Skyrme's energy functional and neutron star matter

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Tony Hilton Royle Skyrme

1922 - 1987

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Hartree-Fock Calculations with Skyrme's Interaction. I. Spherical Nuclei*

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and

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(Received 15 November 1971)

$$v_{12} = t_0(1 + x_0 P_\sigma)\delta(\vec{r}_1 - \vec{r}_2) + \tfrac{1}{2}t_1[\delta(\vec{r}_1 - \vec{r}_2)k^2 + k'^2\delta(\vec{r}_1 - \vec{r}_2)] + t_2\vec{k}'\cdot\delta(\vec{r}_1 - \vec{r}_2)\vec{k} + iW_0(\vec{\sigma}_1 + \vec{\sigma}_2)\cdot\vec{k}'\times\delta(\vec{r}_1 - \vec{r}_2)\vec{k},$$

$$v_{123}^{(3)} = t_3\delta(\vec{r}_1 - \vec{r}_2)\delta(\vec{r}_2 - \vec{r}_3)\,.$$

$$v_{12} = \tfrac{1}{6}t_3(1 + P_\sigma)\delta(\vec{r}_1 - \vec{r}_2)\rho\left(\frac{\vec{r}_1 + \vec{r}_2}{2}\right).$$

"Such a term provides a simple phenomenological representation of many-body effects Skyrme's interaction can be considered as a kind of phenomenological G-matrix which already includes the effect of short-range correlations through its density dependent term."

"Skyrme's interaction is an approximate representation of the effective nucleon force WHICH IS ONLY VALID FOR LOW RELATIVE MOMENTA "

Tensor force includes mixing between 3S1+3D1 (even) and 3P2+3F2 (odd)

Skyrme, Phil.Mag. 1, 1043 (1956), Nucl.Phys. 9, 615 (1958)

$$
\begin{aligned}
V^T = \frac{T}{2}\Bigg\{ & \left[(\sigma_1 \cdot \mathbf{k}')(\sigma_2 \cdot \mathbf{k}') - \frac{1}{3}(\sigma_1 \cdot \sigma_2)\,\mathbf{k}'^2\right]\delta(\mathbf{r}_1 - \mathbf{r}_2) \\
& + \delta(\mathbf{r}_1 - \mathbf{r}_2)\left[(\sigma_1 \cdot \mathbf{k})(\sigma_2 \cdot \mathbf{k}) - \frac{1}{3}(\sigma_1 \cdot \sigma_2)\,\mathbf{k}^2\right]\Bigg\} \\
& + \frac{U}{2}\{(\sigma_1 \cdot \mathbf{k}')\delta(\mathbf{r}_1 - \mathbf{r}_2)(\sigma_2 \cdot \mathbf{k}) \\
& + (\sigma_2 \cdot \mathbf{k}')\delta(\mathbf{r}_1 - \mathbf{r}_2)(\sigma_1 \cdot \mathbf{k}) \\
& - \frac{2}{3}[(\sigma_1 \cdot \sigma_2)\mathbf{k}' \cdot \delta(\mathbf{r}_1 - \mathbf{r}_2)\mathbf{k}]\}. \qquad (12)
\end{aligned}
$$

affects interactions in spin-spin and spin-isospin channels and stability of nuclear matter above saturation density)

Cao, Colo and Sagawa, Phys. Rev. C81, 044302 (2010)

The Skyrme energy density functional derived from the NN potential:

$$H = K + H_o + H_3 + H_{eff}$$

K - kinetic energy term, H_o zero-range term,
H_3 - density dependent term, H_{eff} – effective mass term
n – particle number density, τ – kinetic energy density
Dependent on 8 parameters t_0, t_1, t_2, t_3, x_0, x_1, x_2, x_3

$$H_o = \tfrac{1}{4} t_0 \left[(2 + x_0) n^2 - (2x_0 + 1)(n_p^2 + n_n^2) \right]$$

$$H_3 = \tfrac{1}{24} t_3 n^\alpha \left[(2 + x_3) n^2 - (2x_3 + 1)(n_p^2 + n_n^2) \right]$$

$$H_{eff} = \tfrac{1}{8} \left[t_1 (2 + x_1) + t_2 (2 + x_2) \right] \tau n + \tfrac{1}{8} \left[t_2 (2x_2 + 1) - t_1 (2x_1 + 1) \right] \left(\tau_p n_p + \tau_n n_n \right)$$

In addition, for application in finite nuclei terms dependent on gradient of density and current have to be included:

H_{fin} – finite range simulation term, H_{so} – spin-orbit term
H_{sg} - tensor coupling with spin and gradient

$$H_{fin} = \frac{1}{32}\left[3t_1\left(2+x_1\right)-t_2\left(2+x_2\right)\right]\left(\nabla n\right)^2$$
$$-\frac{1}{32}\left[3t_1\left(2x_1+1\right)-t_2\left(2x_2+1\right)\right]\left[\left(\nabla n_p\right)^2+\left(\nabla n_n\right)^2\right]$$

$$H_{so} = \frac{1}{2}W_0\left[J.\nabla n + J_p.\nabla n_p + J_n.\nabla n_n\right]$$

$$H_{sg} = -\frac{1}{16}\left(t_1x_1+t_2x_2\right)J^2+\frac{1}{16}\left(t_1-t_2\right)\left[J_p^2+J_n^2\right]+H_{coul}+H_{pair}$$

THE PARAMETERS ARE HIGHLY CORRELATED – IN PRINCIPLE INFINITE NUMBER OF COMBINATIONS EXISTS

More modern parameterization of the Skyrme energy density functional
(see for a review Bender et al., Rev. Mod. Phys. 75, 121 (2003))

$$E = E_{\mathrm{kin}} + \int d^3r\,\mathcal{E}_{\mathrm{Sk}} + E_{\mathrm{Coul}} + E_{\mathrm{pair}} - E_{\mathrm{corr}}. \tag{46}$$

$$\mathcal{E}_{\mathrm{Sk}} = \sum_{T=0,1} (\mathcal{E}_T^{\mathrm{even}} + \mathcal{E}_T^{\mathrm{odd}}). \tag{47}$$

time-reversal odd
and even

$$\mathcal{E}_T^{\mathrm{even}} = C_T^{\rho}\rho_T^2 + C_T^{\Delta\rho}\rho_T\Delta\rho_T + C_T^{\tau}\rho_T\tau_T + C_T^{J}\mathrm{J}_T^2 + C_T^{\nabla J}\rho_T\nabla\cdot\mathbf{J}_T, \tag{48}$$

summation over
isospin T

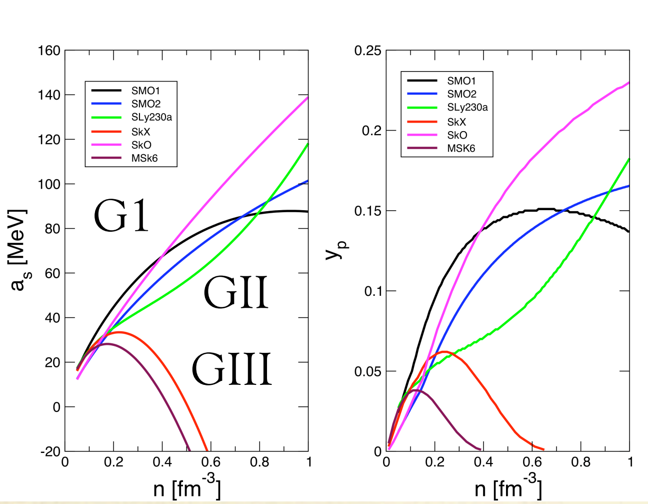
$$\mathcal{E}_T^{\mathrm{odd}} = C_T^{s}\mathbf{s}_T^2 + C_T^{\Delta s}\mathbf{s}_T\cdot\Delta\mathbf{s}_T + C_T^{sT}\mathbf{s}_T\cdot\mathbf{T}_T + C_T^{\nabla s}(\nabla\cdot\mathbf{s}_T)^2 + C_T^{j}\mathbf{j}_T^2 + C_T^{\nabla j}\mathbf{s}_T\cdot\nabla\times\mathbf{j}_T. \tag{49}$$

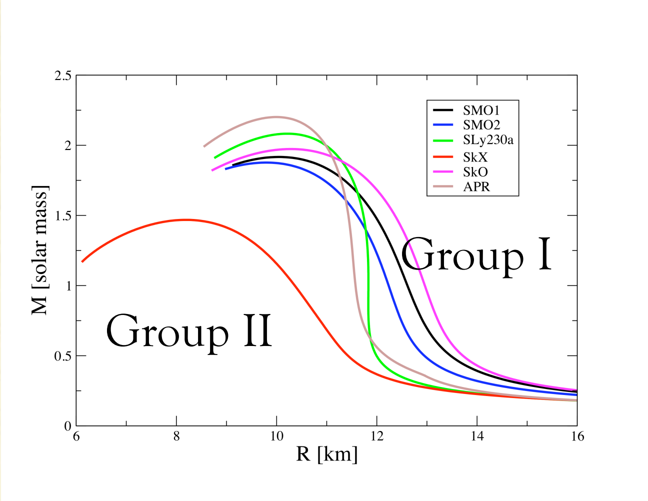
direct correspondence
between coupling
constants C and
the usual parameters
t and x e.g.

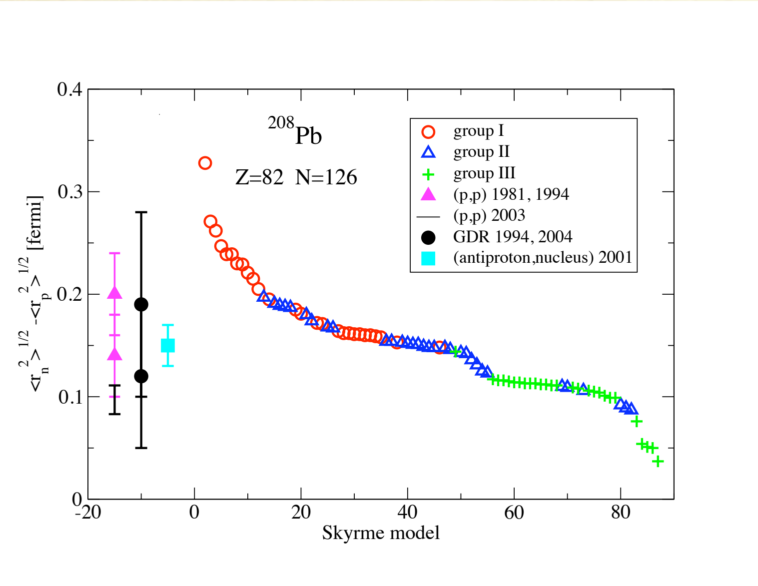
$$C_0^{\rho} = \tfrac{3}{8}t_0 + \tfrac{3}{48}t_3\rho_0^{\alpha},$$

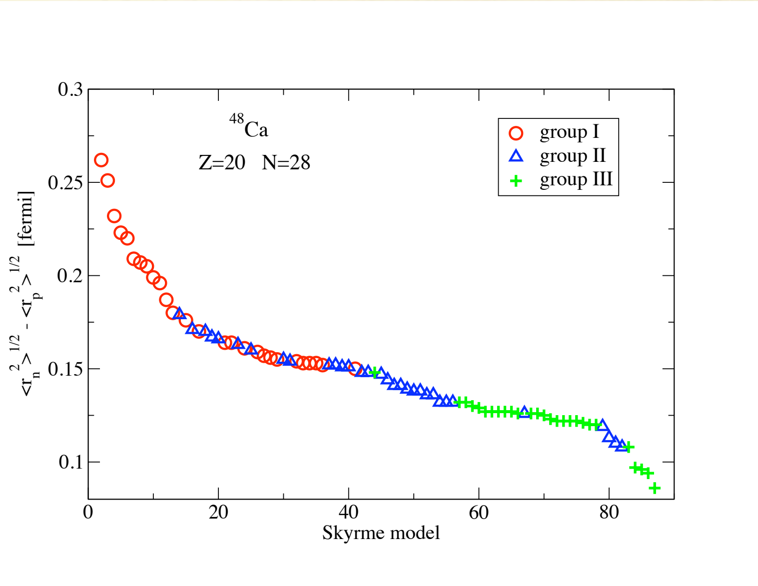
2003 87 Skyrme parameterizations tested: Stone et al PRC C68, 034324

Sorting criteria: density dependence of the symmetry energy and existence of cold neutron stars

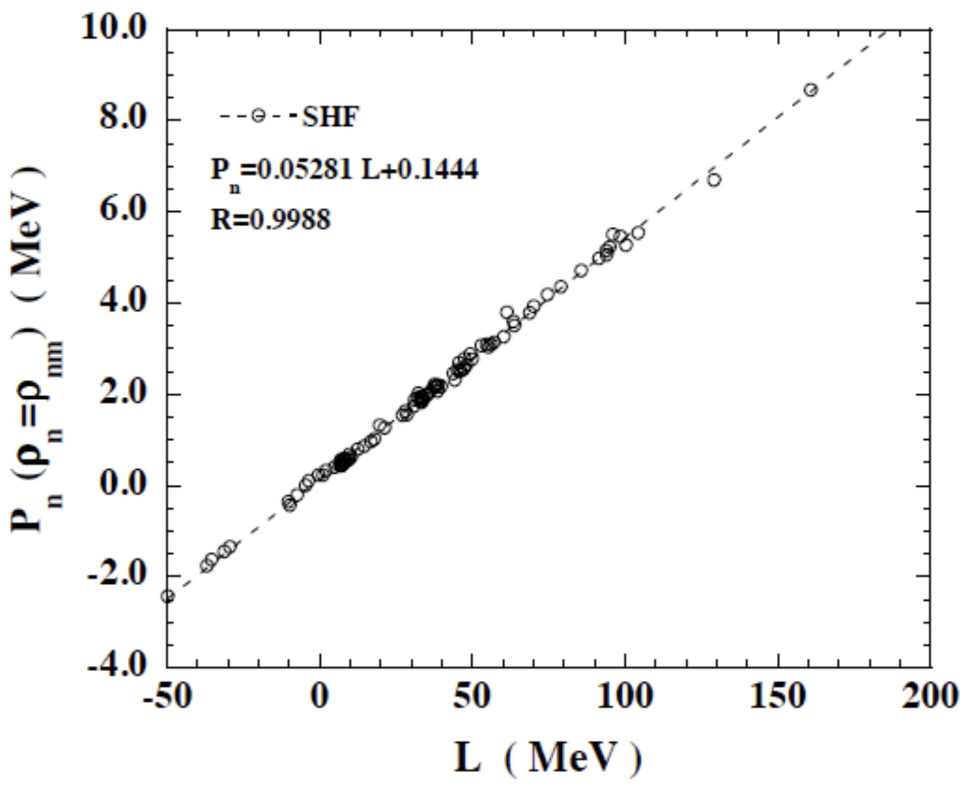








27 Skyrme parameterizations – all of the group I



Yoshida and Sagawa, private communication, 2010

Symmetry energy in nuclear matter:

$$\mathcal{S}(n) = \mathcal{E}(n, I = 0) - \mathcal{E}(n, I = 1).$$

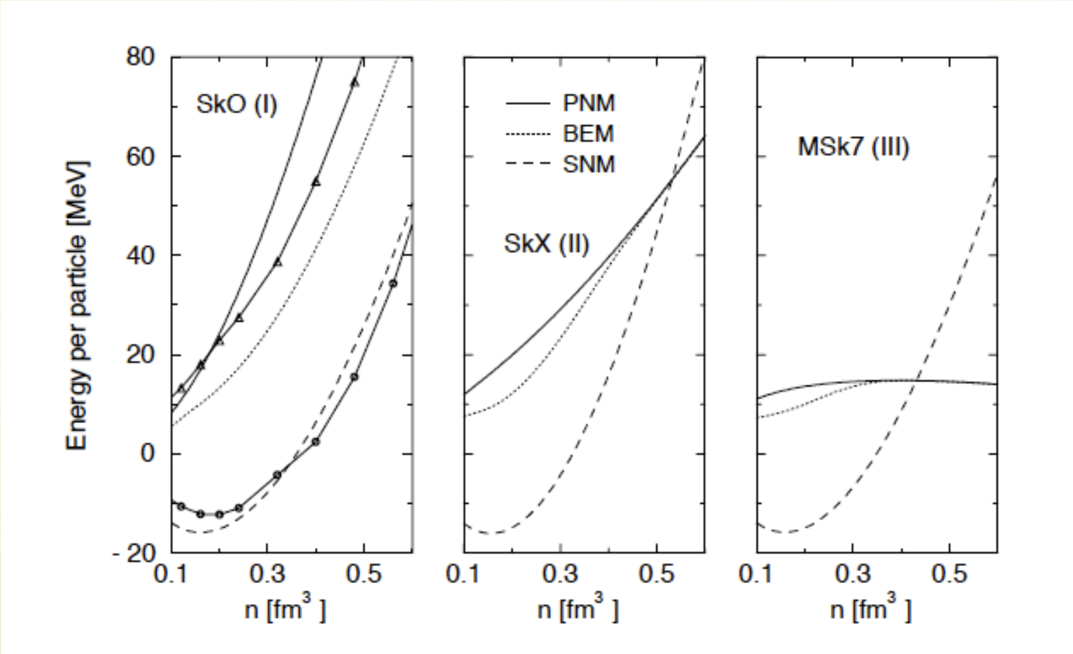
Approximately equal to the the asymmetry coefficient in the semiempirical mass formula:

$$a_{\text{sym}} = \frac{1}{2} \frac{\partial^2 \mathcal{E}}{\partial I^2}\bigg|_{I=0} = \frac{n^2}{2} \frac{\partial^2 \mathcal{E}}{\partial \tilde{n}^2}\bigg|_{\tilde{n}=0}.$$

(i) $\varepsilon(n, I=0)$ a minimum energy of matter at a given density

(ii) all higher derivatives in the expansion are negligible

THE PROBLEM WITH UNDERSTANDING OF THE SYMMETRY ENERGY IS MAINLY CAUSED BY OUR POOR KNOWLEDGE OF THE NN INTERACTION



also mentioned by D.T.Khoa and A. Steiner

2009 144 Skyrme parameterizations 2009 (database Stone, Lee, Danielewicz)

COMPSTAR meeting Coimbra, Portugal, February 2009

$$E_{sym}(\rho) \approx E_{sym}(\rho_0) + \frac{L}{3}\left(\frac{\rho-\rho_0}{\rho_0}\right) + \frac{K_{sym}}{18}\left(\frac{\rho-\rho_0}{\rho_0}\right)^2, \quad (38)$$

where L and K_{sym} are the slope and curvature of the symmetry energy at ρ_0

$$L = 3\rho_0 \frac{\partial E_{sym}(\rho)}{\partial \rho}|_{\rho=\rho_0}, \quad (39)$$

$$K_{sym} = 9\rho_0^2 \frac{\partial^2 E_{sym}(\rho)}{\partial \rho^2}|_{\rho=\rho_0}. \quad (40)$$

$$K_{asym} = K_{sym} - 6L$$

L = 88 ± 25 MeV Vela pulsar glitches Xu et al PRC **77**, 014302 (2008). arXiv:0807.4477v1 [nucl-th].

K_{asym} = -550±100 MeV GMR Li et al PRL 99, 162503 (2007)

K_∞ = 240±20 MeV Moller and Nix

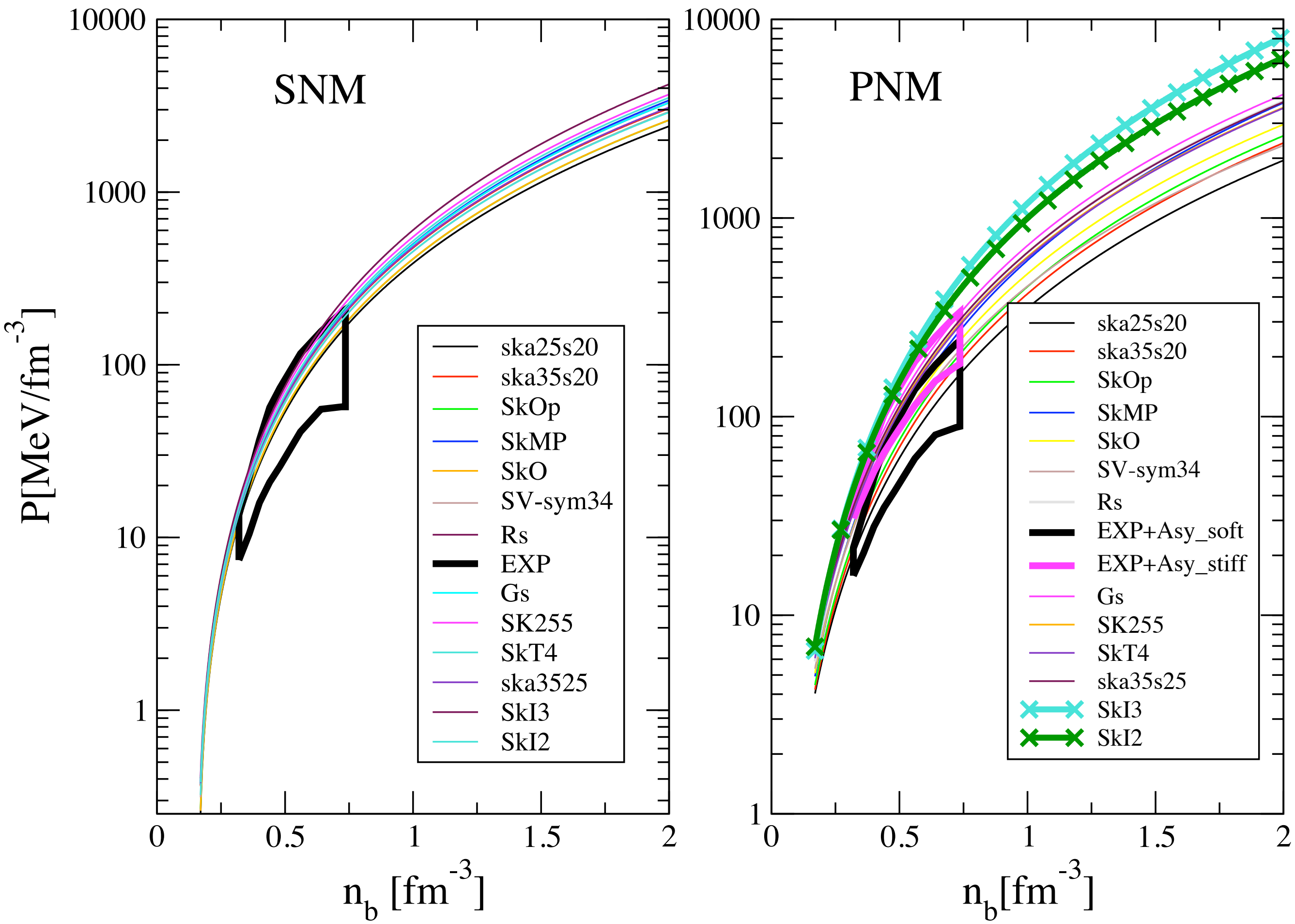
a_s => 27 – 38 MeV Stone and Reinhard

Only 13 out of 144 parameter sets survived

	t0	t1	t2	t3	t4	x0	x1	x2	x3	x4	α
ska25s20	-2180.48	281.49	-160.44	14577.80	0.14	-0.80	0.00	0.06	103.20	103.20	0.25
ska35s20	-1768.83	263.86	-158.34	12904.80	0.13	-0.80	0.00	0.01	100.80	100.80	0.35
SkOp	-2099.42	301.53	154.78	13526.46	-0.03	-1.33	-2.32	-0.15	143.90	-82.89	0.25
SkMP	-2372.24	503.62	57.28	12585.30	-0.16	-0.40	-2.96	-0.27	80.00	80.00	0.17
SkO	-2103.65	303.35	791.67	13553.25	-0.21	-2.81	-1.46	-0.43	176.58	-198.75	0.25
SV-sym34	-1887.37	323.80	351.78	12597.30	-0.23	-0.96	-1.78	-0.72	69.65	20.28	0.30
Rs	-1798.00	335.97	-84.81	11083.90	-0.40	0.00	0.00	-0.87	60.80	60.80	0.30
Gs	-1800.16	336.23	-85.74	11113.50	-0.49	0.00	0.00	-1.03	60.93	60.93	0.30
SK255	-1689.35	389.30	-126.07	10989.60	-0.15	0.12	0.00	-0.74	47.70	0.00	0.36
SkT4	-1808.80	303.40	-303.40	12980.00	-0.18	-0.50	-0.50	-0.50	56.50	56.50	0.33
ska35s25	-1772.73	276.85	-162.36	12899.78	-0.14	-0.80	0.00	-0.49	101.00	101.00	0.35
SkI3	-1762.88	561.61	-227.09	8106.20	0.31	-1.17	-1.09	1.29	94.25	0.00	0.25
SkI2	-1915.43	438.45	305.45	10548.90	-0.21	-1.74	-1.53	-0.18	60.30	60.30	0.25

Second set of constraints: Danielewicz et al – Science

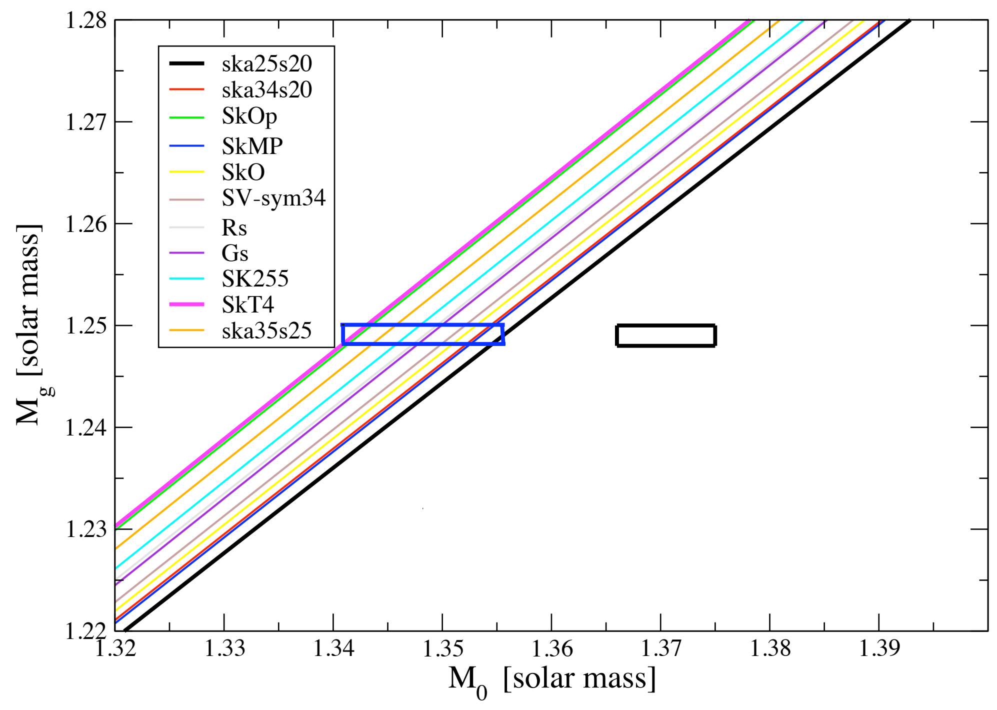
Skyrme sets reduced to 11 – SKI3 and SkI2 did not pass PNM



Third constraint:

Low mass neutron star - *The double pulsar J0737-3039* :

Podsiadlowski et al. Mon.Not.R.Astron.Soc. 361, 1243 (2005)



Prediction of 0.019 – 0.024 M_{solar} loss in the progenitor mass

2010 Testing 195+5 Skyrme parameterizations

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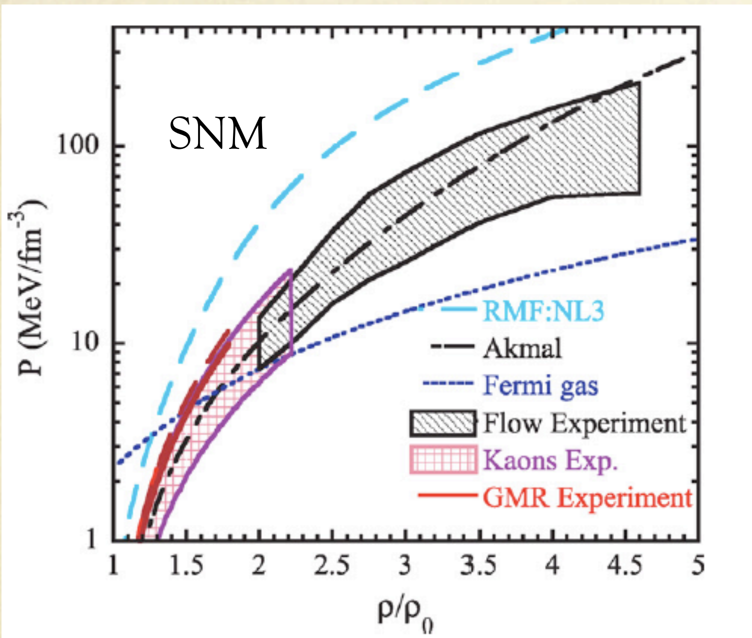
Departamento de Física –Universidade Federal Fluminense
Niterói, RJ, Brazil

Constraints used in the 2009 analysis were extended mainly for low density pure neutron matter (Schwenk and Pethick, PRL 95, 160401)2005 and Epelbaum Eur.Phys.J. A40,199 (2009)



Lynch et al., Prog.Nucl.Part.Phys. 62, 427 (2009)

New constraints on L:

L<70 MeV Newton and Bao_An Li PRC80, 065809 (2009)

L=65±15 MeV Carbone et al., PRC 81, 041301(2010)

L=58±18 MeV Chen et al., arXiv:1004.4672v1

$$E_{sym}(\rho) \sim J\left(\frac{\rho}{\rho_0}\right)^{\gamma}$$

Shetty et al, PRC75, 034602 (2007)

$0.69 < \gamma < 1.05$

11 constraints altogether

Preliminary results:

3 models satisfied all constraints

* SKa25s20
* SKa35s20
GSKI

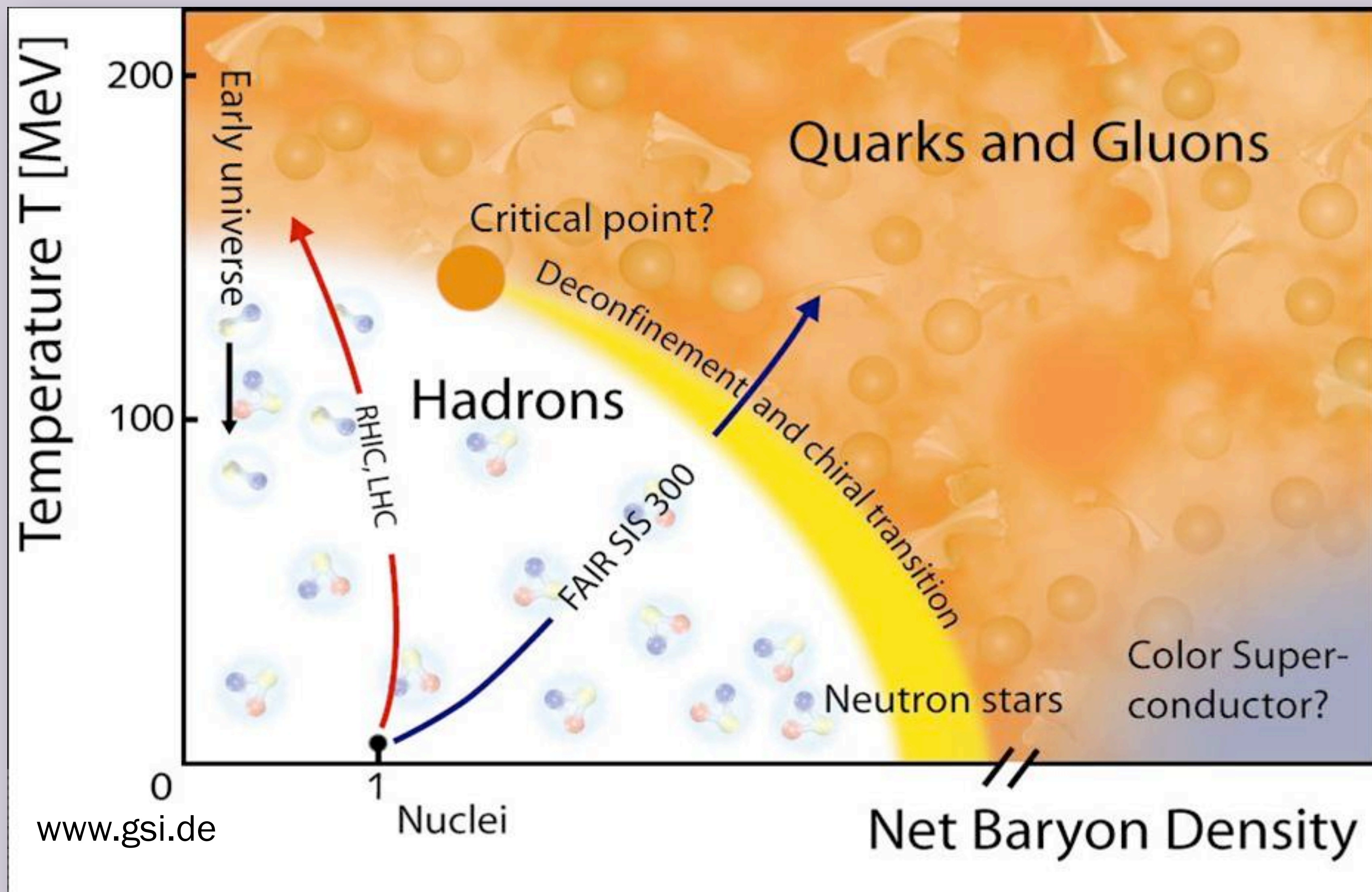
6 models satisfied 10 constraints

* SKa35s25
SKT4
* SKO’
NRAPR
QMC700
QMC750

Key issues: 1. Could we understand why these parameterizations are preferred

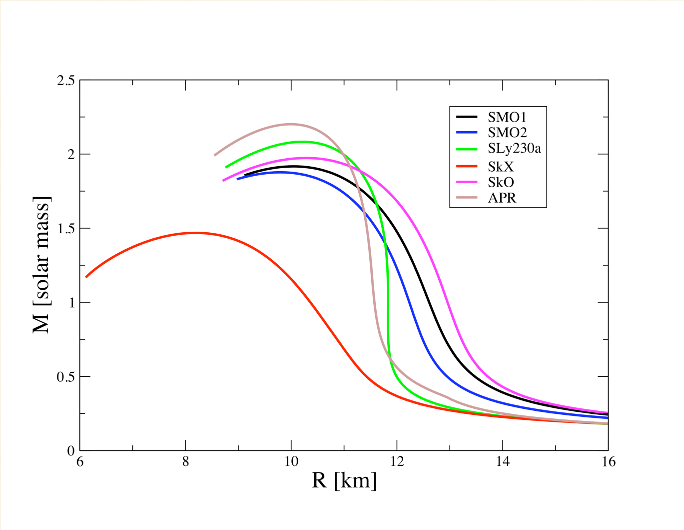
2. Detailed understanding of the WEIGTH and APPLICABILITY of the constraints

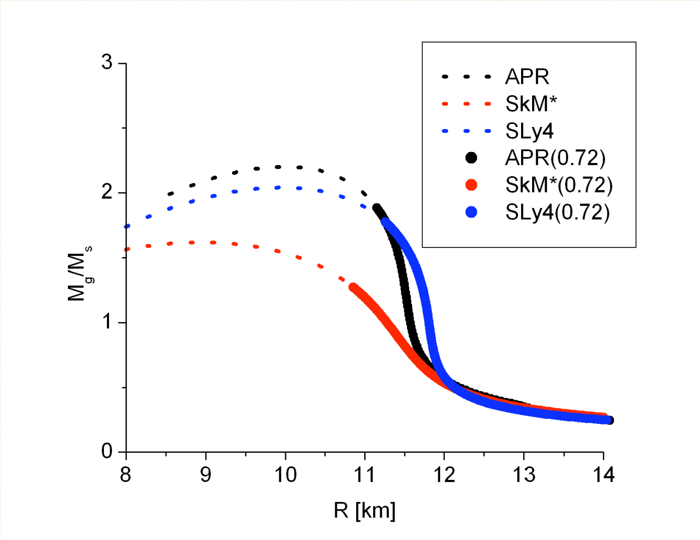
Can we apply directly HIC based constraints directly to neutron star matter?



Can we safely extrapolate Skyrme calculation to many times nuclear saturation density?

"Skyrme's interaction is an approximate representation of the effective nucleon force WHICH IS ONLY VALID FOR LOW RELATIVE MOMENTA " i.e. low densities.





EoS	M_{max}	R[km]	x [n_s]
SMO1	1.92	10.0	7.8
SMO2	1.86	10.2	7.3
SLy230a	2.08	10.2	7.2

EoS	M_{max}	R[km]	x [n_s]
SkX	1.39	7.92	13.4
SkO	1.97	10.4	10.4
APR	2.2.0	10.0	7.1

Answer: Skyrme models should not be used to calculate maximum mass-radius but should be safe up to about about 3 times the saturation density

What to do next?

I. Keep constructing EoS using different models
keep changing when new constraints become available

↓↓

Indefinitely long process

II. Look for significant physics and defendable parameters
Understand relevance of different approaches

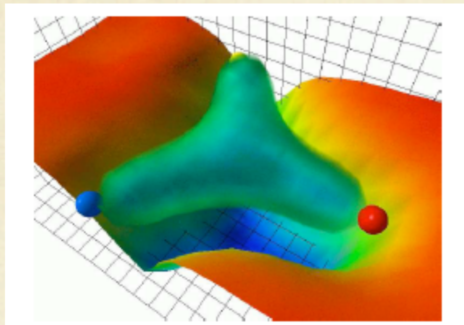
↓

Difficult, long, but we may gain

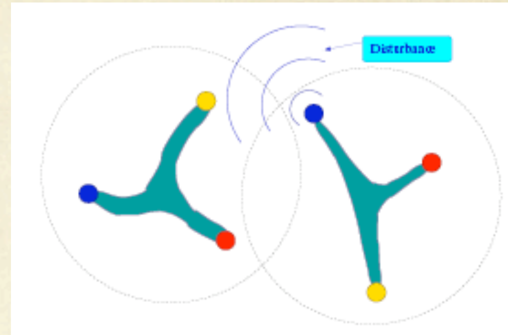
increased predictive power and control over the models

Rewarding at the end

Quark-Meson Coupling Model: Stone, Guichon et al., NPA 792, 341 (2007)



(Leinweber et al., http://www.physics.adelaide.edu.au/cssm/lattice/)



Quark level – baryons modeled by (virtual) quark bags
quarks from different bags interact via meson fields –
interpreted as QCD vacuum fluctuations (outside the bags)

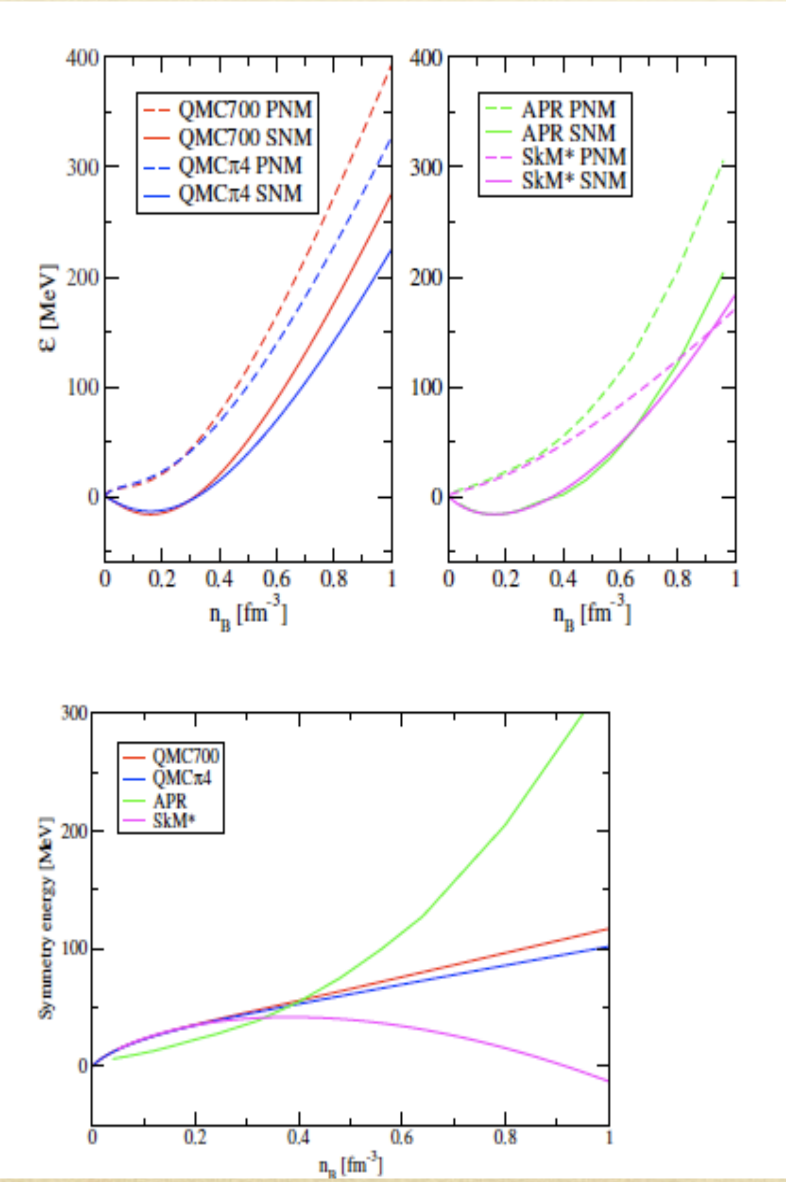
Full Baryon Octet, σ, ω, ρ mesons (experimental meson masses for ρ and ω)

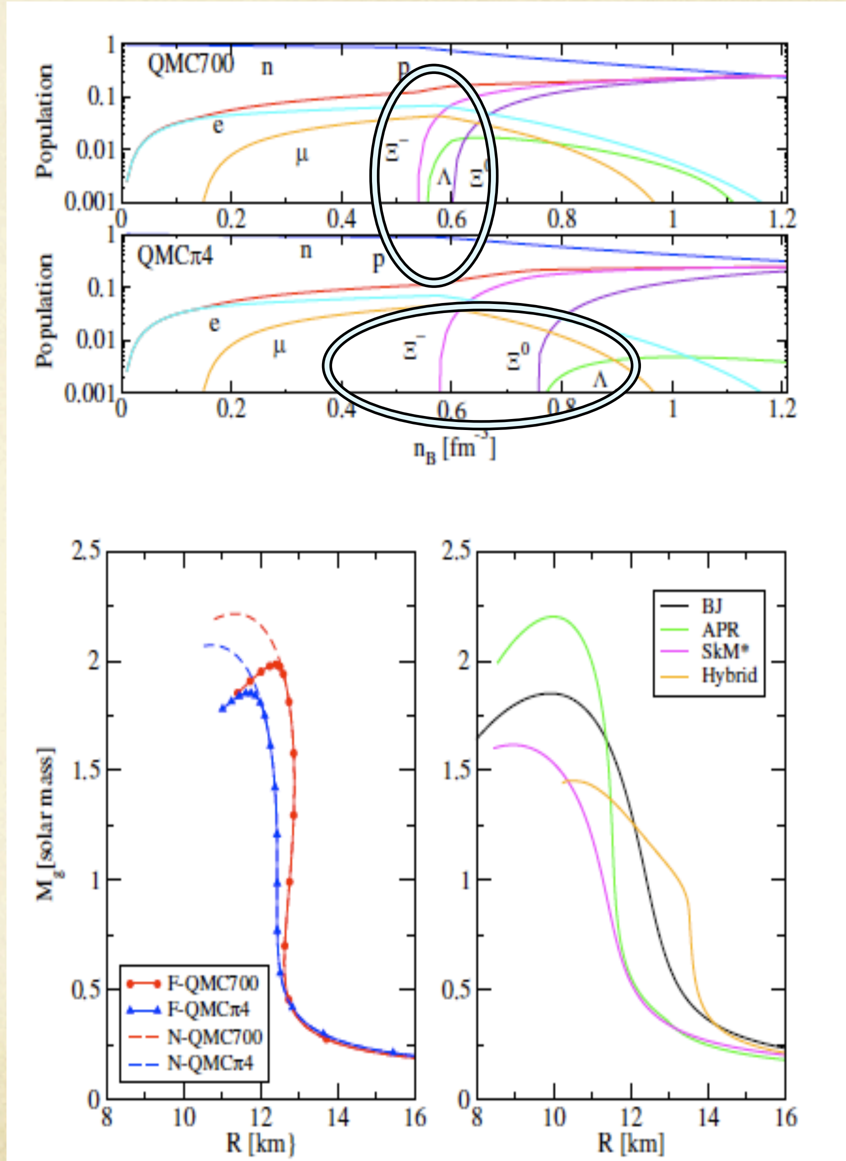
4 parameters, fitted to symmetric nuclear matter and symmetry energy

Works both for finite nuclei and nuclear matter

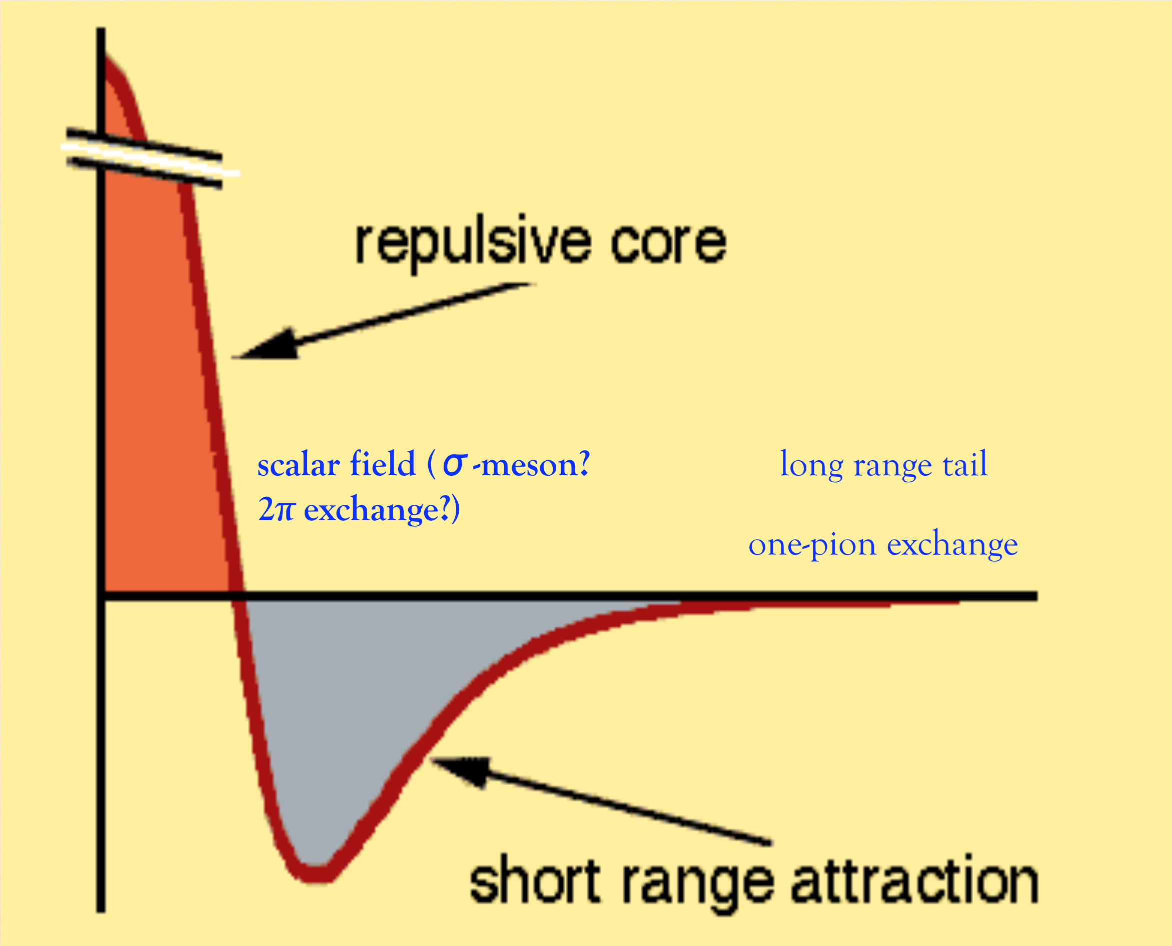
At low densities behaves like a Skyrme model, at higher densities includes relativistic corrections applicable to higher densities provided baryons maintain their identity

Problem? QMC700 K=340 MeV, QMCπ4 K=260 MeV





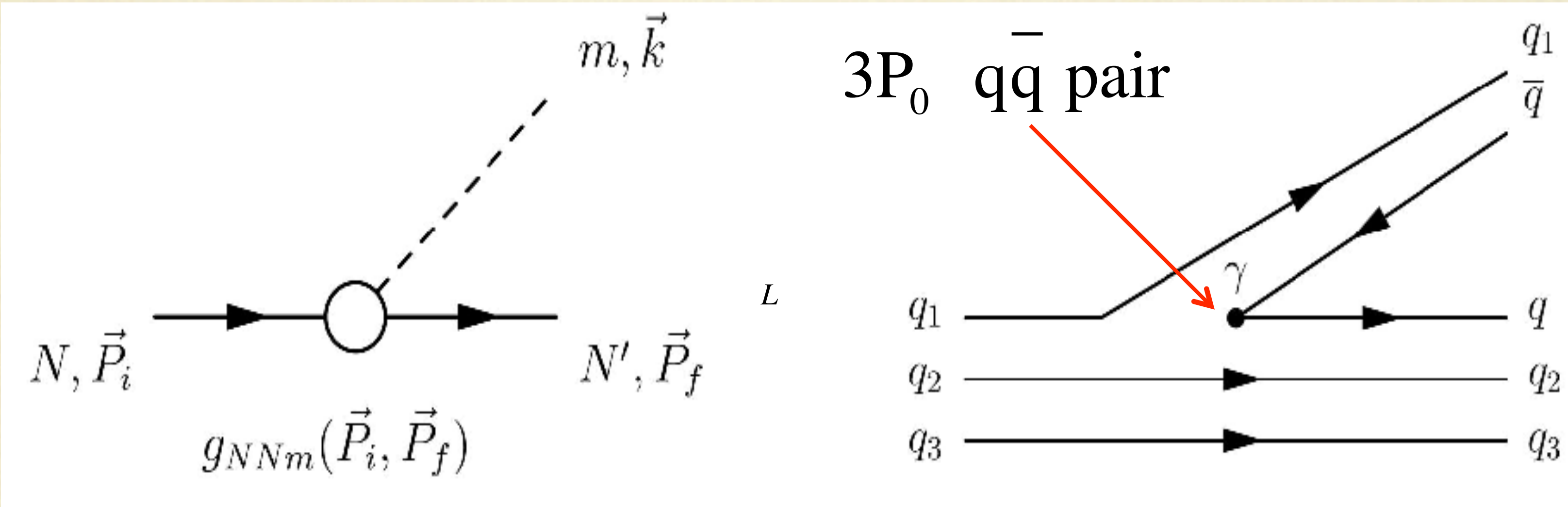
Schematic model of the nucleon-nucleon force



Choose only these three terms instead of all mesons used in meson exchange theory of nuclear forces

Oxford Model meson-exchange

Relativistic effective field theory ← Meson exchange using 3P0 Model



1. Matrix element of quark model quark-antiquark creation operator convoluted with quark model nucleon and meson wave functions yields meson emission amplitudes
2. Mapping these amplitudes on the effective field equivalents yields

COUPLING CONSTANTS AND EFFECTIVE FORM-FACTORS (GAUSSIAN FORM)

Oxford Model - soft core

Colored spin-spin hyperfine contact interaction between
quarks in different nucleons, followed by constituents interchange
Barnes et al. PRC 48, 539 (1993):

$$H_I = \sum_{i<j;i,j=1}^{3} -\frac{8\pi\alpha_s}{3m_i m_j}\,\mathbf{s_i}.\mathbf{s_j}\,\delta(\mathbf{x}_{ij})\sum_{a=1}^{8}\frac{\lambda_i^a}{2}\frac{\lambda_j^a}{2},$$

Individual quark-level scattering amplitudes and core model are converted into a effective nucleon-nucleon (NN) potential

the Oxford potential

Parameterization:
(for application in partial wave decomposition)

-widths of nucleon (α) and meson (β) wave functions constrained by the quark model and fixed in all partial wave channels for all terms in the Lagrangian;

$\alpha = \beta = 0.4$ for all terms [sigma, pion, (omega)]

N-pion coupling constant: fixed to 13.0

N-sigma coupling constant: set to 6.0

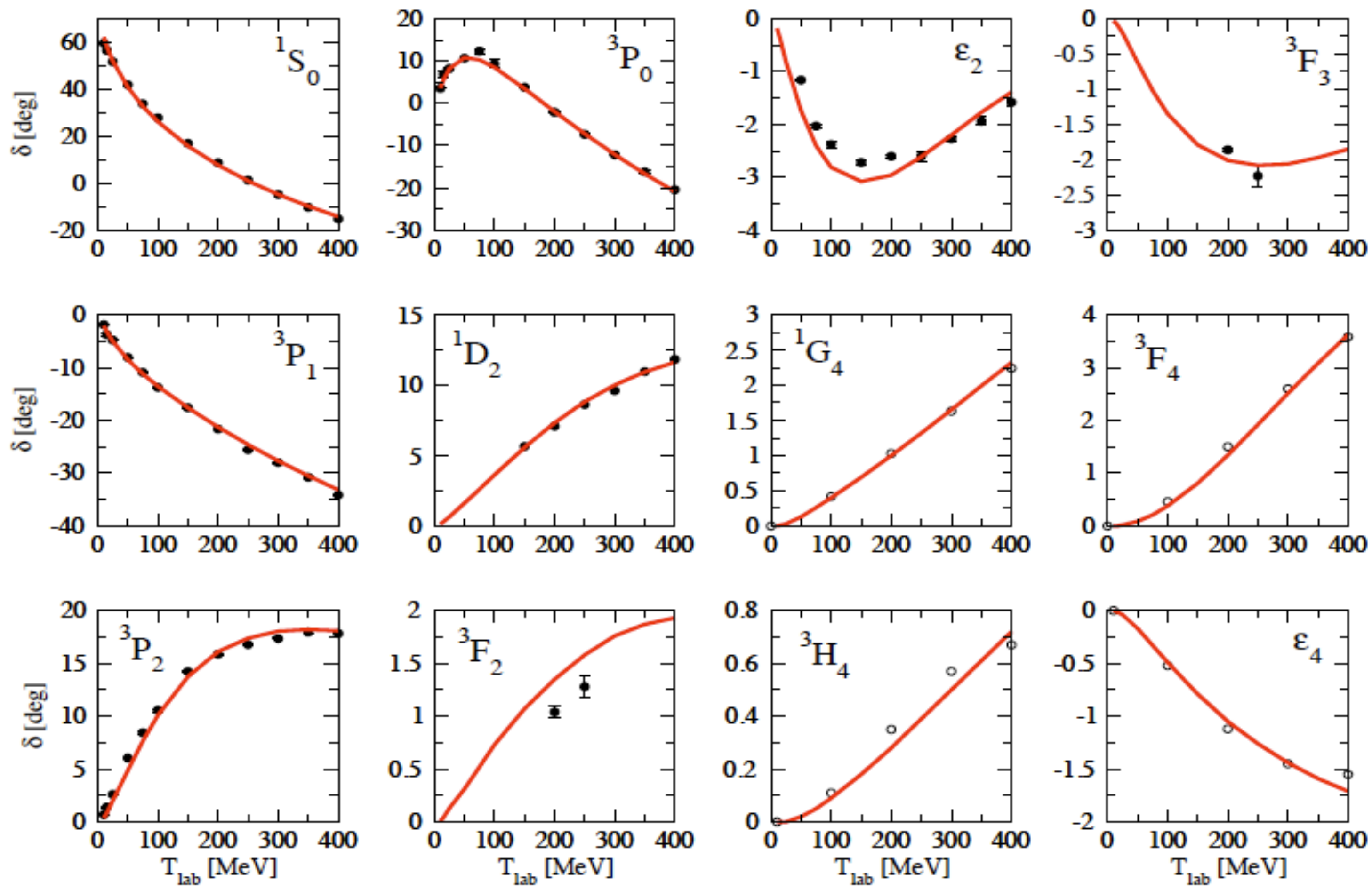
mass of pion: fixed to $m_\pi^{(+/-)} = 139.57$ MeV $m\pi_0 = 134.76$ MeV

mass of sigma: varied with partial wave channel
350 – 650 MeV

ONE variable parameter per partial wave channel

parameters have physical meaning!

T=1 (triplet np channel) NN Phase Shifts



Deuteron properties (bound NN state)

Property	Oxford	CD-Bonn	Nijmegen	Experiment	
B_d [MeV]	2.2245	2.224575	2.224575	2.224575(9)	(fit)
r_m [fm]	1.976	1.966	1.9676	1.971(6)	(prediction)
Q_d [fm^2]	0.285	0.270	0.271	0.2859(3)	(prediction)
P_D [%]	5.76	4.85	5.67		(prediction)

Triton calculation in progress...

This is just a beginning……