

Investigation of the symmetry energy in EOS by isoscaling in heavy ion reactions

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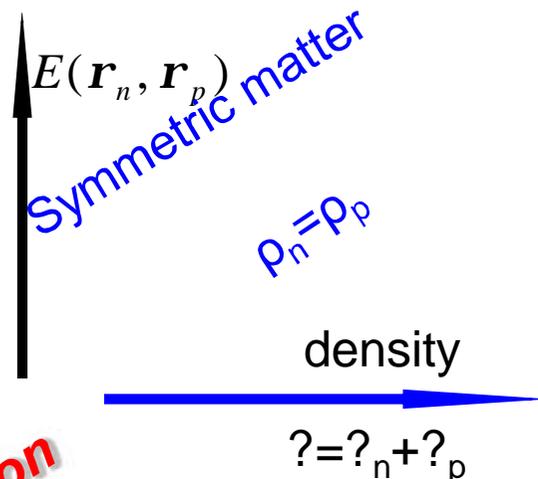
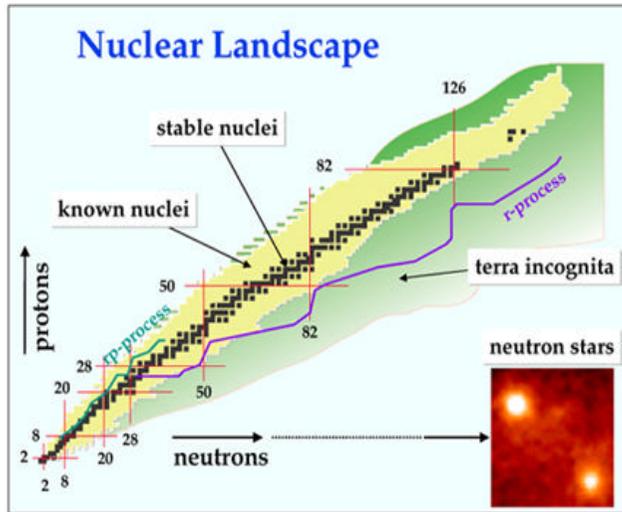
□ Summary

- ◆ Sequential decay effect on isoscaling and constrain of symmetry energy
- ◆ Other observables to study the isospin degree



Background and motivation

◆ Isospin and symmetry energy in nuclear EoS



A new dimension

Isospin asymmetry $d = (\rho_n - \rho_p) / \rho$

Liquid-drop model

$$a_v A - a_s A^{2/3} - a_4 \frac{(N - Z)^2}{A} - a_c \frac{Z(Z - 1)}{A^{1/3}} + a_p \frac{\Delta(N, Z)}{A^{1/2}}$$

Symmetry energy term

EOS of Isospin Asymmetric Nuclear Matter

$$E(\mathbf{r}, \mathbf{d}) = E(\mathbf{r}, 0) + E_{\text{sym}}(\mathbf{r})d^2 + O(d^4), \quad \mathbf{d} = (\mathbf{r}_n - \mathbf{r}_p) / r$$

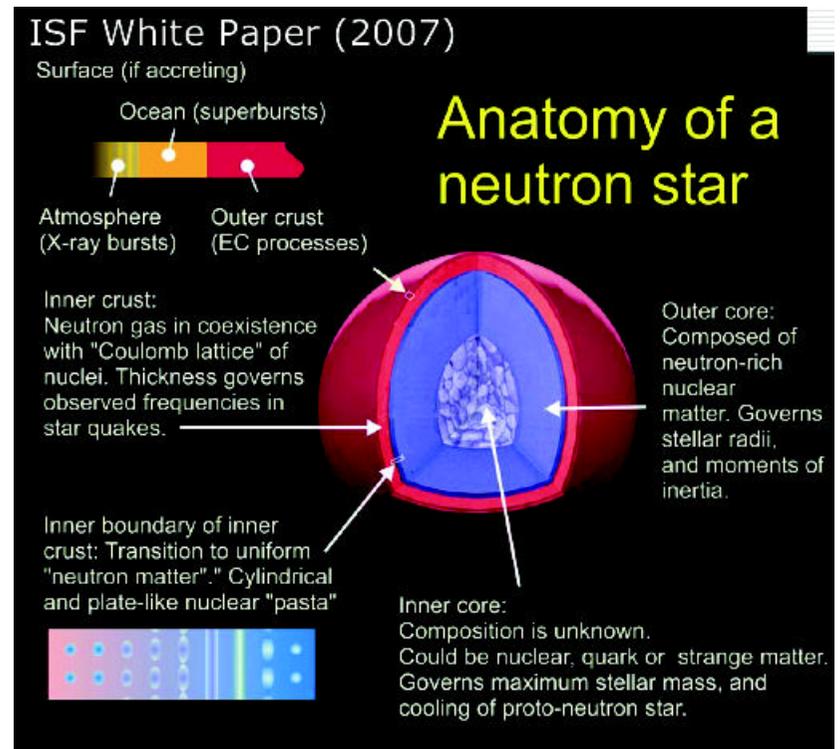
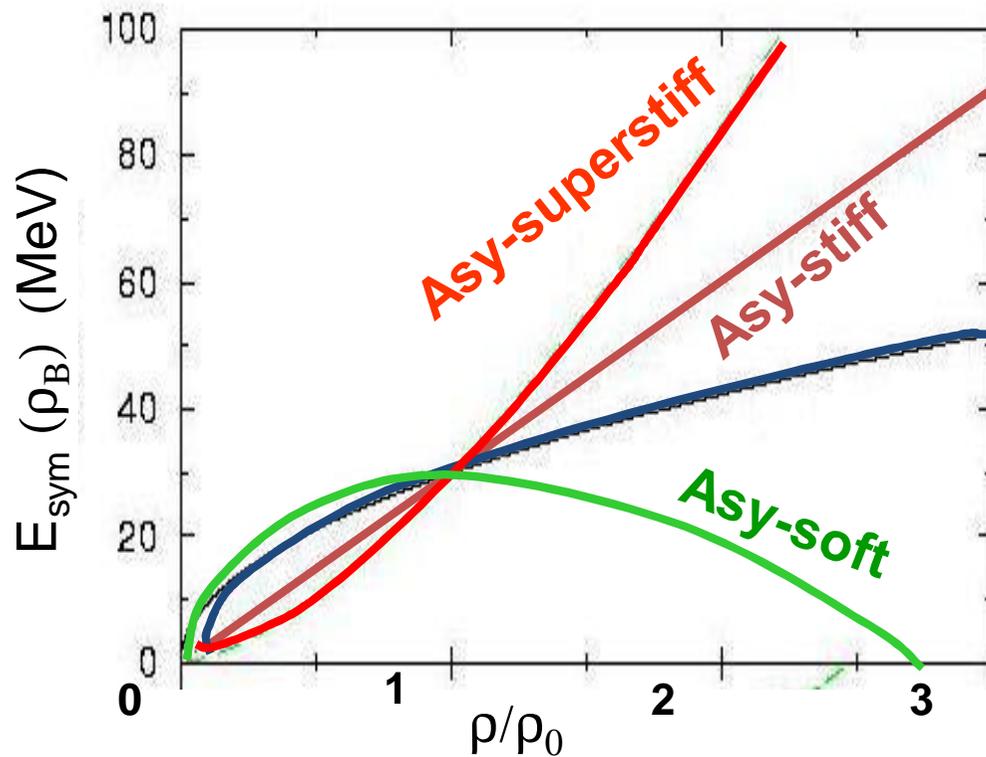
The Nuclear Symmetry Energy

Symmetric Nuclear Matter

$$E_{\text{sym}}(\mathbf{r}) \equiv \frac{1}{2} \frac{\partial^2 E(\mathbf{r}, \mathbf{d})}{\partial d^2}$$

Symmetry energy term

Critical term for the asymmetry nuclear EOS, neutron star(mass, radius, property)



◆ Observables to measure the symmetry energy in nuclear EoS

At sub-saturation densities

- Sizes of n-skins of unstable nuclei from total reaction cross sections
- **Proton-nucleus elastic scattering in inverse kinematics**
- Parity violating electron scattering studies of the n-skin in ^{208}Pb at JLab
- **n/p ratio of FAST, pre-equilibrium nucleons**
- Isospin fractionation and isoscaling in nuclear multifragmentation
- **Isospin diffusion/transport**
- Neutron-proton differential flow
- **Neutron-proton correlation functions at low relative momenta**
- t/ ^3He ratio

Towards high densities

- **p⁻/p⁺ ratio, K⁺/K⁰ ?**
- Neutron-proton differential transverse flow
- **n/p ratio of squeezed-out nucleons perpendicular to the reaction plane**
- Nucleon elliptical flow at high transverse momenta

Model dependent probes or Hypothesis dependent probes



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Constrain symmetry energy from experiment

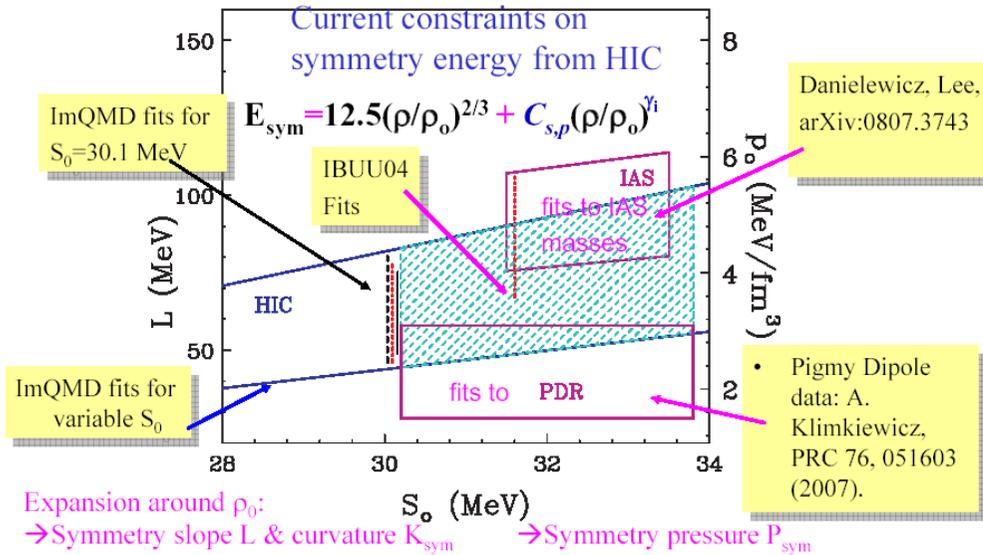
Isoscaling, Isospin diffusion, double n/p Ratio, MSU data, TAMU data at sub-saturate density

$$S \sim 31.6 (r/r_0)^g \quad 0.69 \leq \gamma \leq 1.05$$

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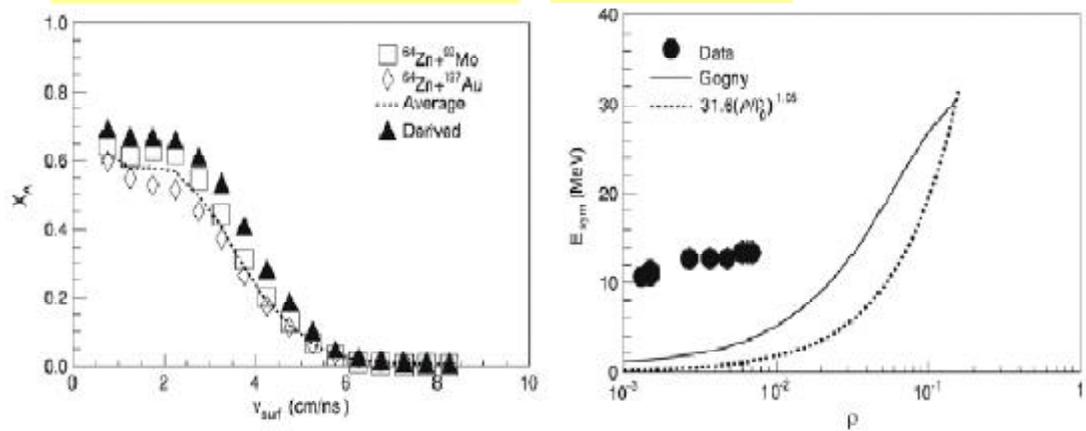
$$S = 12.5 (r/r_0)^{2/3} + 17.6 (r/r_0)^{g_i} \quad 0.4 \leq \gamma_i \leq 1$$

ImQMD

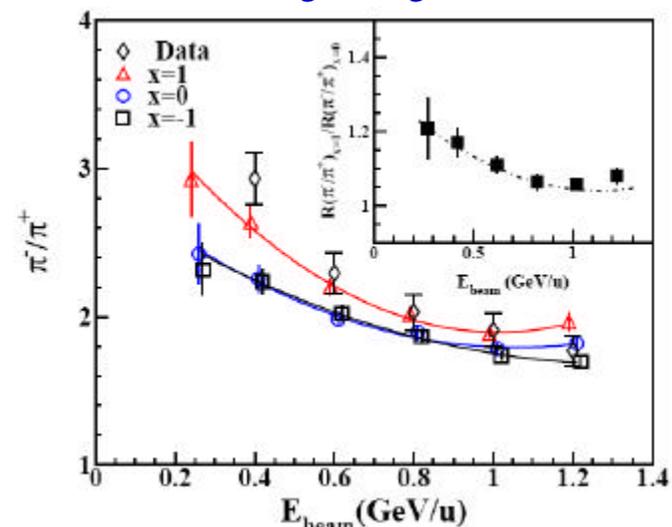


$$E_{sym} = S_0 + \frac{L}{3} \left(\frac{\rho_B - \rho_0}{\rho_0} \right) + \frac{K_{sym}}{18} \left(\frac{\rho_B - \rho_0}{\rho_0} \right)^2 + \dots$$

$$L = 3\rho_0 \left. \frac{\partial E_{sym}}{\partial \rho_B} \right|_{\rho_B = \rho_0} = \frac{3}{\rho_0} P_{sym}$$



High density, FOPI data, Xiao/Li/Chen/Yong/Zhang, PRL102, 062502(2009)



Very low density, TAMU data, Horowitz and Schwenk, Nucl. Phys. A 776 (2006) 55
S. Kowalski, et al., PRC 75 (2007) 014601.
J. B. Natowitz et al, PRL 104, 202501 (2010)

Ideally, Isoscaling measure the hot fragments of the reaction, with the assumption of that secondary decay of the two similar reactions are same, the secondary decay so can be neglected.

But some investigation show the isoscaling difference from hot fragments and cold fragments, our purpose is to study the influence of statistical secondary sequential decay on isoscaling and constrain of the symmetry energy coefficient from isoscaling measurement.

A statistical sequential decay model GEMINI was selected to observave the secondary decay effect.

We simplyly analyze the results from an experimental method, to compare with the experimental data.



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Statistical sequential decay influence on isoscaling and constrain of symmetry energy

Statistical binary sequential decay model (GEMINI)

Calculates the decay of compound nuclei by sequential binary decays, all possible divisions from light-particle emission to symmetric divisions are considered. GEMINI employs a Monte Carlo technique to follow the decay chains of individual compound nuclei through sequential binary decays until the resulting products are unable to undergo further decay.

The decay width for the evaporation of fragments with $Z \geq 2$ is calculated using the Hauser-Feshbach formalism. Light particle decay width is given by

$$\Gamma_{J_2}(Z_1, A_1, Z_2, A_2) = \frac{2J_1 + 1}{2\pi r_0} \sum_{l=|J_0 - J_2|}^{J_0 + J_2} \int_0^{E^* - B - E_{rot}(J_2)} T_l(\mathbf{e}) r_2(U_2, J_2) d\mathbf{e}$$

The symmetry energy term due to the neutron-proton excess is represented in calculating the binding energy for heavy systems ($Z > 12$), For very light systems ($A \leq 12$), binding energies were calculated from the experimental masses.

R.J. Charity et al, Nucl. Phys. A 483, 371, (1988).

R.J. Charity, computer code GEMINI, see <http://wunmr.wustl.edu/pub/gemini>

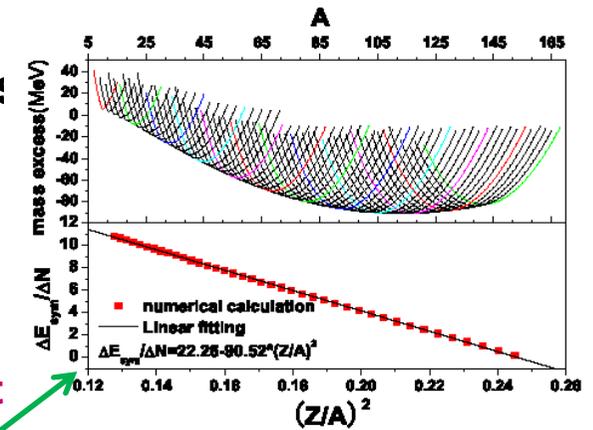
R. Charity et al, PRC56, (1997) 873; H. J. Krappe et al., PRC20, (1979) 992; P. Möller et al., NPA361, (1981) 117



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- ◆ Isolate from dynamical effect, at the saturate density
- ◆ Decay from equalibrated compound nuclei(source) with different isospin asymmetry
- ◆ Couples of group and pairs of compound nuclei(source)
 - Different Atomic number and mass region (Z=30,50)
 - Different isospin asymmetry (N/Z=1.0,1.1,1.2,1.4)
 - Different excitation energy ($E_{ex}=1, 1.4, 2, 2.4, 3\text{MeV/u}$)
- ◆ First step decay only (Well determined source and temperature)
 - one step of decay calculated, then stop the simulation
 - check isoscaling and constrain of symmetry energy coefficient with determined process
- ◆ Full step decay chains included
 - all decay step considered, until no particle or gamma ray emission
 - test the secondary decay effect on isoscaling and symmetry energy constrain



$$\begin{aligned}
 M_{macro}^{(0)} = & M_n N + M_p Z - \left(a_v (1 - k_v I^2) A + a_s (1 - k_s I^2) \right) \\
 & \times \left\{ A^{2/3} - 3 \left(\frac{a}{r_0} \right)^2 + \left(\frac{r_0}{a} A^{1/3} + 1 \right) \left[2A^{2/3} + 3 \frac{a}{r_0} A^{1/3} + 3 \left(\frac{a}{r_0} \right)^2 \right] e^{2r_0 A^{1/3}/a} \right\} \\
 & + \frac{3}{5} \frac{e^2}{r_0} \left[\frac{Z^2}{A^{1/3}} - \frac{5}{2} \left(\frac{b}{r_0} \right)^2 \frac{Z^2}{A} - \frac{5}{4} \left(\frac{3}{2\pi} \right)^{2/3} \frac{Z^{4/3}}{A^{1/3}} \right] + W (|I| + d) - a_{el} Z^{2.39} + \begin{cases} \Delta - \frac{1}{2}\delta, & N \text{ and } Z \text{ odd} \\ \frac{1}{2}\delta, & N \text{ or } Z \text{ odd} \\ -(\Delta - \frac{1}{2}\delta), & N \text{ and } Z \text{ even} \end{cases}
 \end{aligned}$$

◆ The isoscaling behavior in first step decay and constrain of symmetry energy coefficient in GEMINI calculation

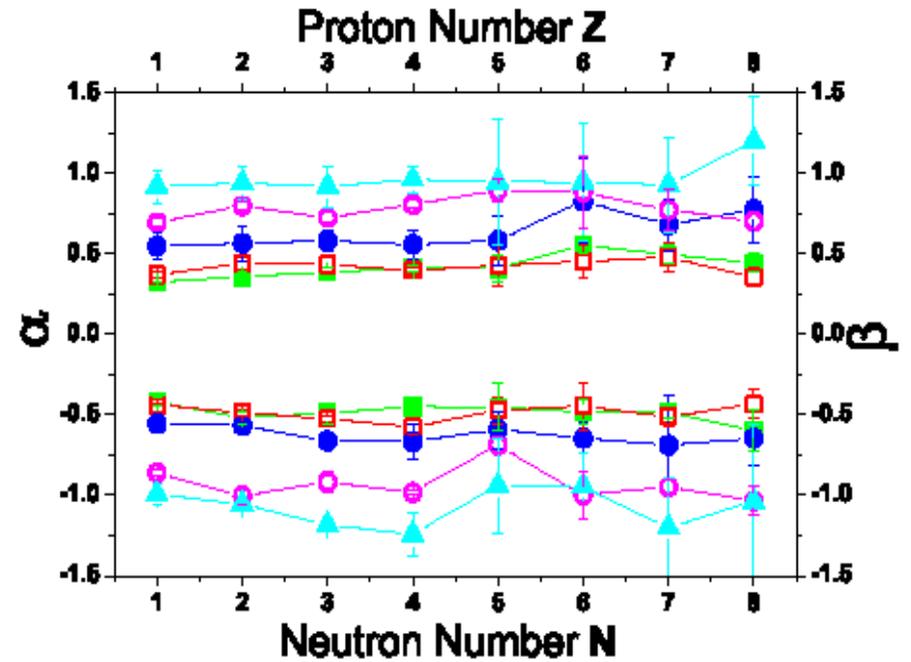
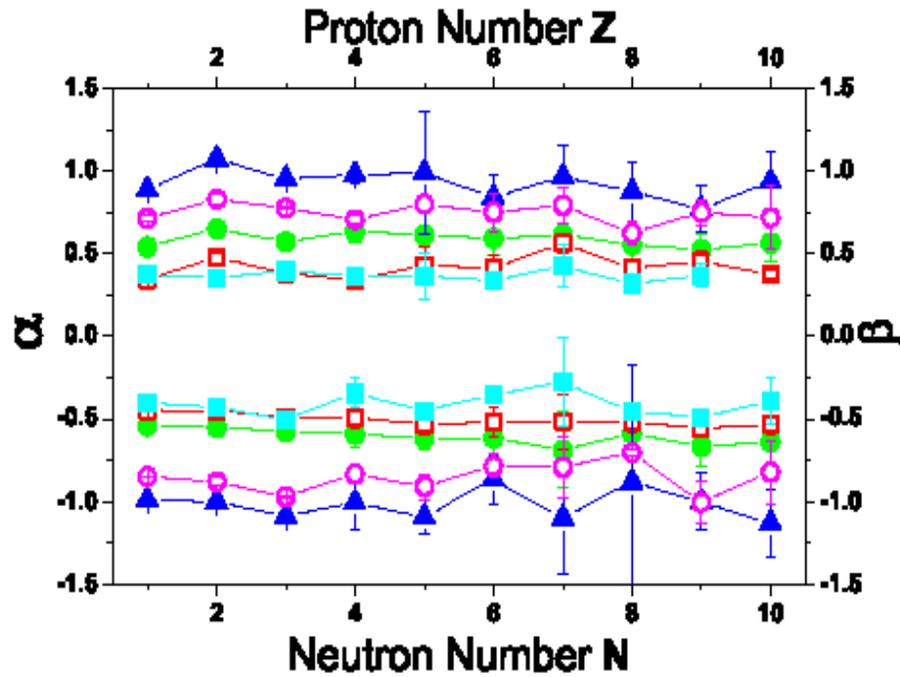


FIG. 2: (Color online) Isoscaling parameters α (positive values) and β (negative values) as a function of the fragment proton number Z or neutron number N from source pairs with the fixed proton number $Z_s = 50$ at excitation energies $E_{ex} = 2$ MeV/nucleon. Symbols in figure correspond to $Y_{A_s=115}/Y_{A_s=110}$ (solid squares), $Y_{A_s=110}/Y_{A_s=105}$ (open square), $Y_{A_s=105}/Y_{A_s=100}$ (solid circles), $Y_{A_s=115}/Y_{A_s=105}$ (open circles), $Y_{A_s=110}/Y_{A_s=100}$ (solid up triangles).

FIG. 3: (Color online) Isoscaling parameters α (positive values) and β (negative values) as a function of the fragment proton number Z or neutron number N from source pairs with the fixed proton number $Z_s = 30$ at excitation energies $E_{ex} = 2$ MeV/nucleon. Symbols in figure correspond to $Y_{A_s=69}/Y_{A_s=66}$ (solid squares), $Y_{A_s=66}/Y_{A_s=63}$ (open square), $Y_{A_s=63}/Y_{A_s=60}$ (solid circles), $Y_{A_s=69}/Y_{A_s=63}$ (open circles), $Y_{A_s=66}/Y_{A_s=60}$ (solid up triangles).

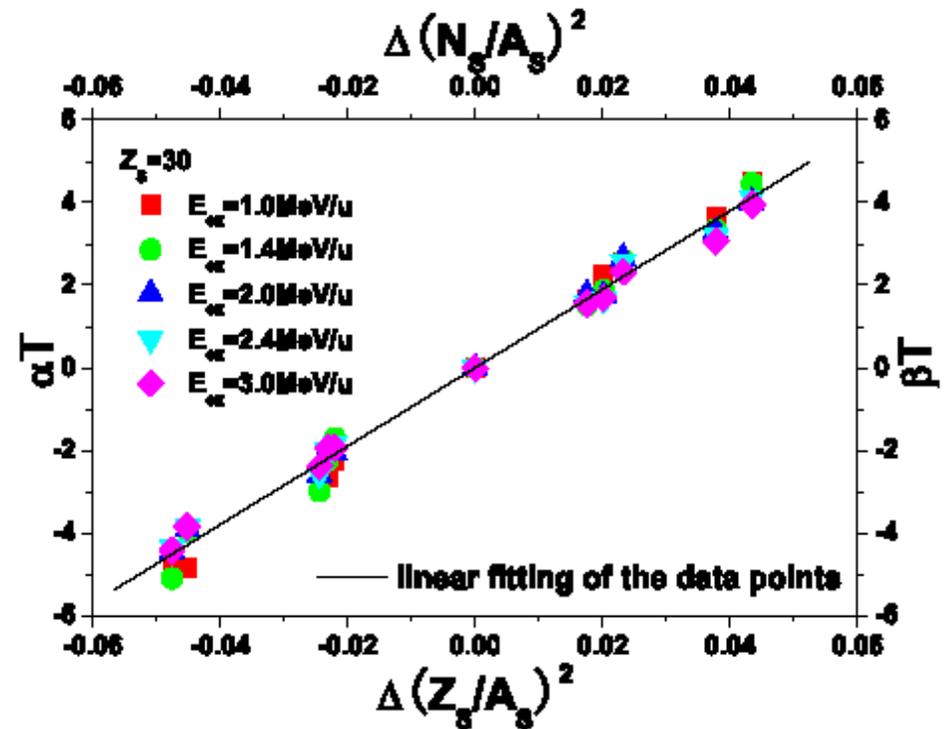
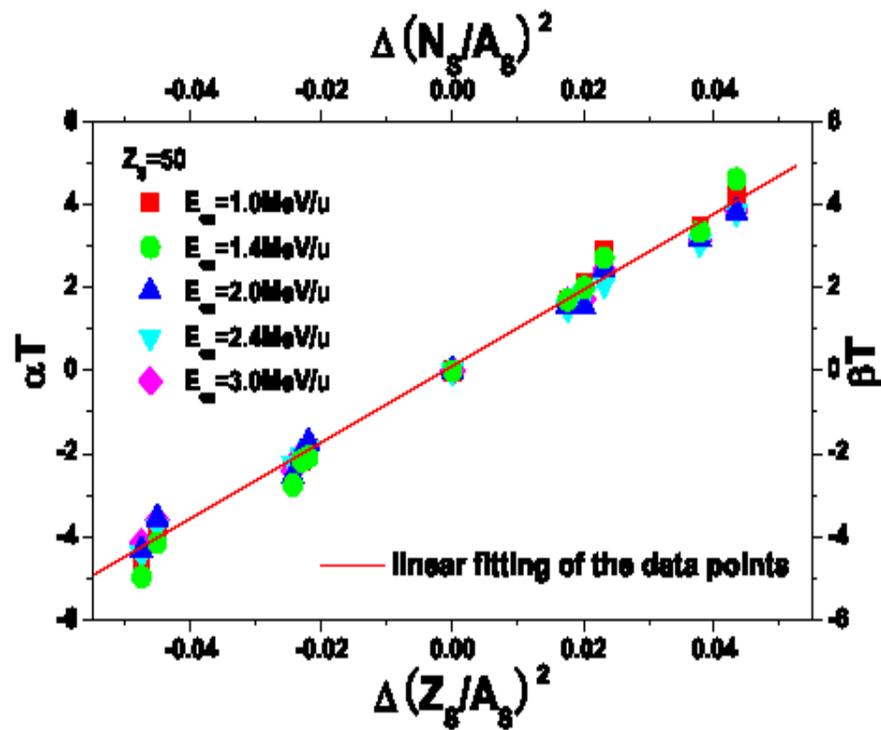


FIG. 6: (Color online) $\alpha \cdot T$ (positive parts) and $\beta \cdot T$ (negative parts) as a function of $(Z_s/A_s)_1^2 - (Z_s/A_s)_2^2$ or $(N_s/A_s)_1^2 - (N_s/A_s)_2^2$ of the sources for various source pairs with atomic

FIG. 7: (Color online) $\alpha \cdot T$ (positive parts) and $\beta \cdot T$ (negative parts) as a function of $(Z_s/A_s)_1^2 - (Z_s/A_s)_2^2$ or $(N_s/A_s)_1^2 - (N_s/A_s)_2^2$ of the sources for various source pairs with atomic

$$\alpha = \frac{4C_{sym}}{T} \left[\left(\frac{Z}{A} \right)_{s1}^2 - \left(\frac{Z}{A} \right)_{s2}^2 \right] \equiv \frac{4C_{sym}}{T} \Delta \left(\frac{Z}{A} \right)_s^2$$

$$\beta = \frac{4C_{sym}}{T} \left[\left(\frac{N}{A} \right)_{s1}^2 - \left(\frac{N}{A} \right)_{s2}^2 \right] \equiv \frac{4C_{sym}}{T} \Delta \left(\frac{N}{A} \right)_s^2$$

$C_{sym} = 23.0 \pm 0.7 \text{ MeV}$ (Fig. 6) $23.8 \pm 0.4 \text{ MeV}$ (Fig. 7)

W.D. Tian et al, PRC76,024607(2007)

Every step of the sequential decay, above relation work



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◆ Influence of the sequential decay

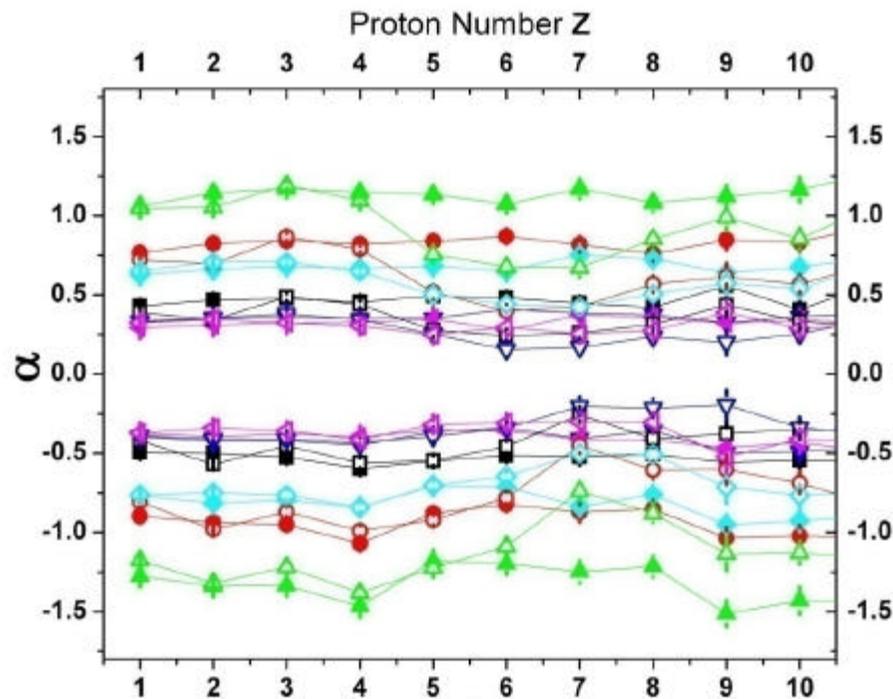


FIG. 1: (Color online) Comparison of Isoscaling parameters α (positive values) and β (negative values) as a function of the fragment proton number Z or neutron number N from source pairs of $Z_s=50$ at excitation energies $E_{ex}=2.4\text{MeV/nucleon}$, all solid symbols are only the first step secondary decay products, open symbols are full step secondary decay products, Symbols in the figure correspond to $Y_{A_s}=105/Y_{A_s}=100$ (squares), $Y_{A_s}=110/Y_{A_s}=100$ (circles), $Y_{A_s}=115/Y_{A_s}=100$ (up-triangles), $Y_{A_s}=110/Y_{A_s}=105$ (down-triangles), $Y_{A_s}=115/Y_{A_s}=105$ (Diamonds), $Y_{A_s}=115/Y_{A_s}=110$ (left-triangles).

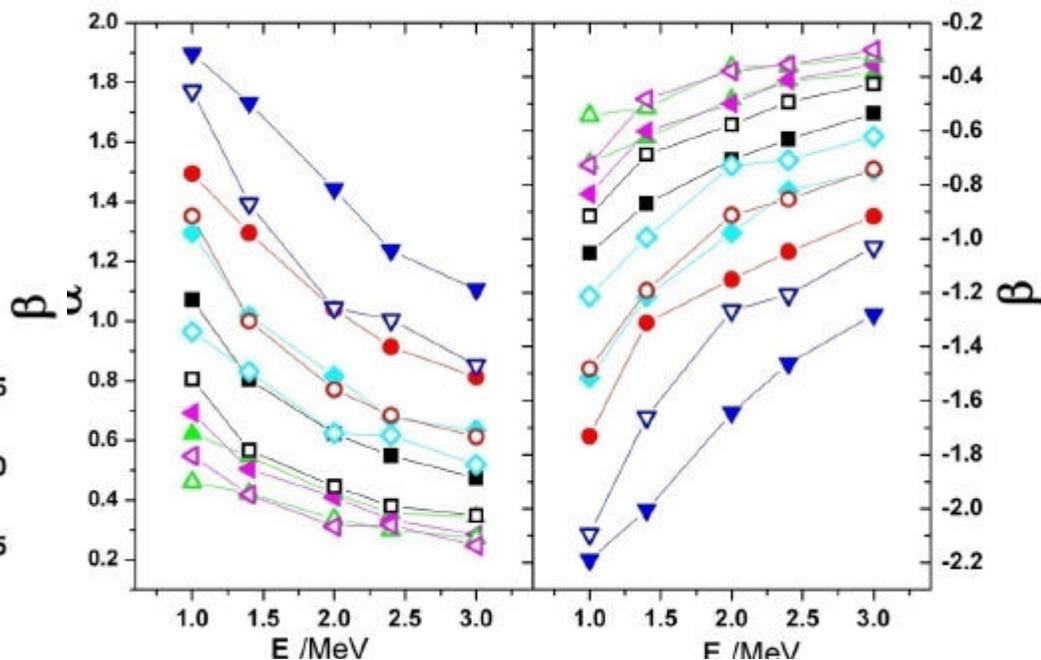


FIG. 2: (Color online) Comparison of Isoscaling parameters α (left panel) and β (right panel) as a function of the source excitation energy from source pairs of $Z_s=30$, all solid symbols are only the first step secondary decay products, open symbols are full steps secondary decay products, Symbols in the figure correspond to $Y_{A_s}=63/Y_{A_s}=60$ (squares), $Y_{A_s}=66/Y_{A_s}=60$ (circles), $Y_{A_s}=66/Y_{A_s}=63$ (up-triangles), $Y_{A_s}=69/Y_{A_s}=60$ (down-triangles), $Y_{A_s}=69/Y_{A_s}=63$ (Diamonds), $Y_{A_s}=69/Y_{A_s}=66$ (left-triangles).

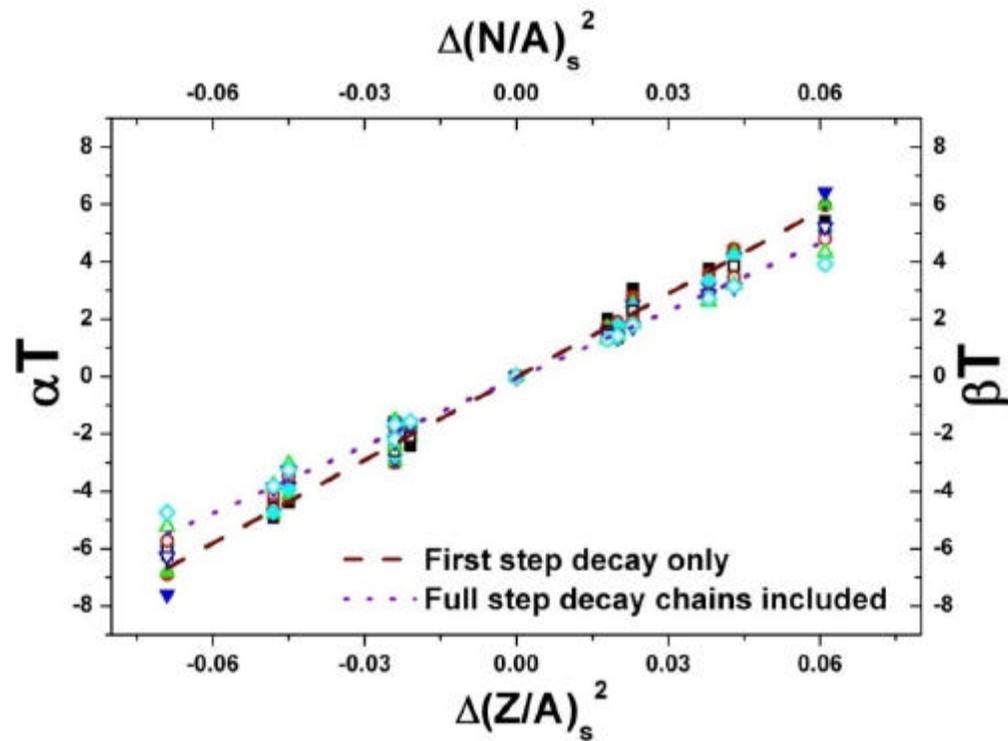


FIG. 3: (Color online) Linear fitting of Isoscaling parameters α (positive values) and β (negative values) as a function of the source isospin difference $\Delta(Z/A)_s^2$ from source pairs of $Z_s=30$, all solid symbols are only the first step secondary decay products, open symbols are full steps secondary decay products, Symbols in the figure correspond to Excitation energies $E_{ex}=1.0$ (squares), 1.4 (circles), 2.0 (up-triangles), 2.4 (down-triangles) and 3.0 MeV (Diamonds), the dash and dot lines are the linear fitting of the first step decay only and full step decay chains included case respectively.

$$\beta = \frac{4\gamma}{T} \left[\left(\frac{Z}{A} \right)_{s1}^2 - \left(\frac{Z}{A} \right)_{s2}^2 \right] \equiv \frac{4\gamma}{T} \Delta \left(\frac{Z}{A} \right)_s^2$$

$$\beta = \frac{4\gamma}{T} \left[\left(\frac{N}{A} \right)_{s1}^2 - \left(\frac{N}{A} \right)_{s2}^2 \right] \equiv \frac{4\gamma}{T} \Delta \left(\frac{N}{A} \right)_s^2$$

Full step decay chains included
Intermediate sources exist tracing
the decay process with different
temperature and isospin asymmetry

the most probable distribution source
temperature and isospin asymmetry
is not the initial values.

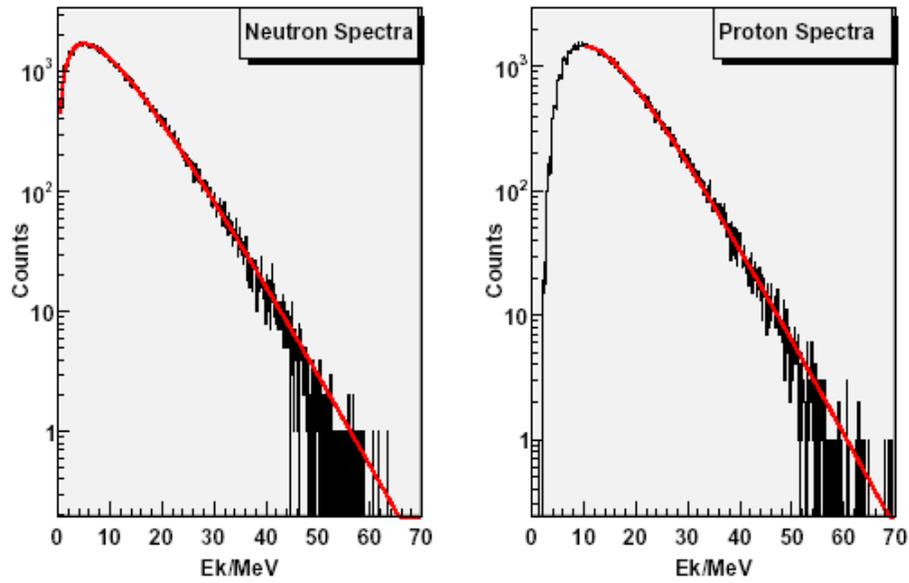


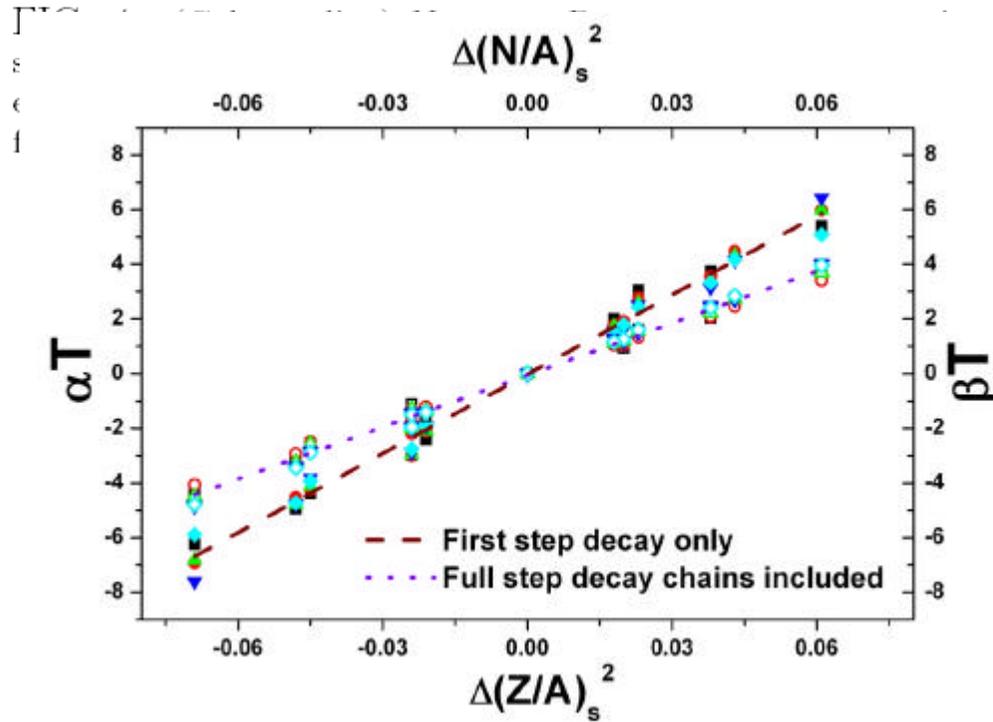
TABLE I: Source temperature extracted in different systems and by different methods.

Source		E_{ex} (MeV)	T_i^a	T_s^b (MeV)	T_r^c (MeV)
Z_s	A_s				
30	60/63	1.0	2.8/2.9	2.4/2.5	2.0/2.0
		1.4	3.4/3.5	3.0/3.1	2.3/2.4
		2.0	4.1/4.2	3.9/4.0	3.5/3.5
		2.4	4.5/4.6	4.4/4.5	4.0/4.0
		3.0	5.1/5.1	5.1/5.2	4.6/4.6
30	66/69	1.0	2.9/2.9	2.6/2.6	2.1/2.2
		1.4	3.5/3.5	3.2/3.2	2.6/2.6
		2.0	4.2/4.2	4.0/4.0	3.6/3.6
		2.4	4.6/4.7	4.6/4.6	4.0/4.0
		3.0	5.2/5.3	5.2/5.3	4.7/4.7

^aSource initial temperature calculated directly from the GEMINI code [22].

^bSource temperature fitted from emission neutron and proton spectra for the first step decay only.

^cReduced source temperature fitted from emission neutron and proton spectra for the full step decay chains included.



$$C_{sym} = 24.2 \pm 0.3 \text{ MeV}$$

Sequential \downarrow decay effect

$$\gamma = 15.84 \pm 0.18 \text{ MeV.}$$

$$\alpha = \frac{4C_{sym}}{T} \left[\left(\frac{Z}{A} \right)_{s1}^2 - \left(\frac{Z}{A} \right)_{s2}^2 \right] \equiv \frac{4C_{sym}}{T} \Delta \left(\frac{Z}{A} \right)_s^2$$

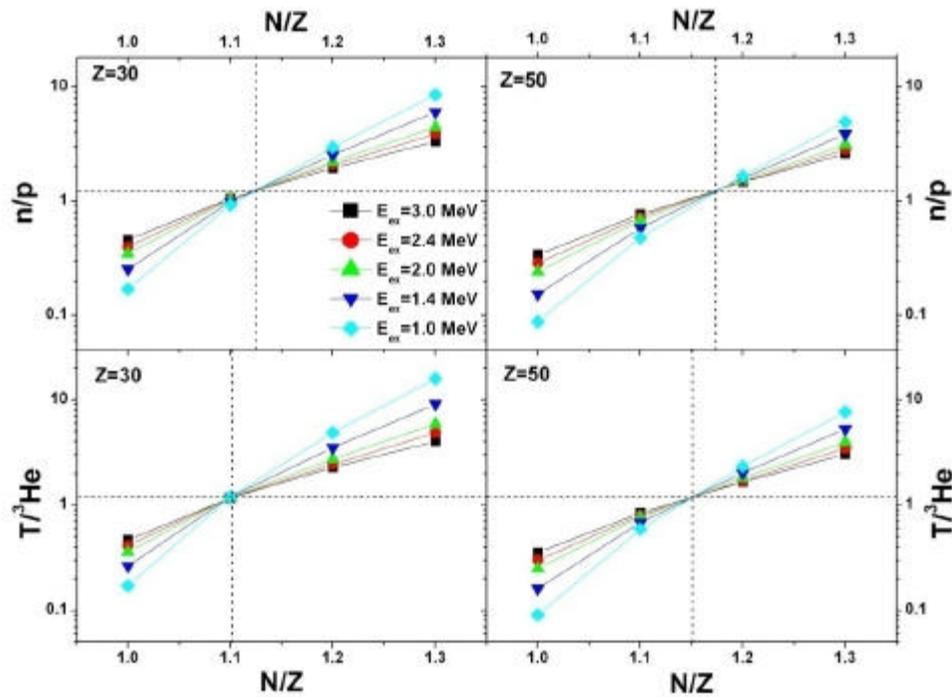
Reduced after sequential decay

Reduced after sequential decay

Should decrease after every step sequential decay, since the intermediate sources approach the stable line or the evaporation attract line,

But does this decrease compensate the change of a and T ?

◆ n/p and T/³He in GEMINI calculation

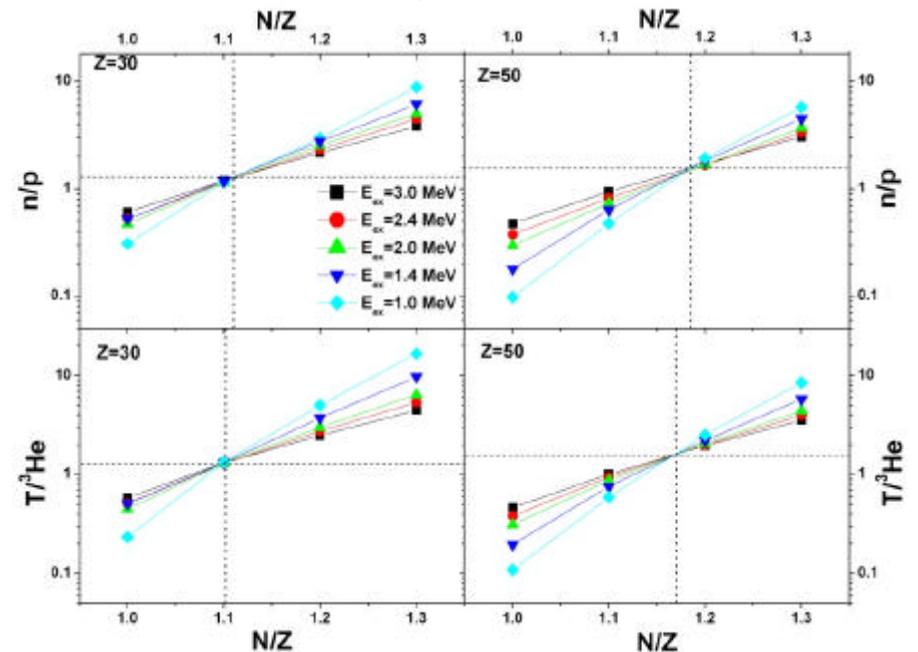


First Step decay only

Strong dependent on source isospin asymmetry N/Z, weak dependent on temperature

No large influence from sequential decay

Full Step decay chains



□ GDR study via QMD simulation for isospin asymmetry reaction

The prompt dipole γ -ray emission in GDR origin from the isospin collective dynamics

By using radioactive beams or isospin asymmetry beams,
seeking for the enhancement of the sensitivity the Iso-EoS.

Isospin dependent Quantum Molecular Dynamics model(QMD)

$$U(\rho, \tau_z) = \alpha \left(\frac{\rho}{\rho_0} \right) + \beta \left(\frac{\rho}{\rho_0} \right)^\gamma + \frac{1}{2} (1 - \tau_z) V_c + C_{\text{sym}} \frac{(\rho_n - \rho_p)}{\rho_0} \tau_z$$

the initial dipole moment

$$D(t=0) = \frac{NZ}{A} |R_Z(t=0) - R_N(t=0)| = \frac{R_P + R_T}{A} Z_P Z_T \left| \left(\frac{N}{Z} \right)_T - \left(\frac{N}{Z} \right)_P \right|,$$

prompt photon emission probability $E_g = \hbar\omega$

$$\frac{dP}{dE} = \frac{2}{3\pi} \frac{e^2}{E\hbar c} \left| \frac{d\overline{V}_k}{dt}(E) \right|^2,$$

$$\frac{d\overline{V}_k}{dt}(E) = \int_0^\infty \frac{d\overline{V}_k}{dt}(t) e^{i(Et/\hbar)} dt$$

V. Baran *et al.*, *Nucl. Phys. A* **679**, 373 (2001).

M. Papa *et al.*, *Phys. Rev. C* **68**, 034606 (2003).



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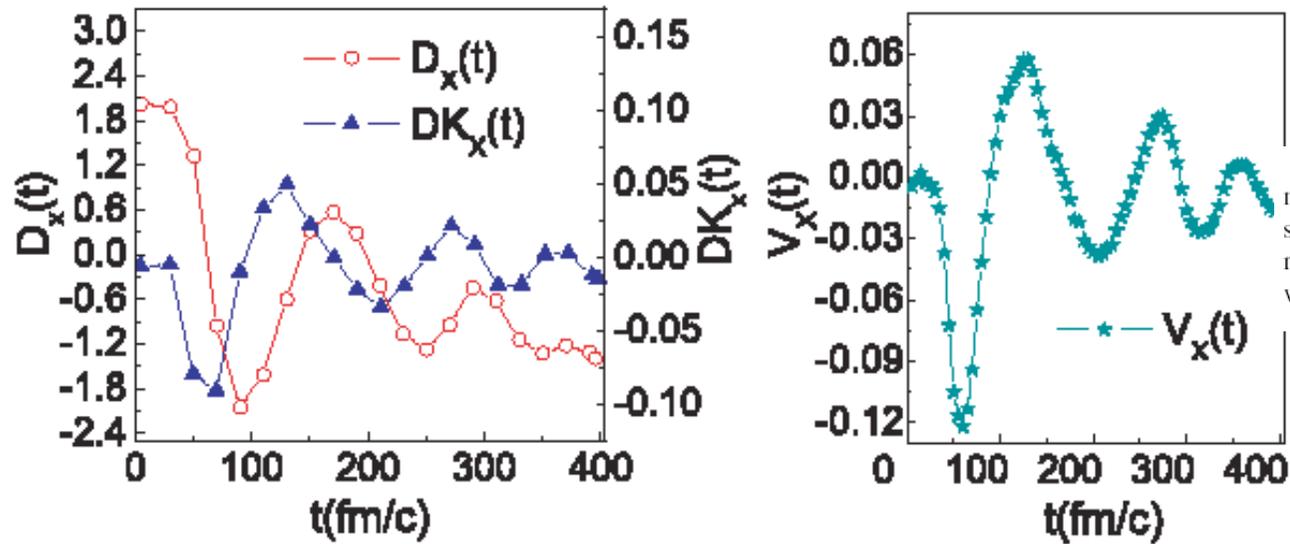
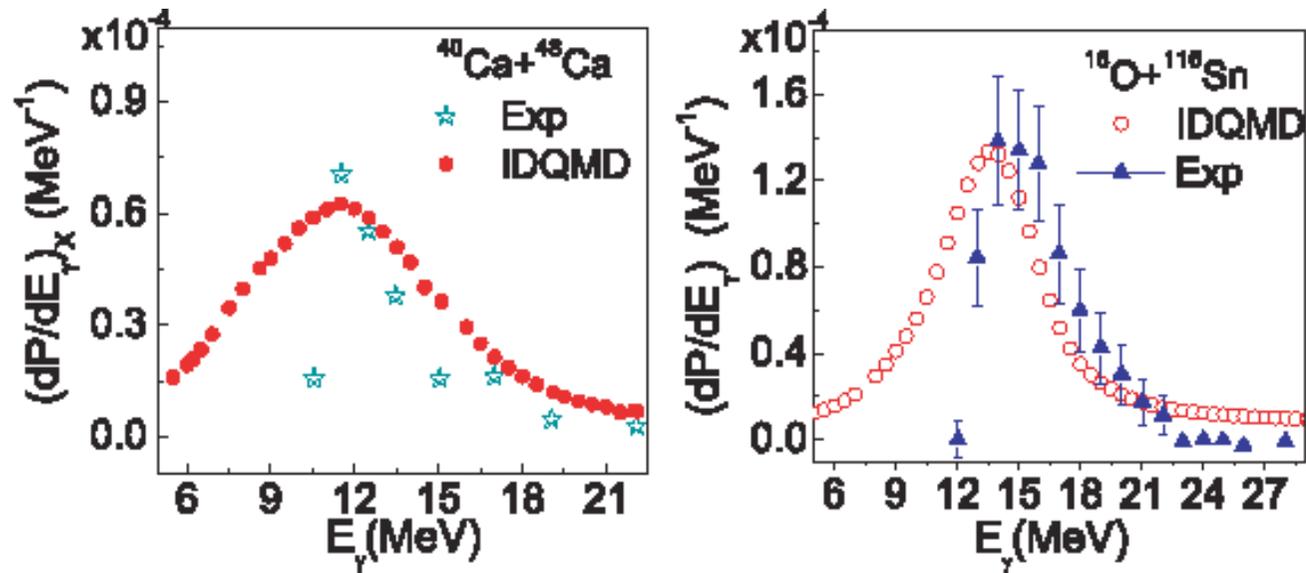


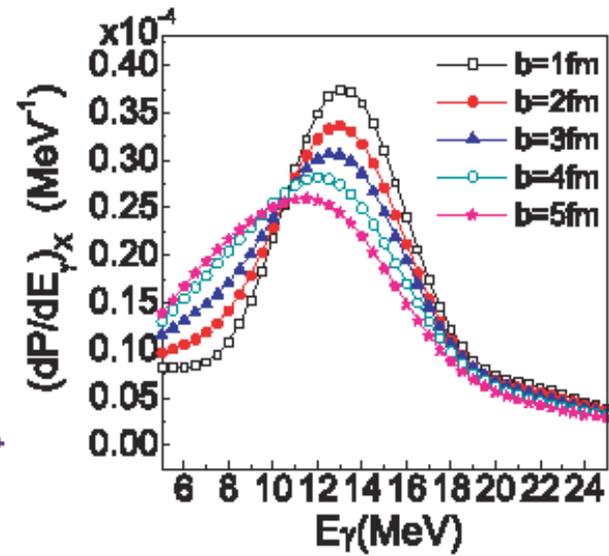
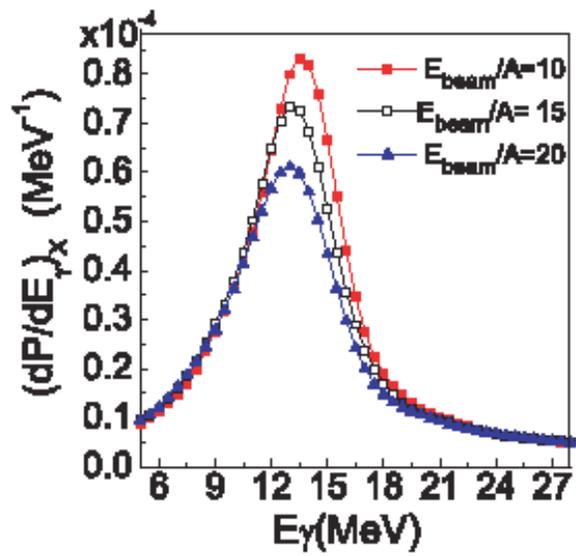
FIG. 1. (Color online) Left: Evolution of the giant dipole moment with time in coordinate space [$D_x(t)$] and momentum space [$DK_x(t)$]. Right: Dynamical dipole mode [$V_x(t)$]. The reaction system is $^{40}\text{Ca} + ^{48}\text{Ca}$ collision at 10 MeV/nucleon with $b = 1$ fm and soft EOS parameters.

Comparison with experimental data

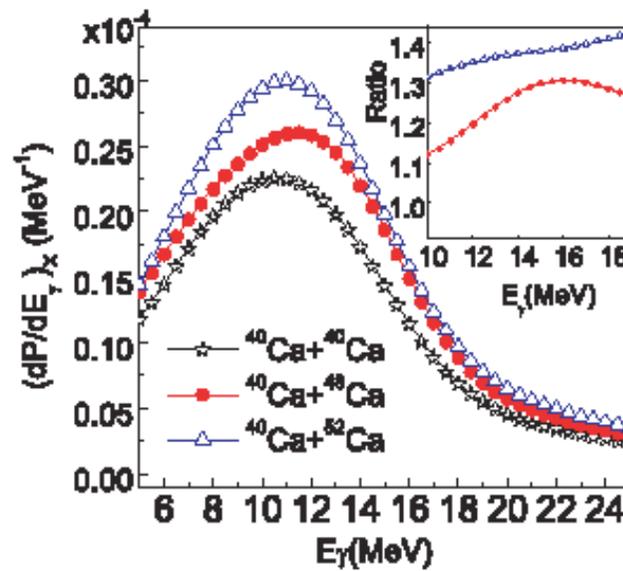
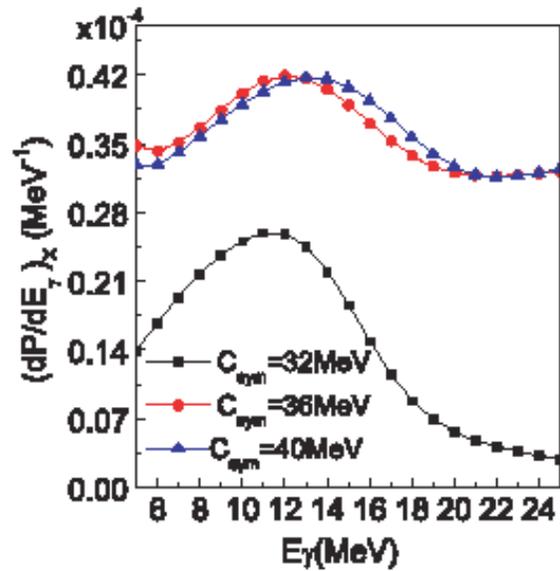


Corsi *et al.*, *Phys. Lett. B* **679**, 197 (2009).

M. Papa *et al.*, *Phys. Rev. C* **72**, 064608 (2005).



Dipole dynamical emission spectra of the $^{40}\text{Ca}+^{48}\text{Ca}$ system at different incident energies and impact parameter b



Dipole dynamical emission spectra of the $^{40}\text{Ca}+^{48}\text{Ca}$ system with different symmetry energy strength and isospin asymmetry N/Z

H. L. Wu et al, Phys.Rev. C 81, 047602 (2010)



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□ Summary

Sequential decay was investigated, It was found that isoscaling still can be observed, secondary sequential decrease the isoscaling parameters.

- 1) If the initial source parameters are used to constrain the symmetry energy coefficient, C_{sym} extracted from cold fragments is different from the hot fragments.
- 2) Use an experimental method to extract the source temperature, that C_{sym} constrain from cold fragment is reduced further.
- 3) The isospin difference from the sources was also changed in the secondary decay procedure, which will affect C_{sym} .

Secondary sequential effect affect the constraint of symmetry energy coefficient C_{sym} , which need to be corrected for the cold fragments.

n/p and $T/{}^3\text{He}$ show strong dependence on source isospin N/Z .

γ rays of the dynamical dipole resonance is sensitive to the isospin freedom and symmetry energy, it can be a signal to measure the symmetry energy.



Thanks



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Wendong Tian RIKEN, Wako, July, 26-28, 2010

