The systematic dependence of $E(2_1^+)$ on N_pN_n, N_B and p-factor for A=120-200 mass region nuclei

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Abstract

The systematic dependence of energy $E(2_1^+)$ and energy ratio $R_{4/2}(= E(4_1^+)/E(2_1^+))$ on neutron number (N), number of valence proton and valence neutron (N_pN_n), total boson number (N_B) and p-factor ($p = \frac{N_pN_n}{N_p+N_n}$) for A=120-200 mass region nuclei. The whole data is divided into four quadrants (Q). The first quadrant (Q-I) of $N > 82$ is of Z=50-82, $82 \leq N < 104$ shell space with particle like proton-bosons and neutron-bosons forming the p-p space. Second quadrant (Q-II) of $82 \leq N \leq 104$ is of Z=50-82 shell space, with hole like proton-bosons space and particle like neutron-bosons space forming the h-p space. Third quadrant (Q-III) of $104 \leq N < 126$ of Z=50-82 shell space, with hole like proton-bosons and neutron-bosons forming h-h space. The fourth quadrant (Q-IV) of $N < 82$ of Z=50-82 shell space with particle like proton-bosons and hole like neutron-bosons forming the p-h space. In brief, quadrant I and III for p-p and h-h bosons space, and II, IV for p-h and h-p bosons space respectively. A simple exponential dependence of $E(2_1^+)$ and a smooth variation of $R_{4/2}$ on N, N_pN_n, N_B and p-factor is obtained.

Introduction

he study of the collective nuclear structure with the neutron number N, and proton number Z, total

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boson number $N_B = N_p + N_n$ where the N_p be the valence proton number and N_n is valence neutron number. The product of N_pN_n gives the understanding of the nuclear interactions involved. The important role of the N_pN_n early was discussed by the de-Shalit and Goldhaber [1]. Hamamoto [2] first pointed out that the square roots of the ratios of measured and the single particle B(E2) values was proportional to the N_pN_n. Casten [3] used the N_pN_n product to study the nuclear structure of the excited states ($E(2_1^+)$) in very simple pattern. Casten et al. [4] presented a review on the evaluation of the nuclear structure on the bases of N_pN_n product, this phenomenon has been to be called the N_pN_n scheme. Gupta et al. [5] presented a systematics dependence of γ-g B(E2) ratios on the N_pN_n product in the different parts of the major shell space Z=50-82, N=82-126 and demonstrated that the interband B(E2) ratios were smooth function of N_pN_n. Further, Gupta et al. [6] pointed that the limitations of F-spin and N_pN_n scheme in reproducing the overall $E(2_1^+)$ systematic in major shell space Z=50-82, N=82-126 into four quadrants. Recently most work related to N_pN_n scheme mainly concentrate on p-factor that is define as $p = \frac{N_pN_n}{(N_p+N_n)}$ [4], and β_2 [7–9], systematic law of $E(2_1^+)$ for heavy nuclei [10]. The systematic energy of first (2_1^+) of all even-even nuclei and odd-even staggering [11, 12].

In present work we focus on the systematics dependence of energy of the first 2_1^+ state of ground band on N_pN_n, N_B and p-factor in even-even nuclei for Z=50-82 and N=82-126 regions by dividing the whole space in four quadrants. The experimental data are taken from ref [13].

I. RESULT AND DISCUSSION

Gupta et al. [14] grouped the A=120-200 nuclei into four quadrants. First quadrant (Q-I) of 82 $\leq$ N $<$104 is of Z=50-82 shell space, with particle like proton-bosons and hole like neutron-bosons forming the p-p subspace. Second quadrant (Q-II) with hole like proton-bosons and particle like neutron-bosons forming the h-p subspace. The third quadrant (Q-III) of 104 $\leq$N $<$ 126 of Z=50-82 shell space, with hole like proton - bosons and hole like neutron-bosons forming the h-h subspace. The fourth hole like neutron-bosons forming the p-h subspace. In brief, quadrant I and III for p-p and h-h bosons space, and II, IV for p-h and h-p bosons space respectively. A simple exponential dependence of $E(2_1^+)$ and a smooth variation of $R_{4/2}$ on N, N_pN_n, N_B and p-factor is obtained. In Fig. 1 for Q-IV ($N < 82$) region we see the variation between $E(2_1^+)$ and N_B, N_pN_n, p-factor. The graph of $E(2_1^+)$ vs N_B data of same Z linked ($N < 82$) at N=78-80. The energy $E(2_1^+)$ decrease constantly, with increasing N_B (and N_n) and nuclei get deformed. But there is no constancy with N_B, for same N_B (any value) the $E(2_1^+)$ varies by large amount e.g. even at $N_B = 9$, $E(2_1^+)$ varies 0.25-0.50. Similarly for N_B=8 the spread in $E(2_1^+)$ =100 keV. Next same data is plotted for N_pN_n in this case the horizontal spread is much less, but for any N_pN_n vertical spread is large. As for N_B, similarly for N_pN_n, $E(2_1^+)$ decrease (definitely increase with increase N_pN_n collectivity). Since the number of n-n, p-p and n-p interactions depend on the product N_pN_n and the deformation of the nucleus may also depend on this product. The dependence on N_pN_n is better viewed on a graphical plot. The $E(2_1^+)$ values do decrease linearly with the increase in N_pN_n. The collectivity is surely dependent on the number of bosons or the product N_pN_n but for small N_p or N_n, the collectivity does not build up, so that the proportionality to N_B or N_pN_n product is not fully attained. For same N_pN_n the higher Z (having lesser N_n) datum lies higher (less deformed), exhibiting greater effect of N_n here. We also see the variation of $E(2_1^+)$ with $p = N_pN_n/(N_p + N_n)$. It is evident that a number dependence on N_B or N_pN_n works better when both numbers N_p and N_n are more than 3 or 4. For p-factor vertical spread is less all data is closer along a single slope line. In Fig. 2 we see the behavior of Q-I ($N > 82$) region. When $E(2_1^+)$ is plotted against N_B, N_pN_n and p-factor then is shows falls and constant trend. The increase in the value of N_pN_n and p-factor the collectivity is increases and energy get decreased. The advent of the sd- Interacting Boson Model, IBM-1 led to the U(6) algebra and its dynamic subgroups of SU(5), SU(3) and O(6), and the assumption of F-spin invariance in the F-spin space. This led to the concept of F-spin and recognition of some F-spin multiplets of $N_B = 11$, 12 (F=5.5 and 6).

The values of $E(2_1^+)$ vary quite fast to the constant N_B or fixed F-spin. For $N < 104$ firstly graph is plotted between $E(2_1^+)$ vs. N_B (see in Fig. 3). For $N_B \geq 11$ data of various Z overlap, vertical spread is small. So we get F-spin $=N_B/2$ identical band multiplets. The overlap increases much and spreading is decreases. Next same data is plotted against N_pN_n in which $E(2_1^+)$ decreases with N_pN_n. In ^{160}Er -^{180}Os case, ^{160}Er and ^{168}Hf have N_pN_n=(14,10) and (10,14), respectively. So that N_pN_n =140 and $F_0 = 1$ in both cases. It suggested in that such (F_0, N_p, N_n) pairs should be similar in structure. Next the same data is also plotted against the p-factor. In this plot the spread is very small and all data is closer along single line because the value of p-factor is large in $N < 104$ region hence this larger value of p-factor condenses the x-scale more to bring together.

Due to which the value of $E(2_1^+)$ get more close and more saturated. Hence identical bands with respect to the N_B, N_pN_n or p-factor is observed in $N < 104$ mass region. Similarly in case of $N \geq 104$ region, when graph is plotted between $E(2_1^+)$ and N_B (see Fig. 4). The energy graph not show any saturation, and more spreading take place, hence $E(2_1^+)$ decreases constantly with N_B. The $E(2_1^+)$ shows more variation with N_B. Next same data is plotted for N_pN_n in as the N_B, similarly as that of N_B the $E(2_1^+)$ decreases with increasing the N_pN_n collectivity (N_p constant N_n increases). There is small spread in both horizontal as well as vertical side. Therefore with increasing the N_pN_n more decrease in the value of $E(2_1^+)$ energy and saturation is attain.

Next the variation with p-factor all data are located in one single line, with increasing the p-factor it condenses the x-scale due to which the data become closer to each other. The Pt nuclei show more fall with the p-factor, it reduces 0.5 to 0.1 MeV.The present study reflects the systematic behavior of $E(2_1^+)$ energy with N_pN_n, N_B and p-factor. An exponential law of energy value of 2_1^+ ground state is shown to be applicable to all three regions i.e. (p-p), (h-p) and (h-h) but (p-h) space is empty.

References

[1] A. de-Shalit and M. Goldhaber, *Phys. Rev.* **92**, 1211 (1953).

[2] I. Hamamoto, *Nucl. Phys. A* **73**, 225 (1965).

[3] R. F. Casten, *Nucl. Phys. A* **443**, 1 (1985).

[4] R. F. Casten, N. V. Zamfir, *J. Phys. G (Nucl. Part. Phys.) 22*, 1521 (1996).

[5] J. B. Gupta, J. H. Hamilton and R. V. Ramayya, *Phys. Rev. C***42**, 1373 (1990).

[6] J. B. Gupta, H. M. Mittal, J. H. Hamilton and R. V. Ramayya, *Int. J. Mod. Phys.* **5**, 1155 (1990).

[7] Y. M. Zhao, R. F. Casten and A. Arima, *Phys. Rev. Lett.* **85**, 720 (2000).

[8] Y. M. Zhao, R. F. Casten and A. Arima, *Phys. Rev. C* **63**, 067302 (2001).

[9] S. Q. Zang, S. G Zhou and F. R. Xu, *High Rnergy Phys. Nucl. Phys.* **28** 516 (2004).

[10] FAn Zhe-Yong, REN Zhong-Zhou and Xu Chang, *Chin. Phys. Lett.* **24** 2555 (2007).

[11] Jin-Hee Yoon, Eunja Ha and Dangwoo Cha, *J. Phys. G (Nucl. Part. Phys.)* **34**, 2545 (2007).

[12] Guanghao Jin, Jin-Hee Yoon and Dangwoo Cha, *J. Phys. G (Nucl. Part. Phys.)* **35**, 035105 (2008).

[13] http:// www.bnl.nndc.gov / nsdf.

[14] J. H. Hamiltonian, A. V. ARmayya and J. B. Gupa, *Bull. Am. Phys. Sco.* **32**, 2130 (1987)

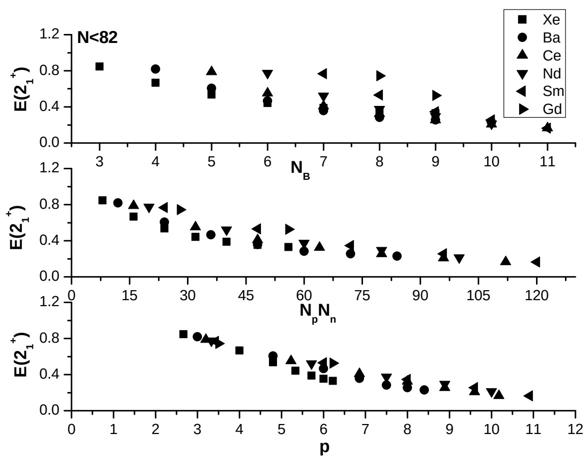

FIG. 1: Variation of $E(2_1^+)$ vs.N_B N_pN_n and p-factor in $N < 82$ region.

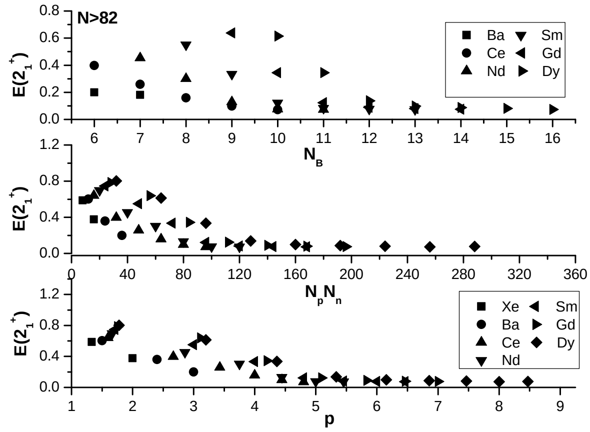

FIG. 2: Variation of $E(2_1^+)$ vs.N_B N_pN_n and p-factor in $N > 82$ region

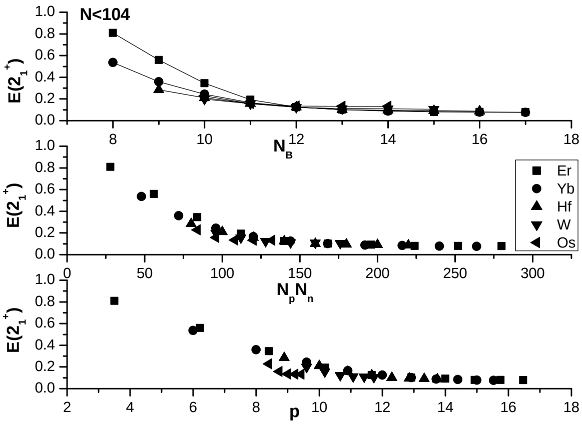

FIG. 3: Variation of $E(2_1^+)$ vs.N_B N_pN_n and p-factor in $N < 104$ region

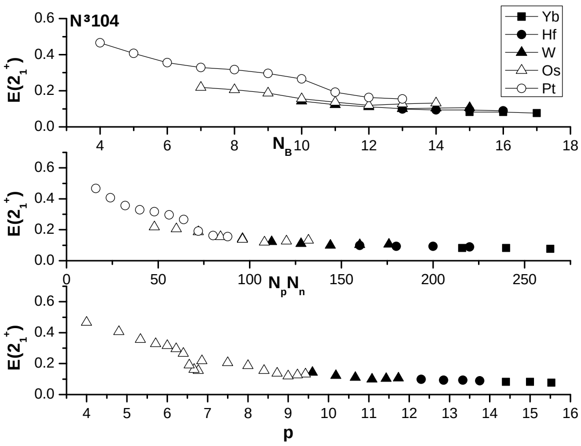

FIG. 4: Variation of $E(2_1^+)$ vs.N_B N_pN_n and p-factor in $N > 104$ region