Neutron specific heat and thermalisation time of neutron stars crust

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HFB calculation & specific heat of superfluid neutrons in non-uniform matter.

Effect of pairing on the thermalisation time of NS crust.


+ Nuclear symmetry energy and core-crust transition in neutron star: a critical study, accepted in EPL, arXiv:nucl-th/0910.5488
Cooling of Neutron stars

Several types of cooling:
- Convective
- Conductive
- Radiative

Time-scales:
- Convective: s
- Conductive: 1-100 y
- Radiative: >10⁵ y

I: Crust thermalisation epoch
II: ν cooling epoch
III: Photon epoch

Cooling of Neutron stars

URCA process (cf talk of Nakatsuka-san):

\[ n \rightarrow p + e^- + \bar{\nu}_e, \quad p + e^- \rightarrow n + \nu_e \]

If proton fraction \( Z/A > 14\% \) (large \( L \), cf talk of Van Giai):
fast cooling by URCA process.

**Fast cooling:**

→ after \( \sim 1 \) year: \( T_{\text{core}} << T_{\text{crust}} \approx 0.5 \) MeV,
→ next \( \sim 10-100 \) years: **thermalisation** of the crust.

\[ \tau \propto \frac{d^2}{D} \]

with \( D = \frac{K}{\sum_i C_{v,i}} \approx C_{v,n} \)

Gamow & Schoenberg, PR 59 (1941)

Lattimer et al., PRL 66 (1991)

Lattimer et al., APJ 425 (1994)
Heat transport equations

GR equation for radiative transport (Thorne 1977):

\[ e^{\phi(r)} L = -K 4\pi r^2 \sqrt{1 - \frac{2Gm(r)}{rc^2}} \frac{d(T e^{\phi(r)})}{dr}, \quad \frac{d(L e^{2\phi})}{dr} = \frac{4\pi r^2}{\sqrt{1 - 2Gm(r)/rc^2}} e^{2\phi} \left( -Q_\nu - ne^{-\phi} \frac{d}{dt} \left( \frac{\varepsilon}{n} \right) \right) \]

Heat transport equation:

\[ \frac{\partial}{\partial r} \left[ K r^2 \frac{e^\phi}{\Gamma(r)} \frac{\partial}{\partial r} (e^\phi T) \right] = r^2 \Gamma(r) e^\phi (C_V \frac{\partial T}{\partial t} + e^\phi Q_\nu) \]

Microscopic inputs:

\[ \begin{align*}
\text{Specific heat:} & \quad C_V(T, ...) \\
\text{Conductivity:} & \quad K(T, ...) 
\end{align*} \]

Depends on the properties of the crust (composition, superfluidity, ...).

Neutrinos emissivity

Solved with the cooling code of D. Page.

Weak or Strong neutron pairing?

Gnedin et al., MNRAS 324 (2001)
Superfluidity in Neutron Stars

(cf talk of Nakatsuka-san):

- **Crust**: - neutron $^1S_0$ superfluidity
- **Core**: - neutron $^3PF_2$ superfluidity
  - proton $^1S_0$ superconductivity
  - “exotic” superfluidity

- **Consequences**: - giant glitches
  - cooling

**Inner crust:**
Lattice of nuclear clusters + unbound particles (e, n)

Neutrons are superfluid in the $^1S_0$ channel, acting inside the nuclear cluster and in the gas.

$\rightarrow$ non-uniform superfluid matter treated in the HFB theory.
$^1S_0$ Pairing in uniform matter
(BCS and beyond)

Theories for uniform matter:
- BCS,
- beyond BCS: BCS+ screening, QMC, AFDMC, ...

\[ \Gamma = \gamma + \phi + \ldots \]

\[ \begin{array}{c}
\text{BCS} \\
\text{We assume two models for pairing:} \\
\text{Strong} \quad \text{(max at 3 MeV)}, \\
\text{Weak} \quad \text{(max at 1 MeV)}. \\
\end{array} \]

**Skyrme Self-Consistent H-F Bogoliubov in coordinate space**

Approx.: 1 single nuclei + n & p (no NSE, cf talk of Gulminelli, Typel)

Negele & Vautherin NPA 207 (1973)
Neutrons specific heat in $^{500}\text{Zr}$

$N=460, Z=40$

Pairing field profile at various temperatures:

Neutron specific heat:

Disappearance of the pairing:

in the neutron gas in the cluster

Neutrons specific heat (HFB)

We propose a general formula fitting these results.

Simple implementation in the cooling calculation.

Temperature profiles in the crust

The presence of non-uniform matter reduces the difference between strong and weak pairing.

Conclusions

- We have described **pairing correlations** in **non-uniform** nuclear matter using HFB theory & calculated $C_v$.
- We propose a formula for the $C_v$ in the crust.
- The $C_v$ have been used in a model for **thermal relaxation** of the crust (fast cooling).
- The crust thermalisation is influenced by the pairing correlations, the non-uniform matter induces some effects: the **difference** of cooling time between strong and weak pairing interaction **is reduced** compared to a calculation in uniform matter.
Nuclear symmetry energy and core-crust transition in neutron star: a critical study

Talk of F. Gulminelli

Density and pressure at the crust-core transition


- Crustal fraction of the moment of inertia
  \[ \frac{\Delta I}{I} = \frac{28\pi P_i R^3}{3M c^2 \xi} \left( 1 - 1.67 \xi - 0.6 \xi^2 \right) \]

- Can be measured from pulsar glitches
- Puts constraints on the NS radius; ex: Vela pulsar
- Results similar to LS
- Sensitivity to $C_{\text{sym}}$ to be studied

Lines: this w
Symbols: LS EOS
Correlation of $P_t$ versus $L$

There is a difference between the pressure in neutron matter and at fixed density:

and the transition pressure $P_t$:

One has to define the transition point.
Here we choose the intersection of the $\beta$-eq. with the spinodal contour.

Filled: 21 Skyrme interactions
Empty: 7 NL RMF, 4 DD RMF
Cross: BHF.

Very weak correlation!!
Correlations of $\rho_t$ and $Y_{p,t}$ versus $L$

Generalized Liquid-drop model:

$$E(\rho, y) = \sum_{n \geq 0} \left( c_{IS,n} + c_{IV,n} y^2 \right) \frac{x^n}{n!}$$

where

$$y = (\rho_n - \rho_p) / \rho$$

$$x = (\rho - \rho_0) / (3\rho_0)$$

$$c_{IS,0} = E_0 = E(\rho_0)$$

$$c_{IS,1} = 0 \quad c_{IS,2} = K_\infty$$

$$c_{IV,1} = L \quad c_{IV,2} = K_{sym}$$
Contributions to $\delta P_t / \delta L$

GLDM:

$$E(\rho, y) = \sum_{n \geq 0} \left( c_{IS,n} + c_{IV,n} y^2 \right) \frac{x^n}{n!}$$

$$P(\rho, y) = \frac{\rho^2}{3\rho_0} \left[ L y^2 + \sum_{n \geq 2} \left( c_{IS,n} + c_{IV,n} y^2 \right) \frac{x^{n-1}}{(n-1)!} \right]$$

Due to the cancelation between the terms, $\delta P_t / \delta L$ is close to 0 $\rightarrow$ very weak correlation.
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Thank you!

Firework in Aizu-Wakamatsu