NuSYM10, July 26- 28, RIKEN, Wako, 2010

Nuclear Symmetry Energy in Compact Star Matter

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Work in progress with
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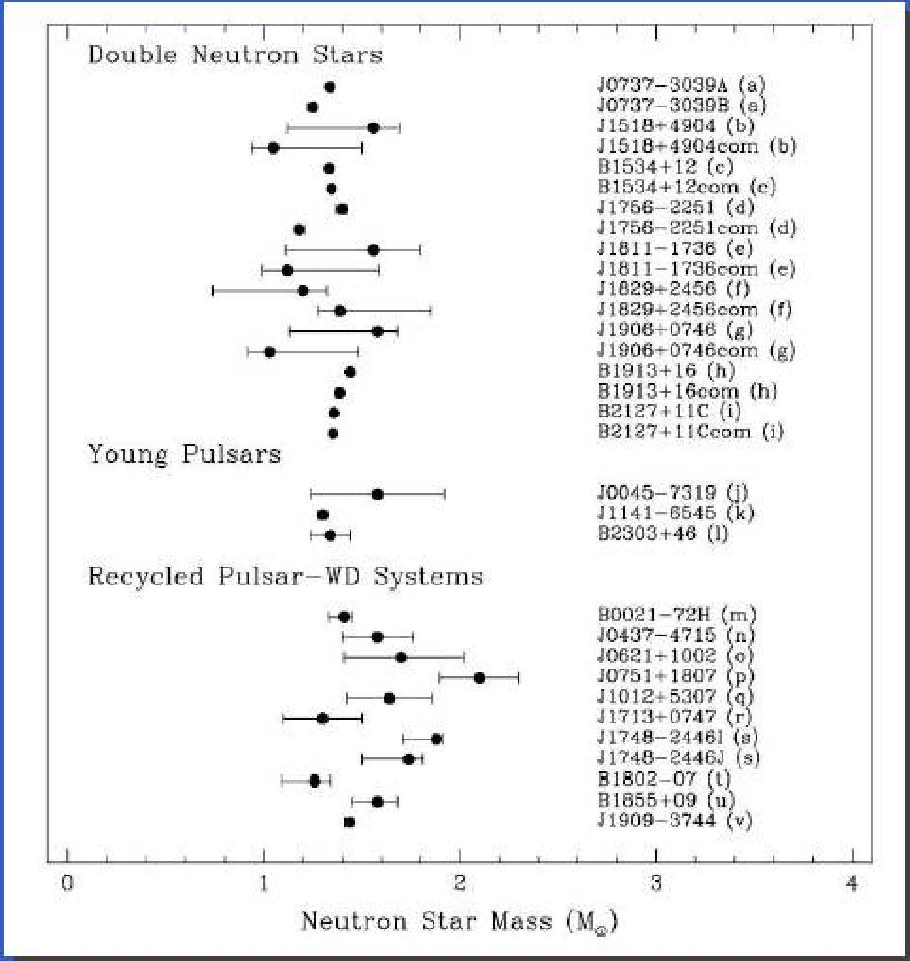
극한조건에서의 강입자 물질 연구사업단
Hadronic Matter under Extreme Conditions

I. Compact Stars

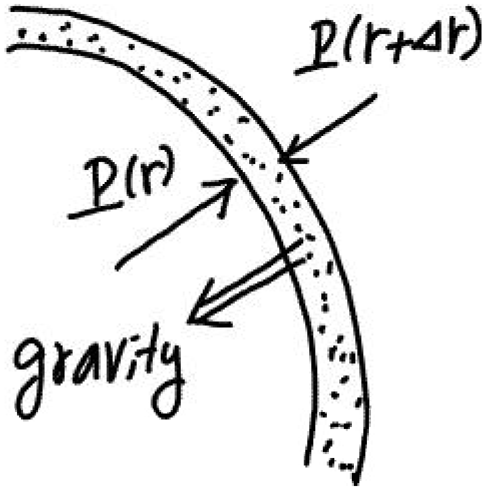
- Stars (neutron stars, quark stars, ..)
  ~ solar mass and ~10km or less in size
- Constituents of compact star
  - Hadrons responsible for mass of star
    - neutrons, protons, hyperons
    - pions, kaons, ...
  - Leptons
    - electrons, muons,
    - neutrinos(e-, mu-)
  - Exotics ?
- ~ Zero temperature, Charge neutrality

Masses of Pulsars (Stairs 2006)



- more than 1800 pulsars known with 140 binary pulsars
- best determined mass: $M = (1.4414 \pm 0.0002) M_\odot$ Hulse-Taylor pulsar (Weisberg and Taylor, 2004)
- mass of PSR J0751+1807 corr. from $M = (2.1 \pm 0.2) M_\odot$ to $M = (1.14 - 1.40) M_\odot$ (Nice et al. 2008)
- mass of PSR J1903+0327 (not finalized yet): $M = (1.67 \pm 0.01) M_\odot$ (Freire et al. 2009)

- Stars and Gravity



TOV equation

- Equation of state(EOS) of compact star

  energy density and pressure

$$P = n^2 \frac{\partial(\epsilon/n)}{\partial n}$$

- Higher nucleon number density inside

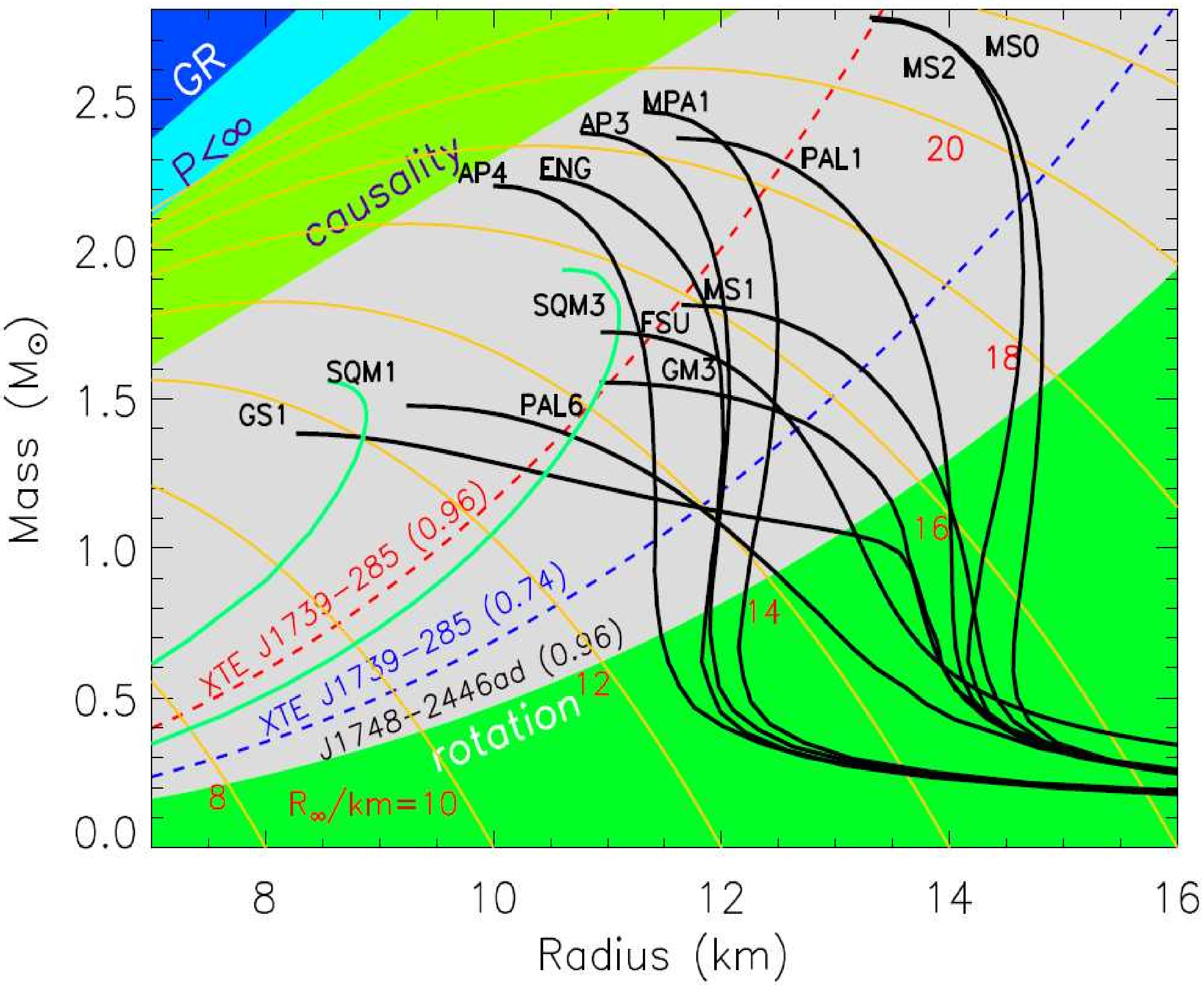
  n =0 → n_0 → 6n_0 →

1. Change of nucleon-nucleon interaction with density
2. Emerging of new hadrons with density

   kaons, hyperons(strange hadrons), ..

- Dense Hadronic Matter at the Core

→ Change of EOS with density



J.M. Lattimer and M. Prakash, Phys.Rept.442:109-165,2007

Ⅱ. Hadronic Interactions

- Fermi pressure of nucleons
  + Strong interaction
- Nucleon-nucleon repulsion and attractions
- N-N interaction is known only upto nuclear density, $n_0 = 0.16\ fm^{-3}$

- Heavier nuclei

Mass formula (Weizsäcker, 1935; Bethe and Bacher, 1936)
Binding energy of a nucleus ($A = Z + N$)

$$\mathcal{B}(N,Z) = b_{\rm vol}A - b_{\rm surf}A^{2/3} - \frac{1}{2}b_{\rm sym}\frac{(N-Z)^2}{A} - \frac{3}{5}\frac{Z^2e^2}{R_c}$$

- Dense nuclear matter

$$E(n, N_p) \simeq m_N + \frac{3}{5}E_F^0\left(\frac{n}{n_0}\right)^{2/3} + S(n)(1-2N_p)^2 + V(n)$$

$$E_F^0 = \frac{(3\pi^2 n_0/2)^{2/3}}{2m} = 37.2\text{MeV}$$

- n > n_0, ?????
- Lattice QCD, Effective theory for hadrons, Many body inetractions, ...

III. Nuclear Symmetry Energy

- Measure of n-p asymmetry in nuclear interaction
- In nuclear matter

$$E(n, N_p) \simeq m_N + \frac{3}{5} E_F^0 \left(\frac{n}{n_0}\right)^{2/3} + S(n)(1 - 2N_p)^2 + V(n)$$

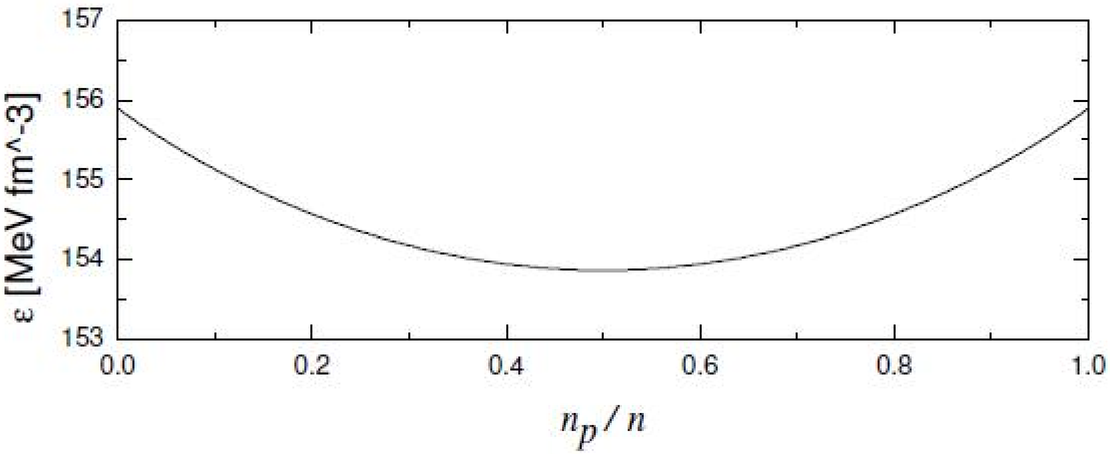
$$S(n) = S_{free}(n) + S_{int}(n)$$

$$S(n_0) \sim 30 \text{ MeV}$$

- Non-interacting n-p system

$$\epsilon_n = \frac{8\pi}{(2\pi)^3}\int_0^{p_F} (p^2 + m_n^2)^{1/2} p^2 dp$$

$$n_n = \frac{8\pi}{(2\pi)^3}\int_0^{p_F} p^2 dp$$



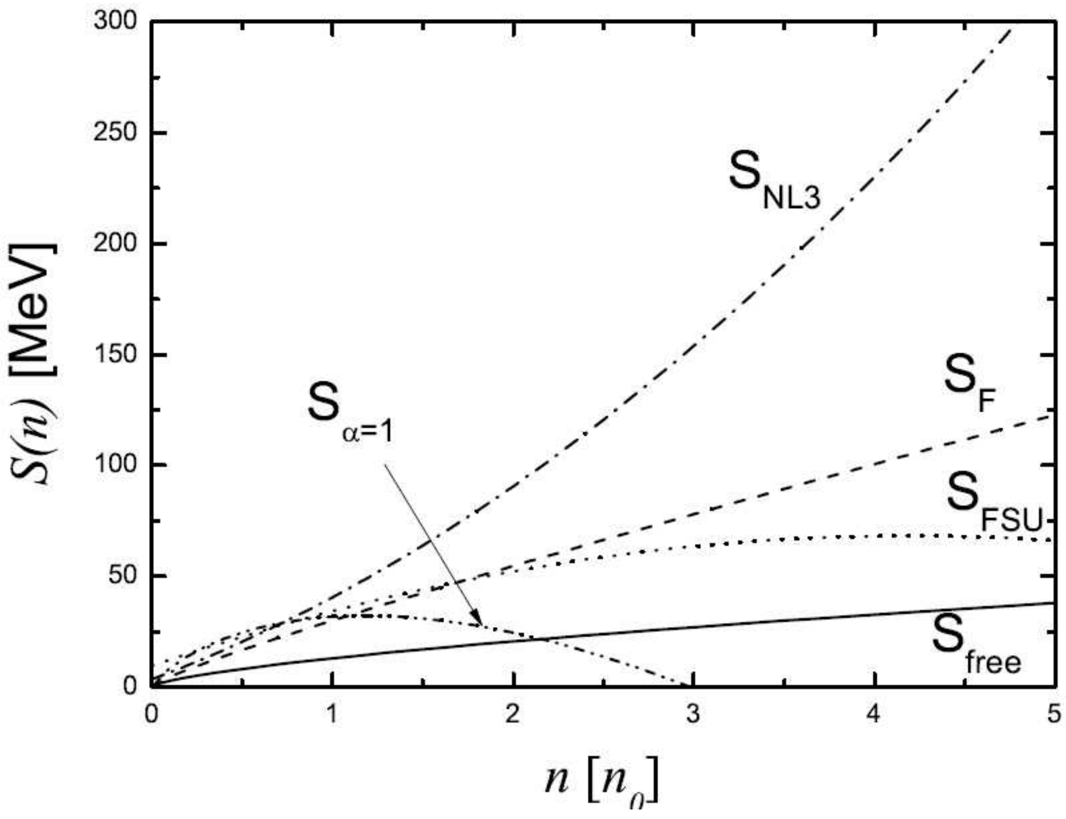
$$S_{free}(n) = \left(2^{2/3} - 1\right)\frac{3}{5}E_F^0\left(\frac{n}{n_0}\right)^{2/3}$$

Phenomenological forms of symmetry energy

$$S_F(n) = (2^{2/3} - 1)\frac{3}{5}E_F^0 \left[\left(\frac{n}{n_0}\right)^{2/3} - F(n)\right] + S_0 F(n)$$

$$S_\alpha = (2^{2/3} - 1)\frac{3}{5}E_F^0 \left(\frac{n}{n_0}\right)^{2/3} + A(\alpha)\frac{n}{n_0} + [18.6 - A(\alpha)]\left(\frac{n}{n_0}\right)^{B(\alpha)}$$

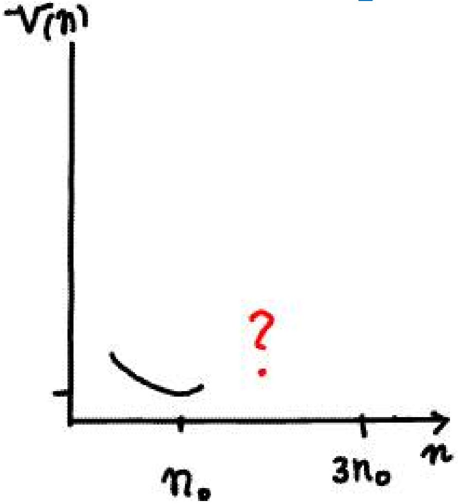
$$S_3(n) \simeq S_0^* + L\rho + \frac{1}{2}K\rho^2$$

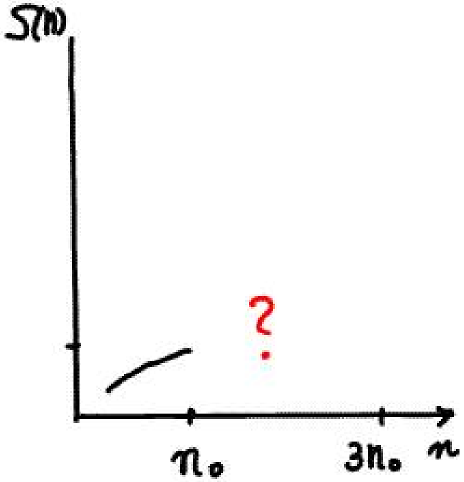


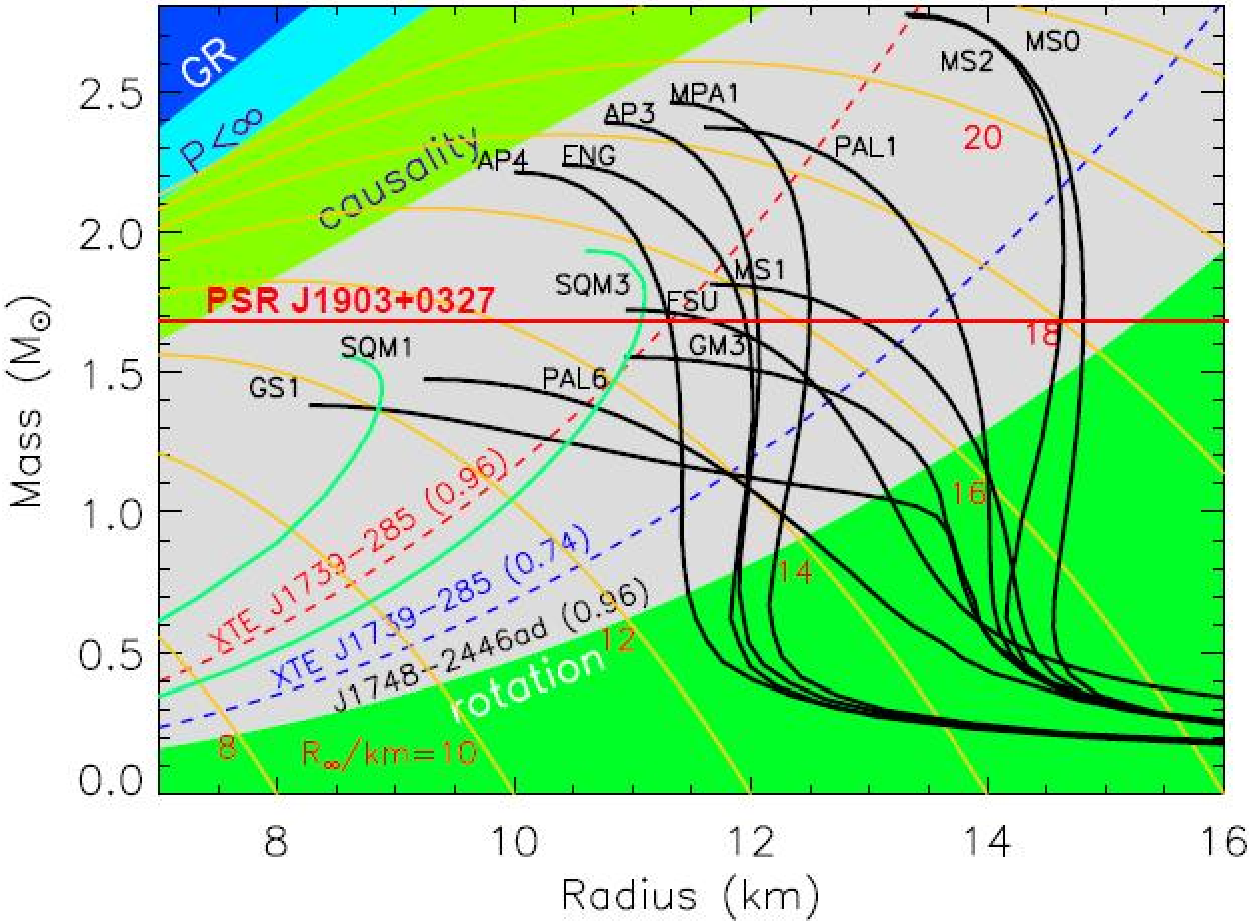
- Up to nuclear density , $n_0 = 0.16\ \mathrm{fm}^{-3}$
- Compact star, $n > n_0$ with central density $n_{center} > 3\ n_0$

$S(n)$? $V(n)$?

- Simple extrapolation of what are known at low density nuclear matter?






GR
P<∞
causality
rotation
PSR J1903+0327
XTE J1739-285 (0.96)
XTE J1739-285 (0.74)
J1748-2446ad (0.96)
MS0
MS2
MPA1
AP3
PAL1
AP4
ENG
MS1
SQM3
FSU
GM3
SQM1
PAL6
GS1
20
18
16
14
12
8
R∞/km=10
Mass (M☉)
Radius (km)
0.0
0.5
1.0
1.5
2.0
2.5
8
10
12
14
16

IV. Nuclear symmetry energy in compact stars

- What is the role of symmetry energy(n-p asymmetry) in compact star?
- Symmetry energy provides a channel for new degrees of freedom in n-p system via weak interaction:

  muon,

  strange particles(kaon, hyperon) ,..

Symmetry energy and weak equilibrium in compact star

- chemical potential of neutron and proton

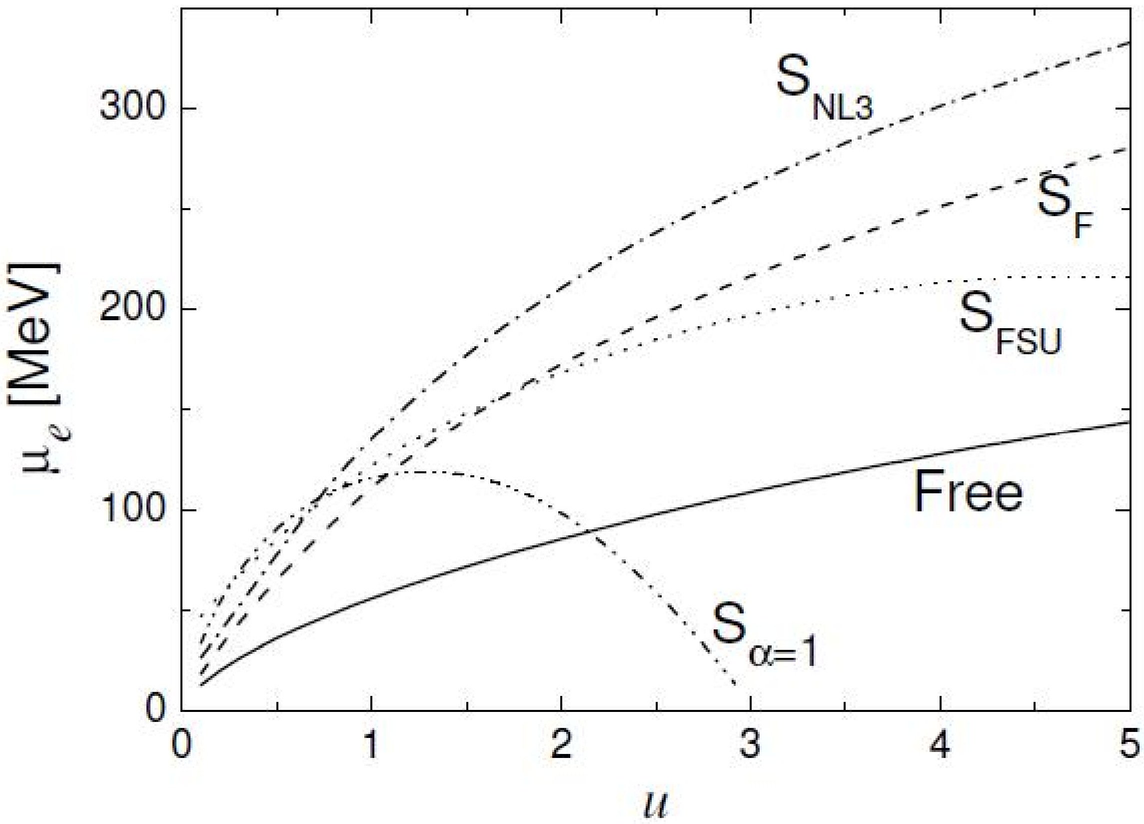
$$\mu_n - \mu_p = 4(1 - 2N_p)S(n)$$

- beta equilibrium with electron

$$n \rightarrow p + e \quad \Longrightarrow \quad \mu_n - \mu_p = \mu_e$$

- charge neutrality condition

$$n_p = n_e$$



S_{NL3}
S_F
S_{FSU}
Free
$S_{\alpha=1}$
μ_e [MeV]
u
0
100
200
300
0
1
2
3
4
5

- Strong interaction

$$\mu_n - \mu_p = 4(1-2N_p)S(n) + [K, \Sigma, \Xi \cdots]$$

- Weak equilibrium with new degrees of freedom,

$$\mu_n - \mu_p = \mu_\mu (m_\mu)$$
$$\mu_{K^-} (m_K^*)$$
$$\mu_e = \mu_{\Sigma^-} - \mu_n$$
$$\mu_{\Xi^-} - \mu_\Lambda$$

- Kaon condensation, hyperon matter, ..

- Hadron interaction at high density

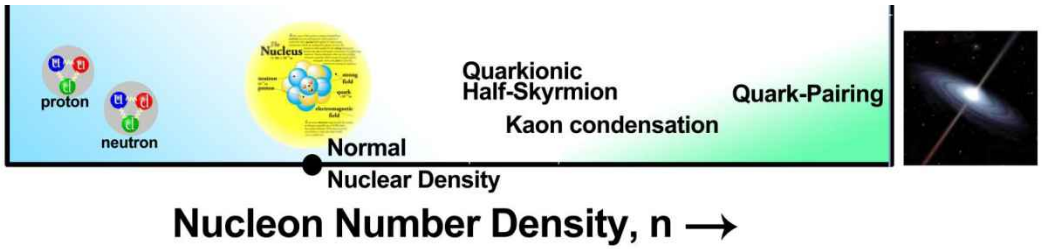
$S(n)$ $[K, \Sigma, \Xi \cdot\cdot]$ $m^*_{K^-}$

- Chiral perturbation theory
- Tensor force with BR scaling
- Thee body forces
- HLS
- Skyrmion on the lattice
- hQCD

....

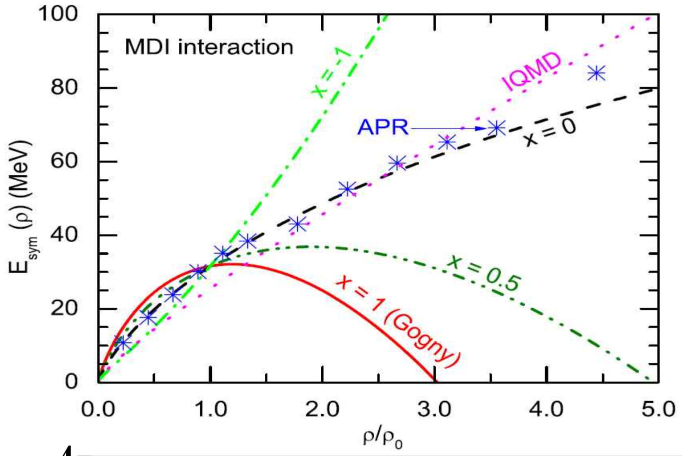
piece-wise effective theories(or polytropic EOS)

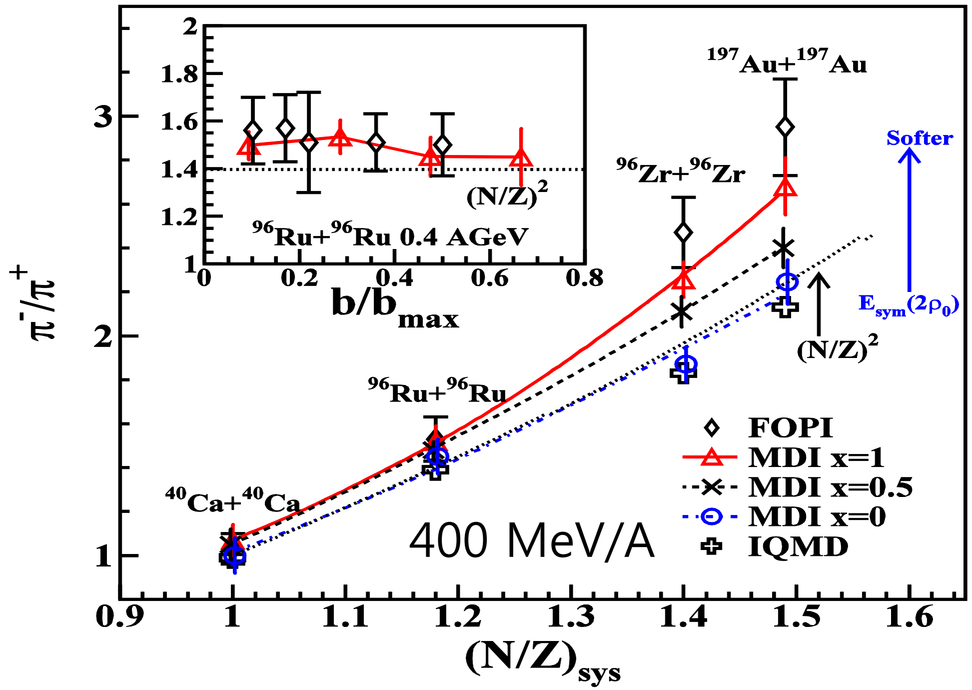
...., n(i)<n< n(i+1), ..

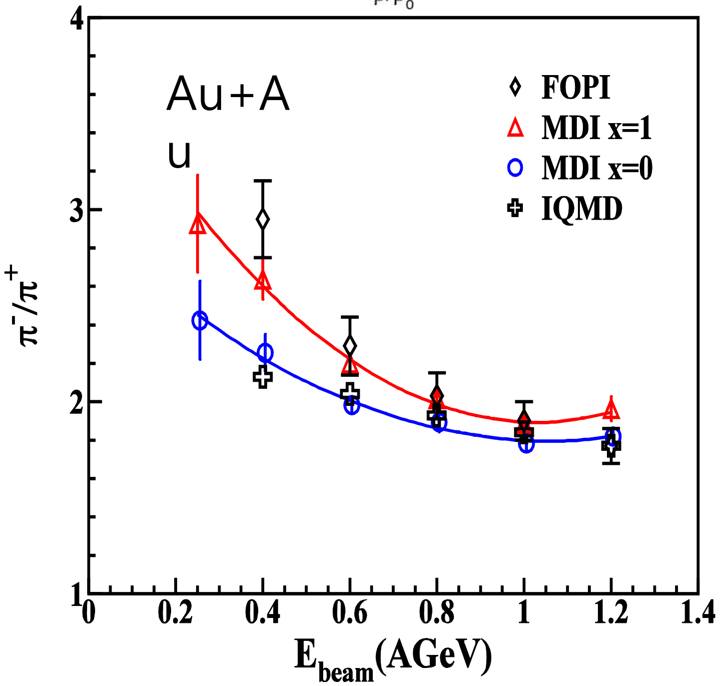


V. Supersoft symmetric energy

Transport model analysis using IBUU04
Z. Xiao, B.A. Li, L.W. Chen, G.C. Yong and M. Zhang, PRL 102, 062502 (2009)







"Supersoft" is a disaster?

M.Rho, Trento, 2010

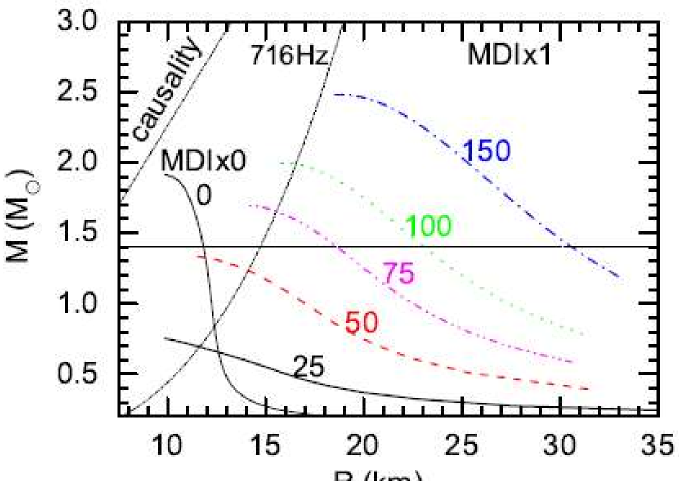
If Nature chose the "supersoft" E_{SS}

- There could be NO stable neutron stars *unless* …!!

  But Nature is full of neutron stars including the Hulse-Taylor binary pulsar.

- Drastic wayout by D.H. Wen *et al,* PRL **103,** 211102 (09):
  Modify Newtonian gravity, which could be *emergent*
  (e.g., *a la* E. Verlinde, arXiv:1001.0785).

But kaons will not condense,

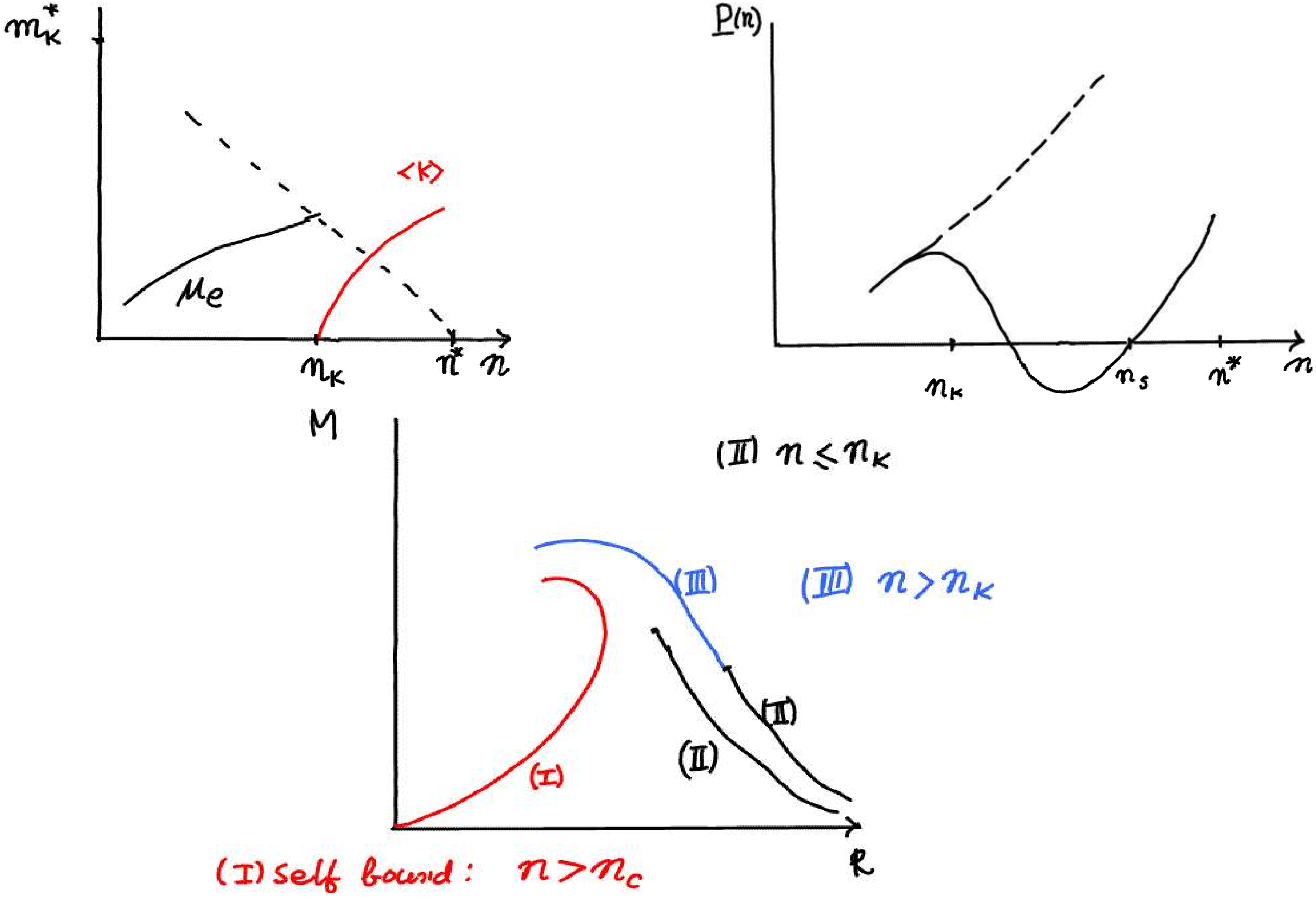


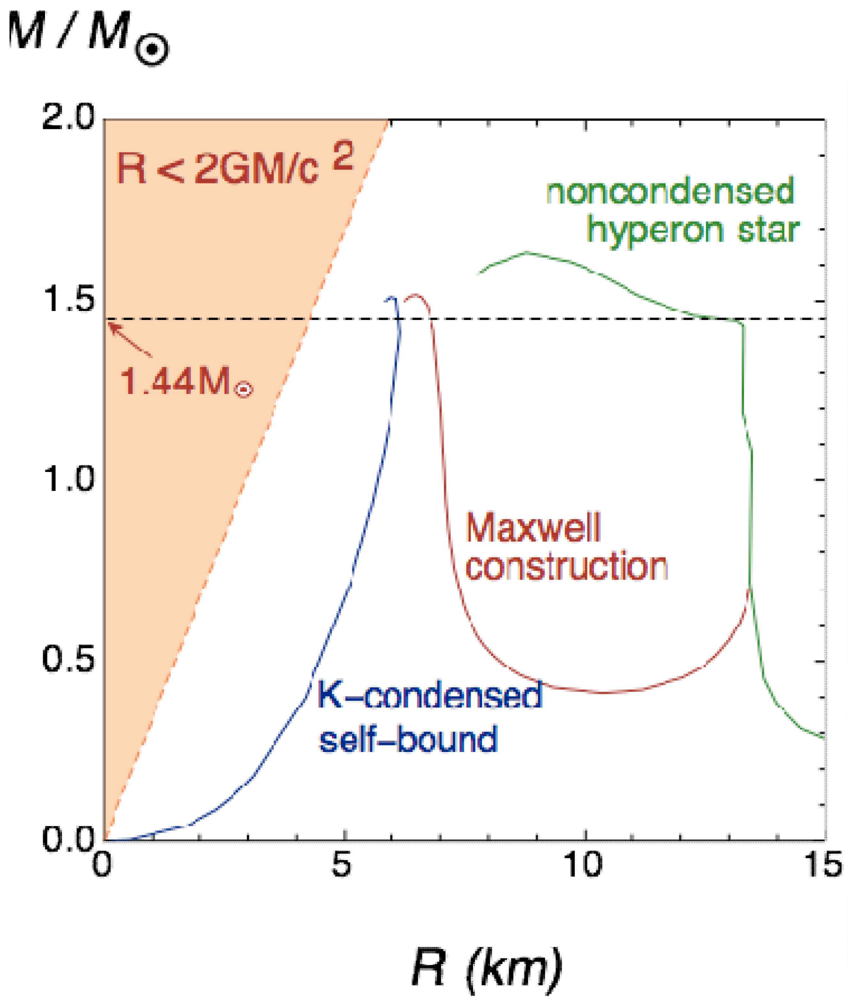
VI. Possible structure with Strangeness

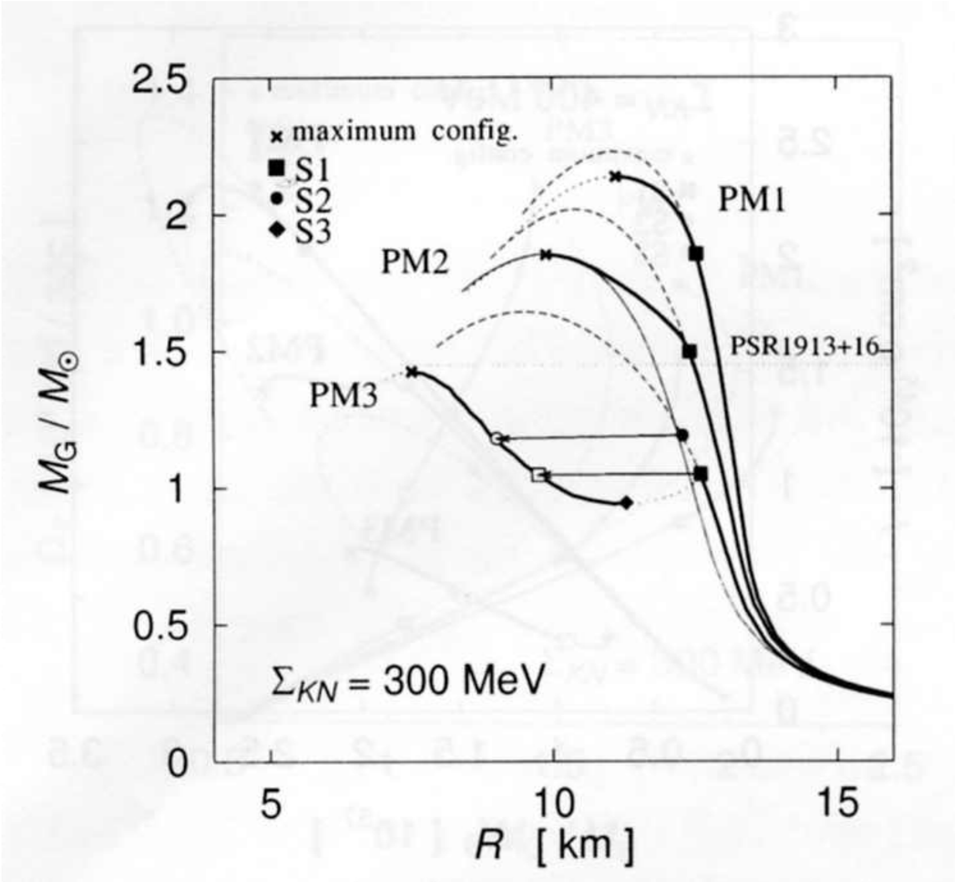
- When the difference between chemical potentials of proton and neutron becomes comparable to kaon mass or hyperon mass in medium, the corresponding strange particles begin populating and the EOS get changed significantly.
- Kaon condensation

  Kaplan and Nelson,
  Bethe and Brown,
  Brown, Rho and Kubodera, …

Stars with kaon condensed matter




M / M⊙
R < 2GM/c 2
noncondensed
hyperon star
1.44M⊙
Maxwell
construction
K-condensed
self-bound
R (km)


maximum config.
S1
S2
S3
PM1
PM2
PM3
PSR1913+16
M_G / $M_\odot$
Σ_{KN} = 300 MeV
R [km]

Muto et al.

Strange quark star with kaon-condensed surface (K. Kim, M. Rho and HKL in progress)

- For a hadron system, where kaon chemical potential becomes much small about electron mass, there is no leptons to balance the positive charge of protons but kaons.
- This is equivalent to strange quark matter with high enough density where strange quark mass can be neglected.
- It can define the transition surface between hadronic phase with kaon condensation and strange quark matter

Hadronic Matter | Quark Matter

$$p \Rightarrow u\,u\,d$$

$$n \Rightarrow u\,d\,d$$

$$K^- \Rightarrow s\,\bar{u}$$

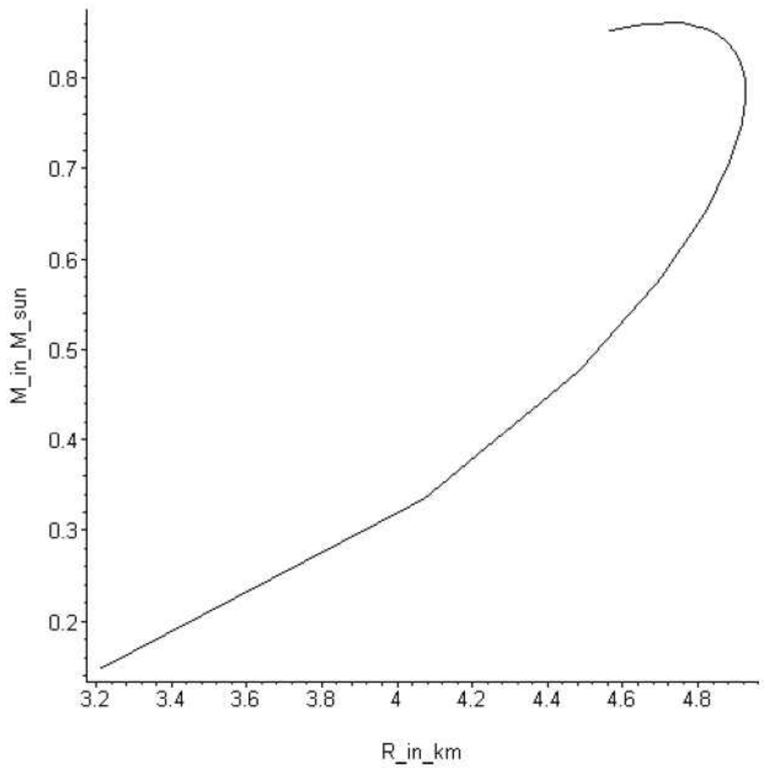
$$\mu_K = 0 \Rightarrow \mu_s = \mu_u$$

$$\mu_n = \mu_p \Rightarrow \mu_u = \mu_d$$

Kaon condensation
At zero chemical potential

SQM

- SQ star with self bound kaon surface



preliminary

VII. Hadronic crystal-structure at higher density?

- Nucleon crystal structure at the core of compact star at higher density
- Solitonic description →Skyrmions on the lattice
- Density dependence of symmetry energy and effective mass of roaming particles through(kaon mass)

Appearance of fractionized skyrmions

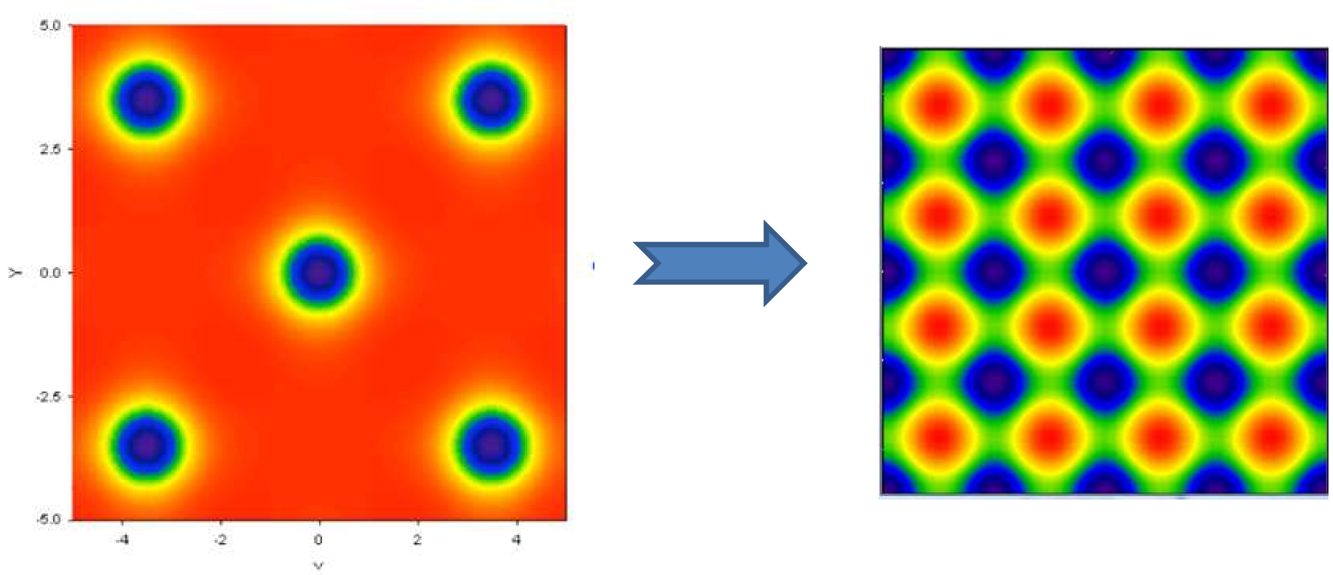
B.Y. Park et al,1999

$\mathbf{U} = \mathbf{e}^{\mathbf{2i}\pi/\mathbf{f}_\pi}$ ⟶ skyrmion

$\mathbf{U} = \xi_\mathbf{L}\xi_\mathbf{R}^\dagger, \quad \xi_{\mathbf{L},\mathbf{R}}$ ⟶ half-skyrmion

$$\mathcal{L}_\xi = \frac{f_\pi^2}{2}\{\mathrm{Tr}\left[|D_\mu\xi_L|^2 + |D_\mu\xi_R|^2\right]$$

- Simulate dense matter by putting skyrmions in FCC crystals and squeeze them: ½-skyrmions in CC appear at $n_{1/2}$

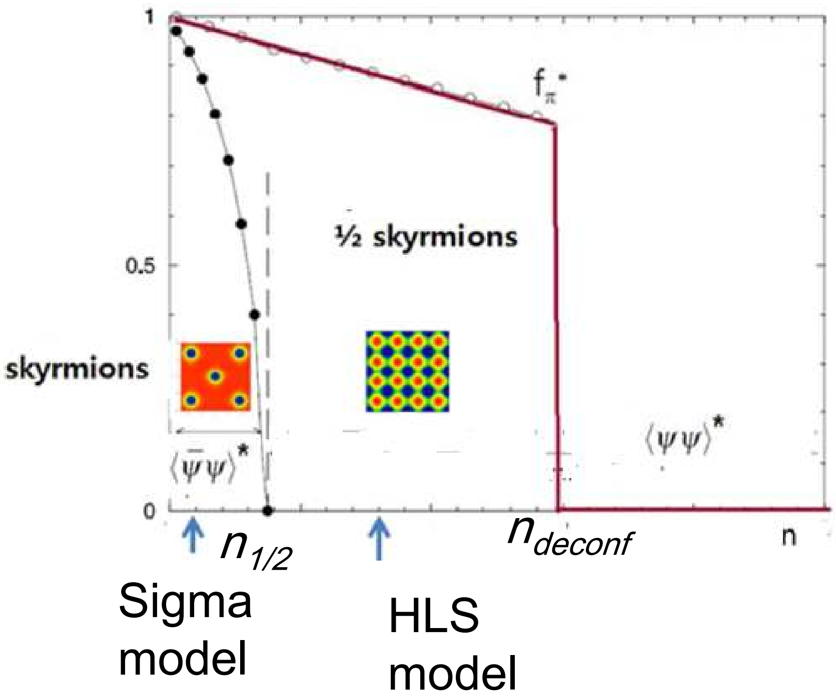


skyrmions

Half-skyrmions

M.Rho, Trento, 2010

"Phase"structure



The ½-skyrmion phase at $n \geq n_{1/2}$ is characterized by

$$\langle \bar{q}q \rangle = 0, \quad f_\pi \neq 0, \quad a = 1, \ \frac{m_\rho^*}{m_\rho} \ll 1$$

i.e. "vector mode"

Rough estimate: $n_{1/2} \sim (1.3 - 2)\, n_0$

M.Rho, Trento, 2010

What does the ½-skyrmion phase do to E_{sym} ?

Symmetry energy $\sim$ 1/N_c

Collective-quantize the (neutron) skyrmion matter

I. Klebanov, 1985

→ Isospin rotation

$$U(\vec{r}, t) = A(t) U_0(\vec{r}) A^\dagger(t)$$

$$E^{\rm tot} = A M_{\rm cl} + \frac{1}{2A\lambda_I} I^{\rm tot}(I^{\rm tot}+1)$$

$$I^{\rm tot} = \frac{1}{2} A\delta \qquad \delta = (n_p - n_n)/(n_n + n_p)$$

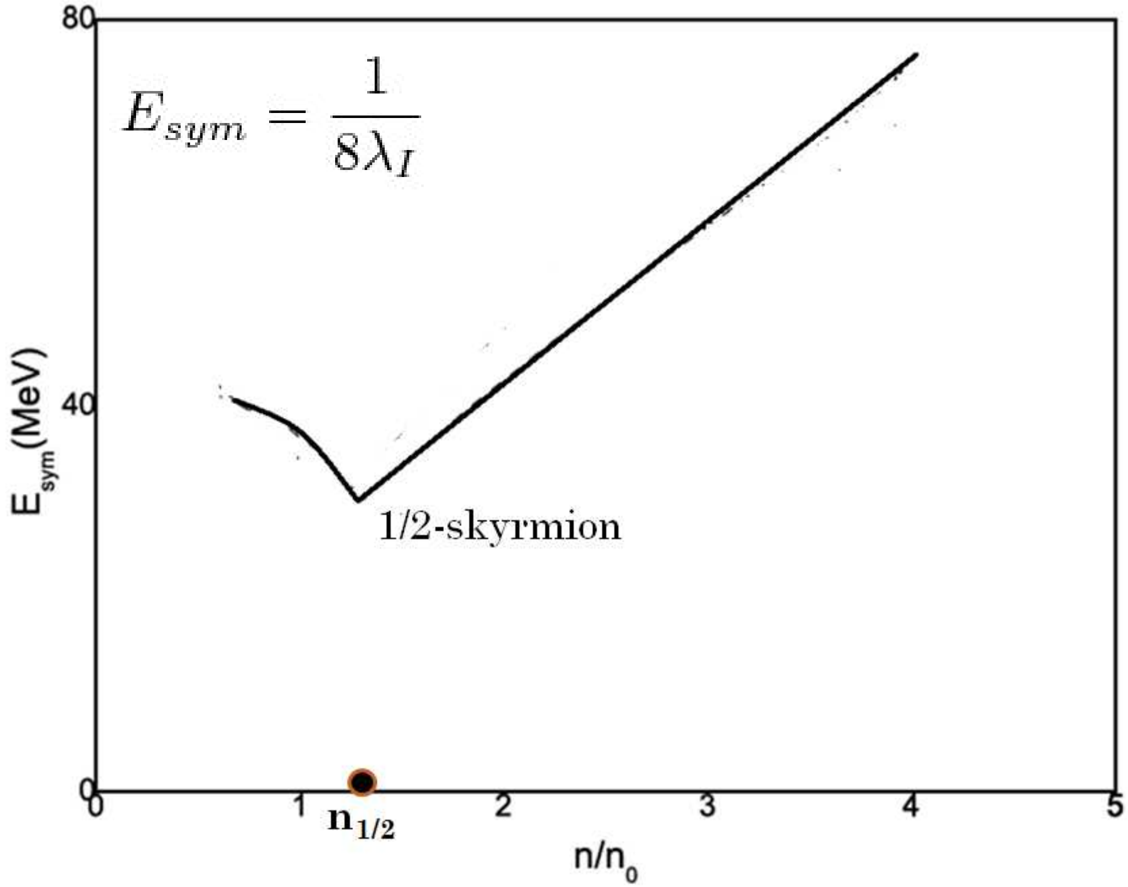
$$E = M_{cl} + \frac{1}{8\lambda_I}\delta^2 \quad \Rightarrow \quad E_{sym} = \frac{1}{8\lambda_I}$$

Moment of inertia

M.Rho, Trento, 2010

E_{sym} from half-skyrmion matter

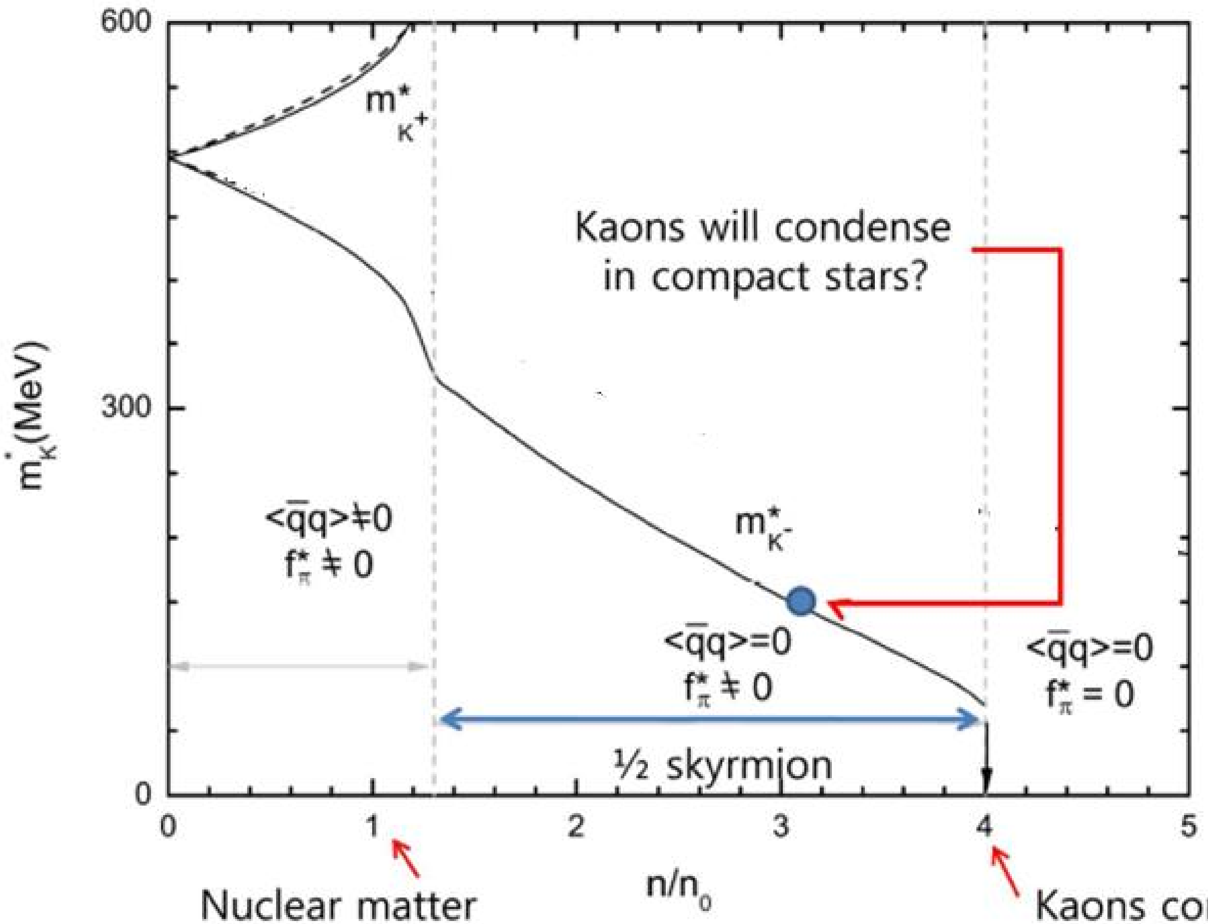
H.K. Lee, B.Y. Park, R. 2010



M.Rho, Trento, 2010

Anti-kaon "roaming" through ½-skyrmion matter: Wess-Zumino term

How to measure isospin spirals?



Nuclear matter
J-Parc

Kaons condense
in nuclear matter

M.Rho, Trento, 2010

How to understand the cusp at $n_{1/2}$ in "standard" nuclear physics?

H.K. Lee, B.Y. Park, R. 2010

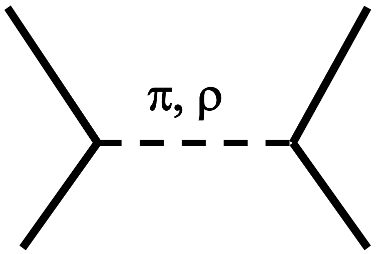
In-medium scaling

M.Rho, Trento, 2010

How to concoct the E_{ss}

Possible mechanism: medium-scaling tensor forces

C. Xu & B.A. Li, arXiv:0910.4803



$$V_M^T(r) = S_M \frac{f_{NM}^2}{4\pi} m_M \tau_1 \cdot \tau_2 S_{12} \left(\left[\frac{1}{(m_M r)^3} + \frac{1}{(m_M r)^2} + \frac{1}{3 m_M r} \right] e^{-m_M r} \right)$$

$$M = \pi, \rho, \; S_{\rho(\pi)} = +1(-1).$$

Tensor forces cancel: Exploit medium-enhanced cancelation to describe the C14 dating by J.W. Holt et al, PRL **100**, 062501 (08)

Assumed scaling: $$\frac{\mathbf{m}_\rho^*}{\mathbf{m}_\rho} \approx \frac{\mathbf{m}_\mathbf{N}^*}{\mathbf{m}_\mathbf{N}} \approx \frac{\mathbf{f}_\pi^*}{\mathbf{f}_\pi} \approx 1 - \alpha \frac{n}{n_0}$$

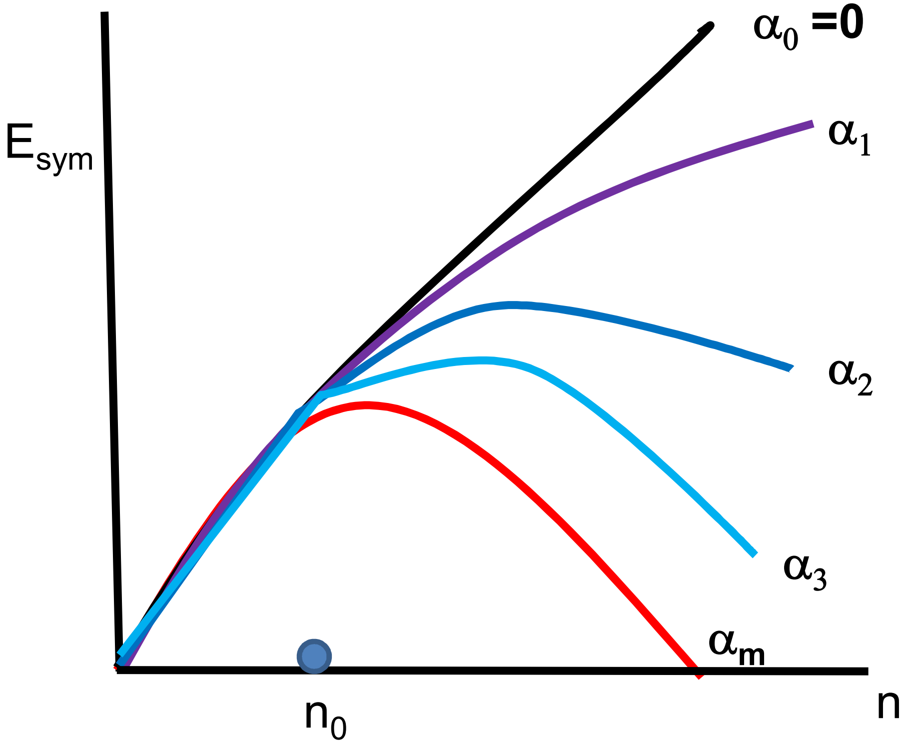
Apply to E_{sym}

M.Rho, Trento, 2010

C. Xu & B.A. Li, arXiv:0910.4803

$$\frac{m_\rho^*}{m_\rho} \approx \frac{m_N^*}{m_N} \approx \frac{f_\pi^*}{f_\pi} \approx 1 - \alpha \frac{n}{n_0}$$ for *all* density (n)

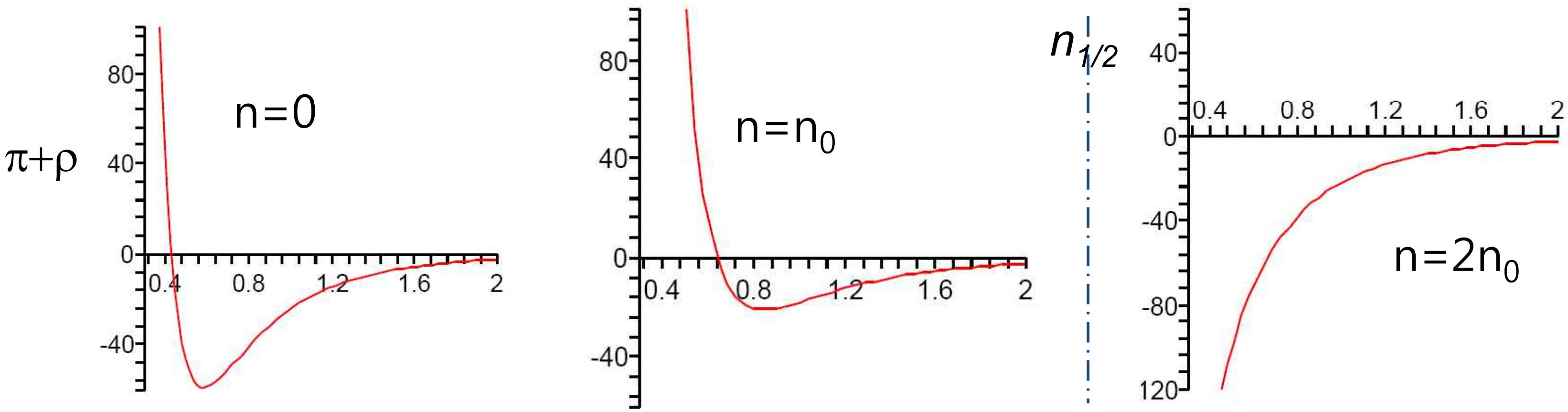
$\Phi_i(n) = 1 - \alpha_i\,(n/n_0)$ $\quad 0 = \alpha_0 < \alpha_1 < \alpha_2 < \alpha_3 \ldots < \alpha_m \approx 0.2$



M.Rho, Trento, 2010

Tensor forces are drastically modified in the ½-skyrmion phase

- For density $n < n_{1/2}$: $\dfrac{m_\rho^*}{m_\rho} \approx \dfrac{m_N^*}{m_N} \approx \dfrac{f_\pi^*}{f_\pi} \approx 1 - \alpha \dfrac{n}{n_0}$ ← "standard" nuclear physics
- For density $n \geq n_{1/2}$: $\dfrac{m_N^*}{m_N} \approx \dfrac{f_\pi^*}{f_\pi} \lesssim 1$, $\dfrac{m_V^*}{m_V} \approx \dfrac{g_V^*}{g_V} \approx \dfrac{\langle \bar{q}q \rangle^*}{\langle \bar{q}q \rangle}$



Decreasing tensor

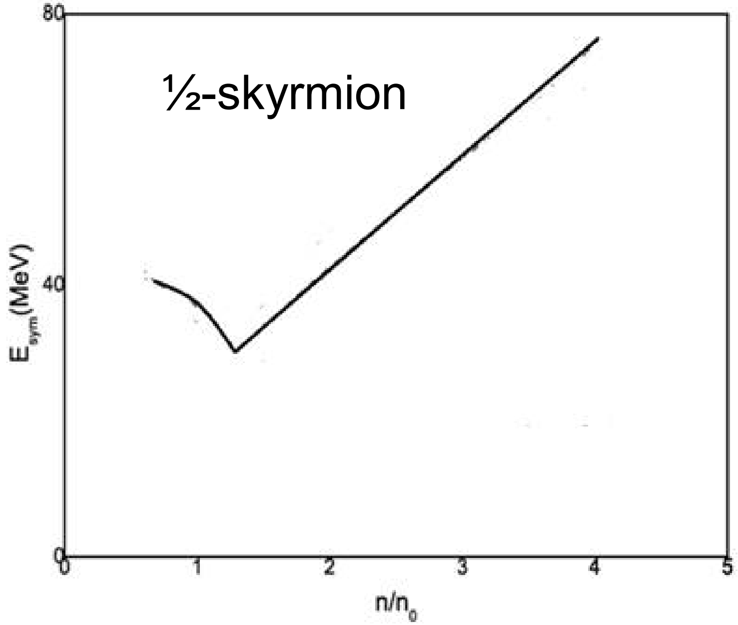
Increas

$$R \equiv \frac{f_{N\rho}^*}{f_{N\rho}} \approx \frac{g_V^*}{g_V} \frac{m_\rho^*}{m_\rho} \frac{m_N}{m_N^*}$$

Above $n_{1/2,}$ the ρ **tensor gets** "killed," enabling the *pions (π^0's) to condense*
→ pionic crystal in dense neutron matter (e.g., Pandharipande and Smith 74).

M.Rho, Trento, 2010

½-skyrmion vs. scaling



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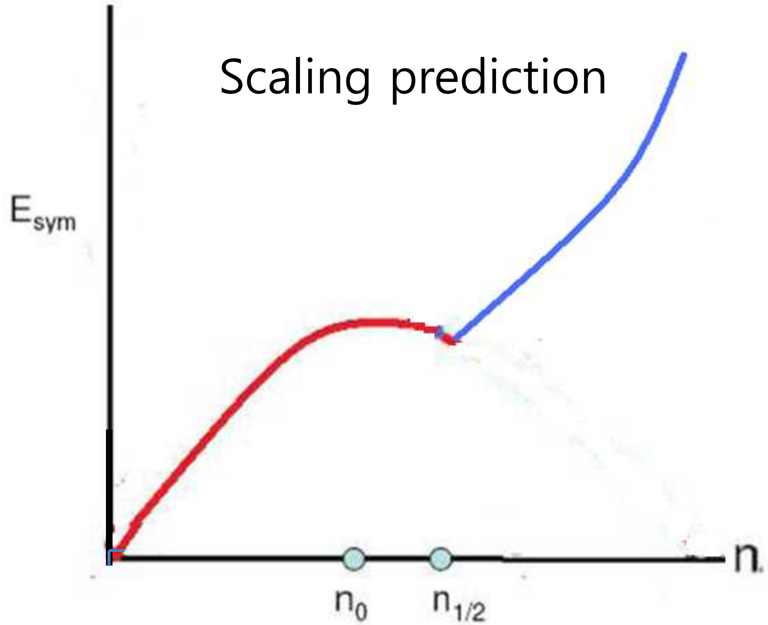

This prediction could be checked or falsified at FAIR or even RIB (e.g., KoRIA) machines

Summary

- Higher density, $n > n_0$ at the core of compact star → new physics
- Symmetry energy (n-p asymmetry) :

  new degrees of freedom into n-p system in weak equilibrium (crust and core)
- Emergence of strangeness at the core
- kaon condensation, hyperon degrees of freedom

Mass, radius and compositions,….

Cooling, GW, GRB, …..