RSC}) \quad \dots \quad \text{np}$$

- ▪ a_{sym} with different density dependence
- $a_{\text{sym}} = 28 \text{ MeV}$ at ρ_0

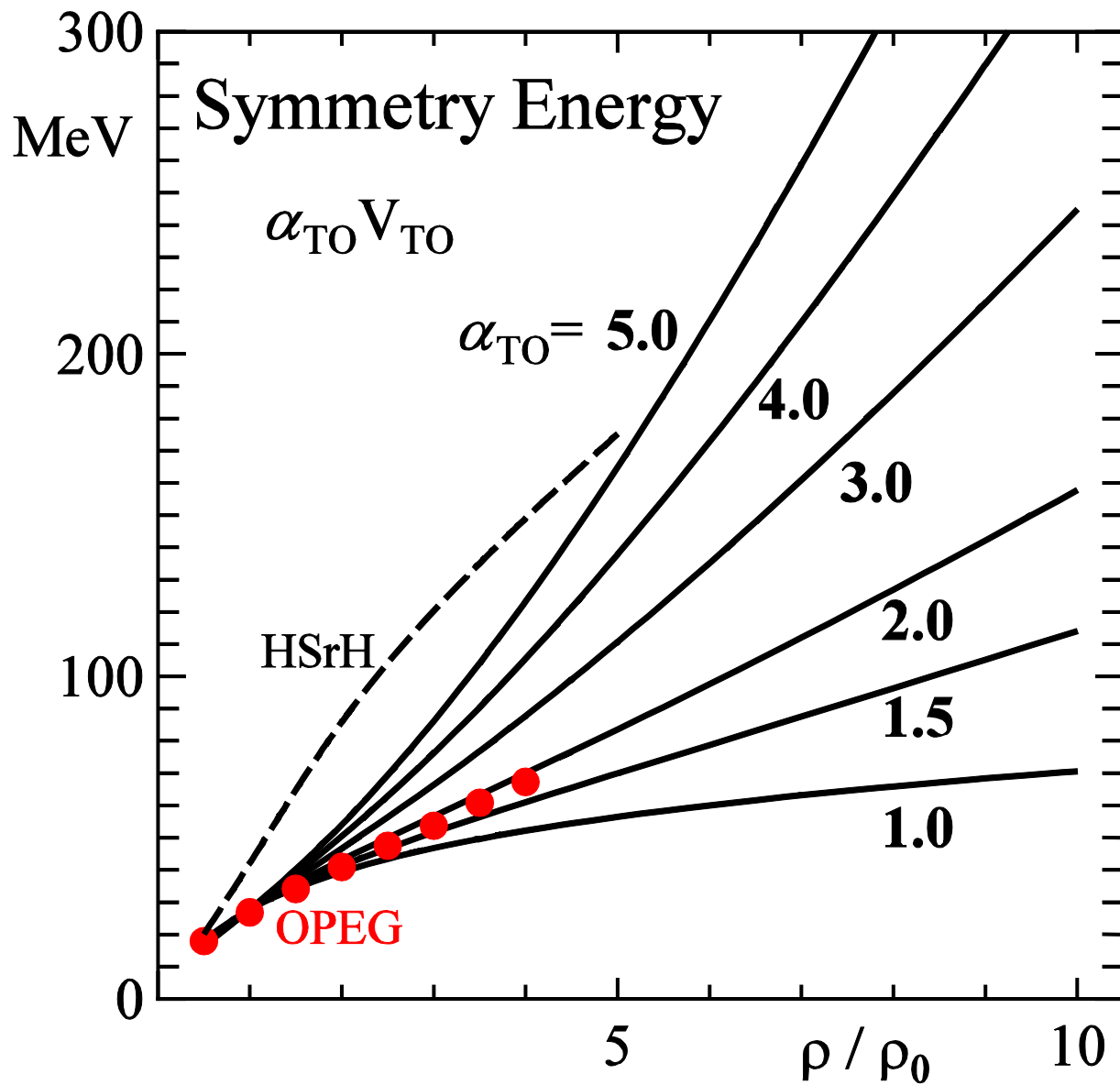
Other parts of the interaction are the same.

○ $\alpha_{\text{TO}} = 2.00$
 $\alpha_{\text{TE}} = 0.970$

OPEG

○ $\alpha_{\text{TO}} = 5.00$
 $\alpha_{\text{TE}} = 0.877$

HSrH



3.3 Results

(1) Threshold Densities : $\rho_t(\Lambda)$, $\rho_t(\Sigma^-)$

$$U_\Lambda(k=0) + \Delta m_\Lambda = k_{Fn}^2/2m_n + U_n(k_{Fn})$$

$$\Delta m_\Lambda = m_\Lambda - m_n \sim 176\text{MeV}$$

$$U_{\Sigma^-}(k=0) + \Delta m_{\Sigma^-} - \mu_{e^-}$$

$$= k_{Fn}^2/2m_n + U_n(k_{Fn})$$

$$\Delta m_{\Sigma^-} = m_{\Sigma^-} - m_n \sim 255\text{MeV}$$

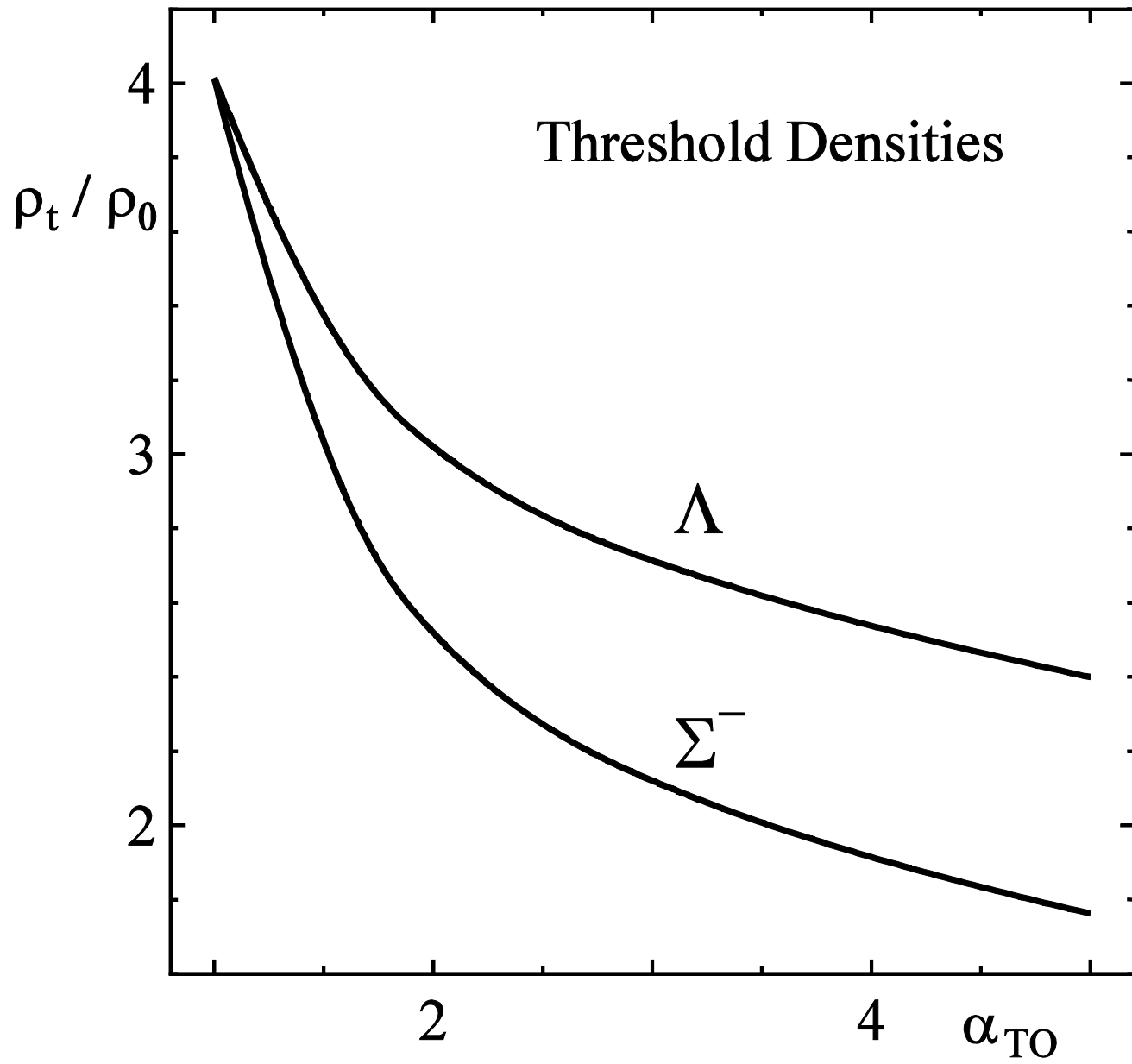
$$\mu_{e^-} = 100 \sim 200\text{MeV}$$

α_{TO}	:	$\rho_t(\Lambda)$	$\rho_t(\Sigma^-)$
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1.0	:	$4.01\rho_0$	$4.01\rho_0$
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2.0	:	$3.00\rho_0$	$2.49\rho_0$
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5.0	:	$2.40\rho_0$	$1.76\rho_0$
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(2) Λ , Σ^- -fractions

$$\underline{\alpha_{\text{TO}} = 2.00}$$

$$y_n \sim 32\%$$

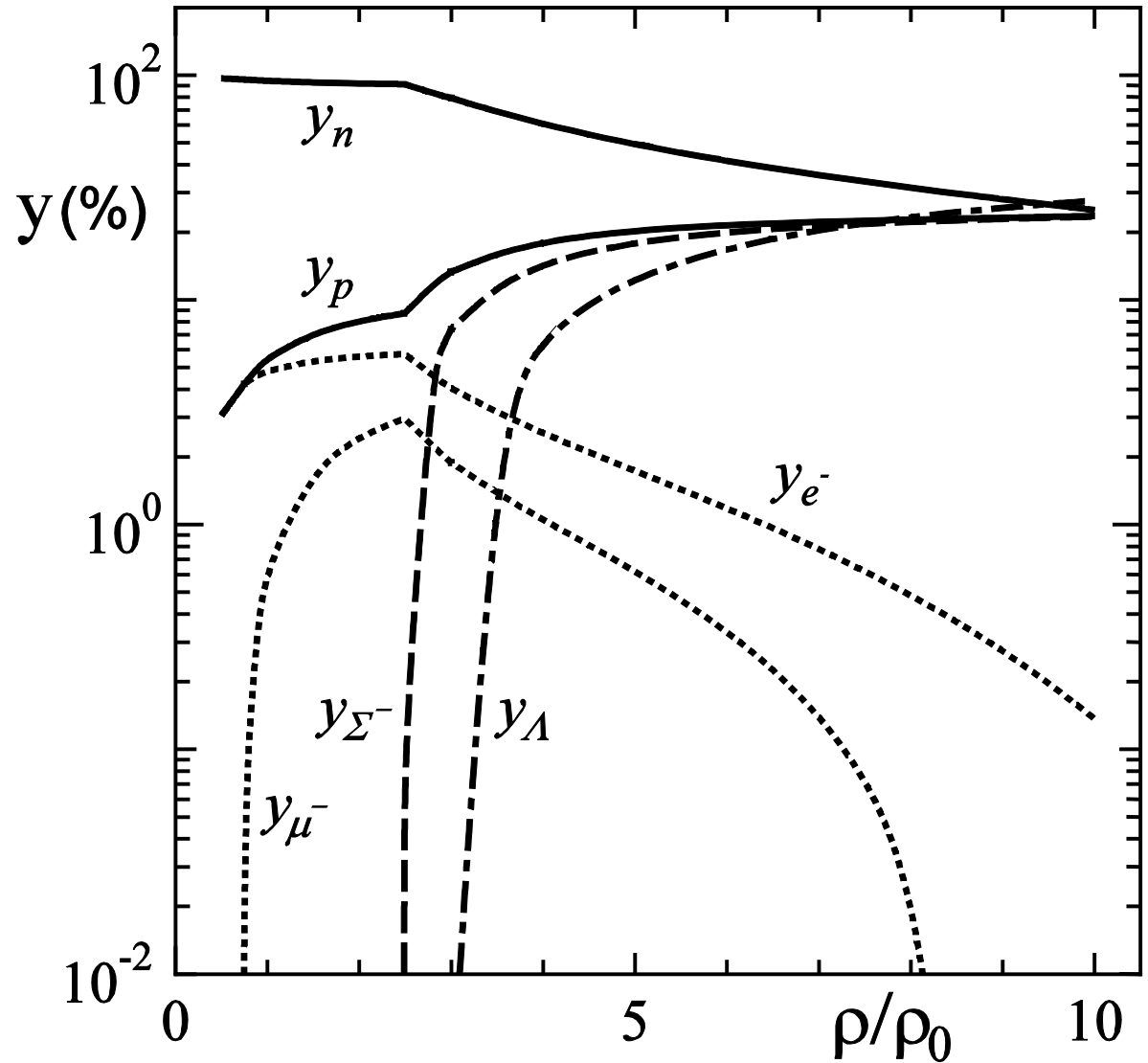
$$y_p \sim 23\%$$

Y-Mixing

$$y_\Lambda \sim 23\%$$

$$y_{\Sigma^-} \sim 22\%$$

$$\text{at } \rho = 8\rho_0$$



$$\alpha_{\text{TO}} = 5.00$$

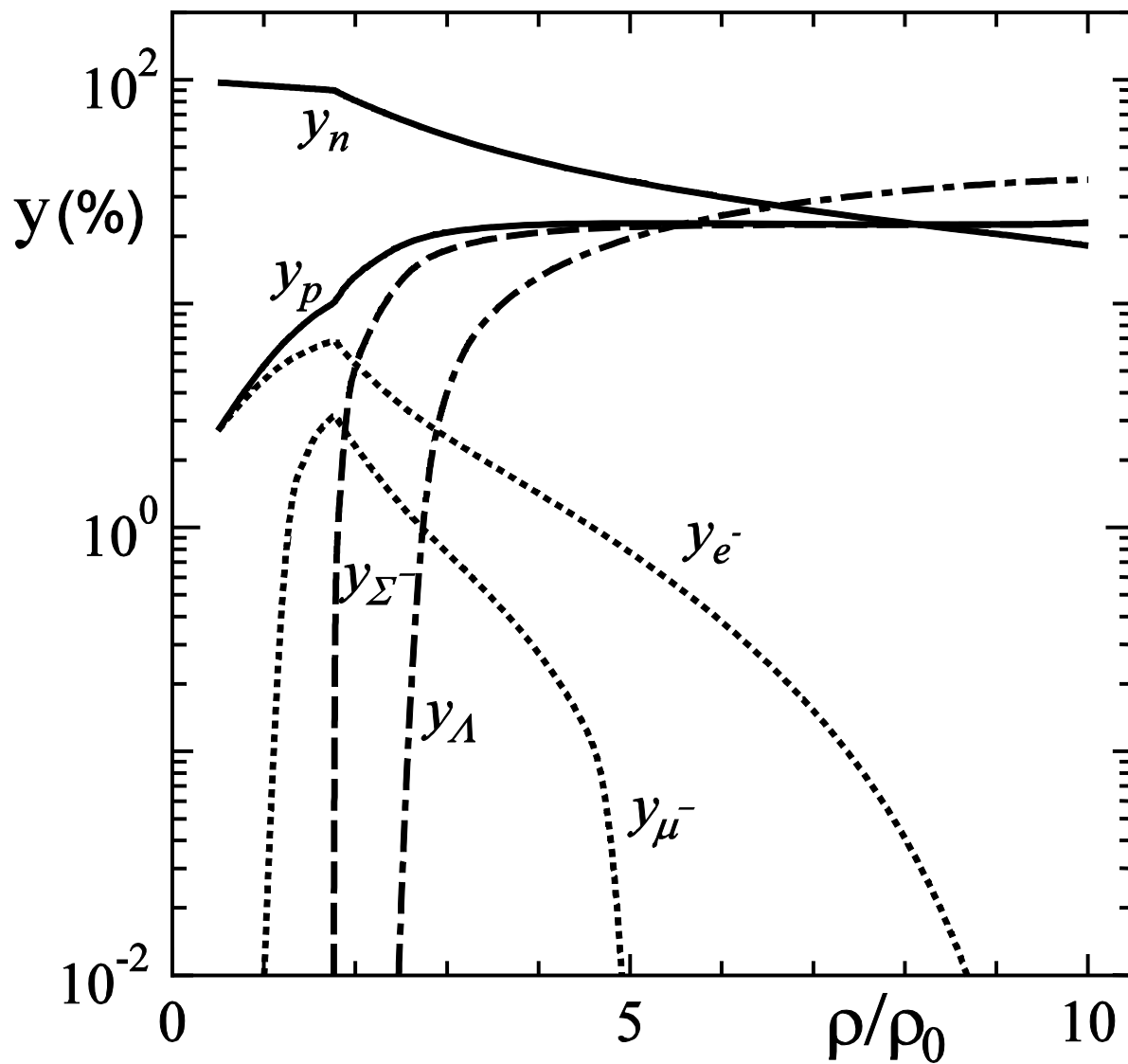
$$y_n \sim 23\%$$

$$y_p \sim 23\%$$

Y-Mixing

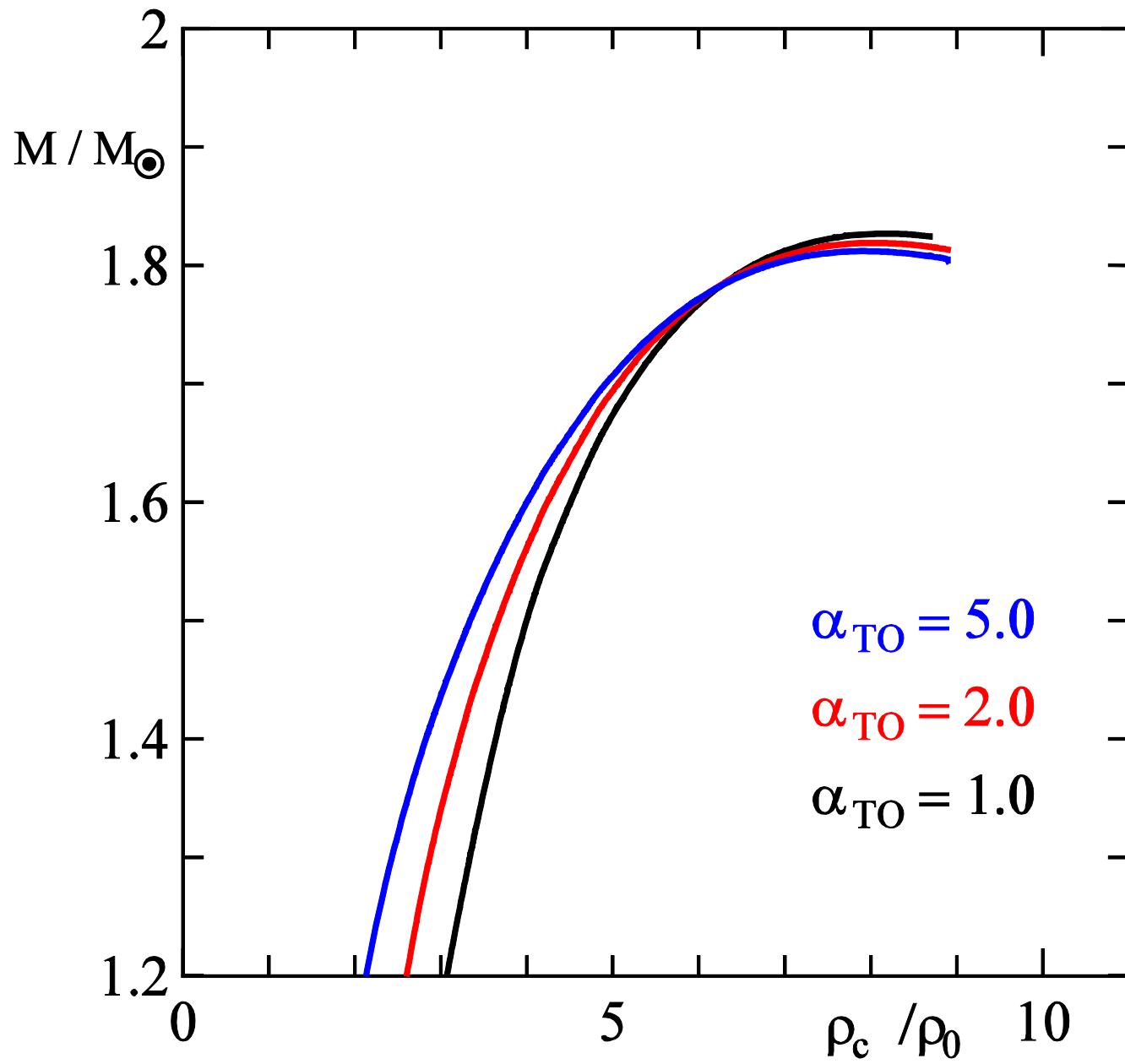
$$y_\Lambda \sim 31\%$$

$$y_{\Sigma^-} \sim 23\%$$



(3) Properties of Neutron Stars with Maximum Mass

α_{TO}	M/M_{\odot}	R(km)	ρ_c / ρ_0	$N_B (10^{57})$
1.0	1.827	9.545	8.259	2.547
2.0	1.819	9.763	8.018	2.513
5.0	1.812	9.826	7.975	2.472



4. Summary

- (1) We estimate how the density dependence of symmetry energy affect on hyperon mixing in neutron stars.
- (2) As the density dependence is steeper, threshold densities decrease largely.

$$\rho_t(\Sigma^-) = (4.01 \rightarrow 2.49 \rightarrow 1.76) \rho_0$$

$$\rho_t(\Lambda) = (4.01 \rightarrow 3.00 \rightarrow 2.40) \rho_0$$

(3) In the central region of neutron star with maximum mass, fractions of baryons change as

$$y_{\Sigma^-} = (19 \rightarrow 22 \rightarrow 23)\% , \quad y_{\Lambda} = (19 \rightarrow 23 \rightarrow 31)\%$$

$$y_n = (42 \rightarrow 32 \rightarrow 23)\% , \quad y_p = (20 \rightarrow 23 \rightarrow 23)\%$$

(4) EOS and properties of neutron stars with maximum mass change slightly.