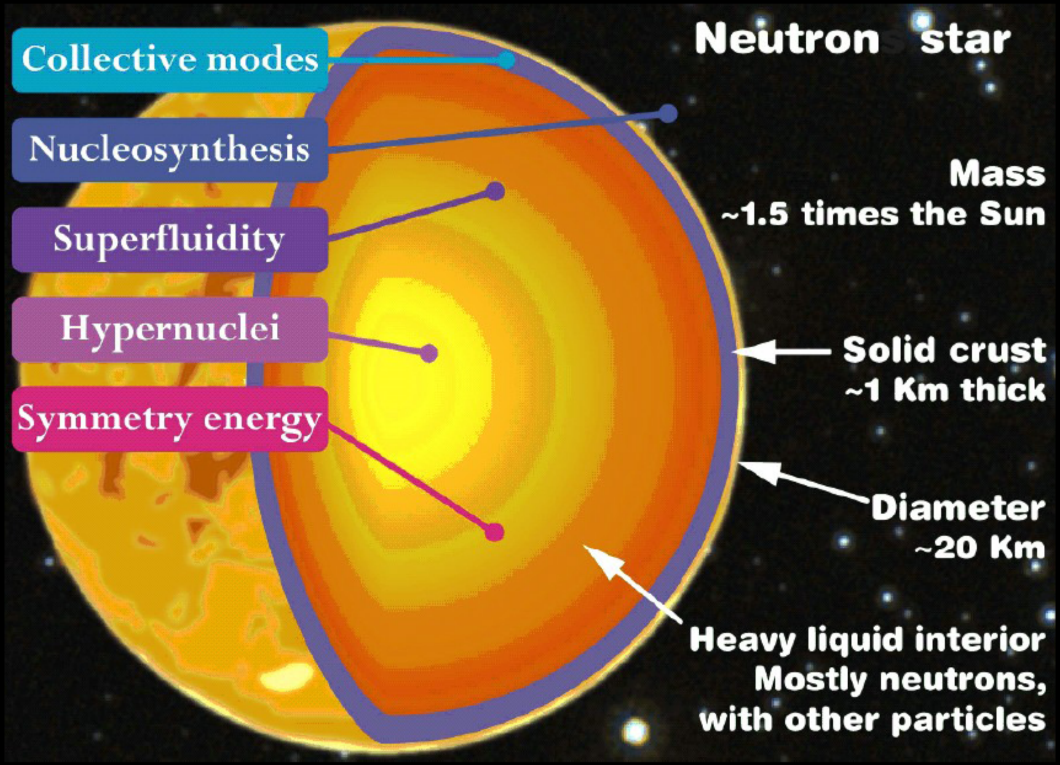
Neutron specific heat and thermalisation time of neutron stars crust

J. Margueron, IPN Orsay, France.



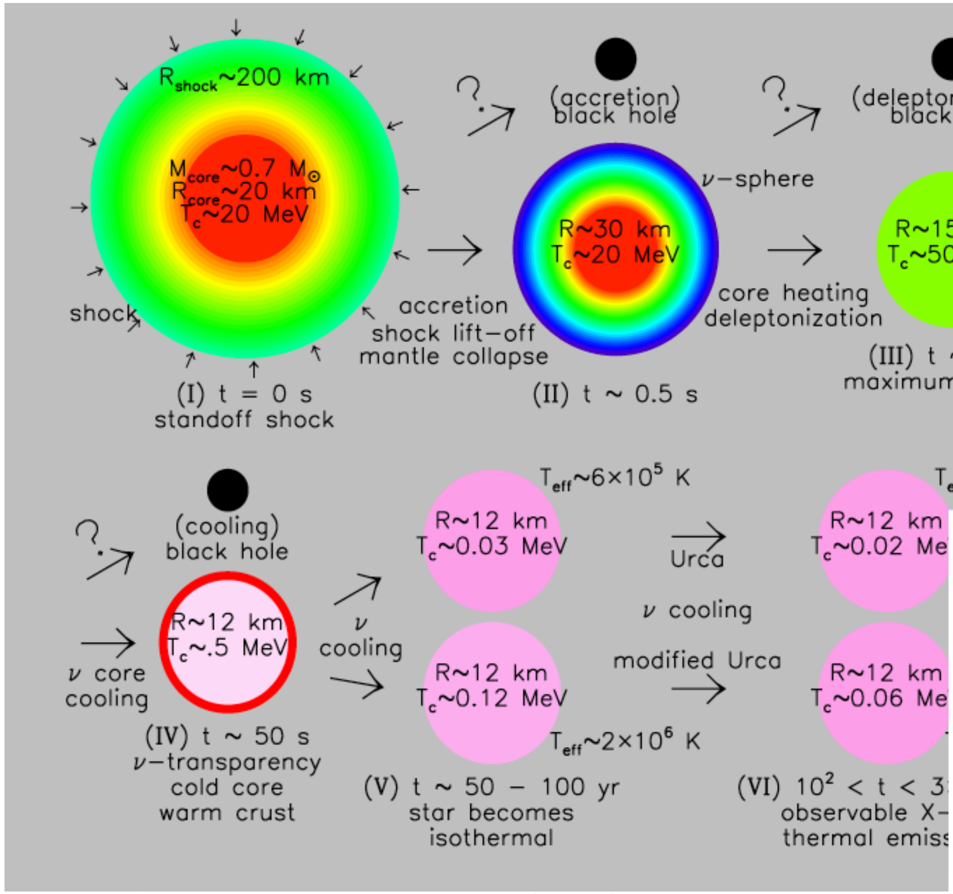
HFB calculation & specific heat of superfluid neutrons in non-uniform matter.

Effect of pairing on the thermalisation time of NS crust.

M. Fortin, F. Grill, J.M., D. Page, N. Sandulescu, arXiv:nucl-th/0910.5488

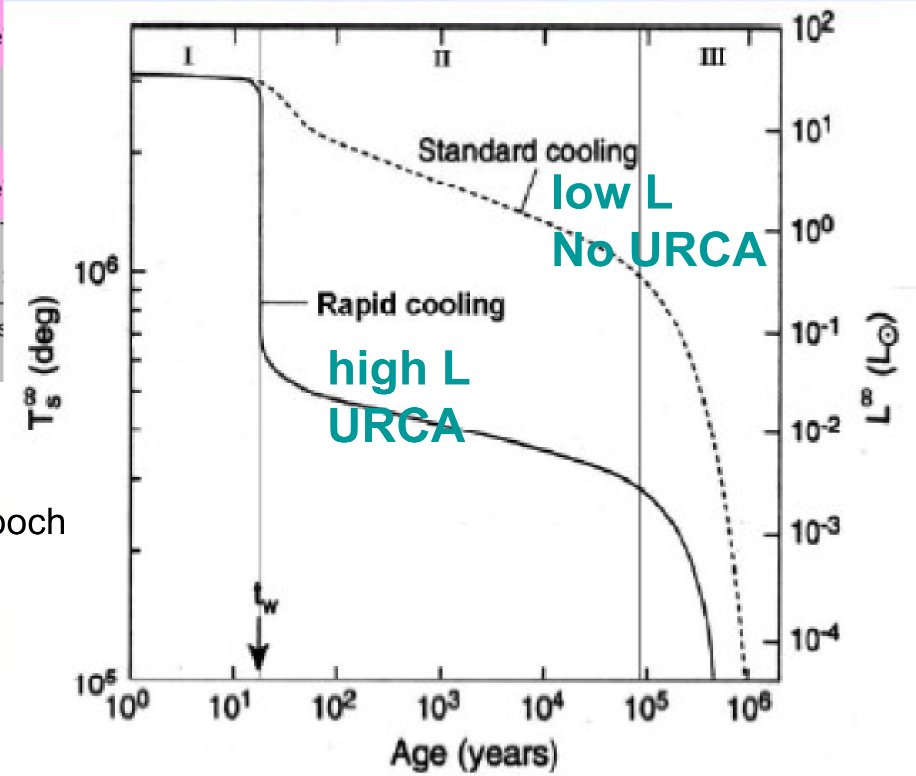
\+ **Nuclear symmetry energy and core-crust transition in neutron star: a critical study**, *accepted in EPL,* arXiv:nucl-th/0910.5488

Cooling of Neutron stars



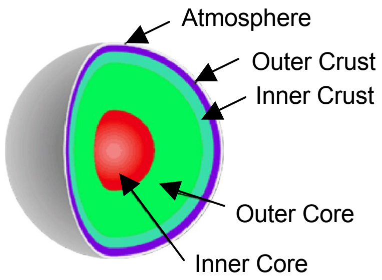
Lattimer & Prakash, Phys. Rep. 442 (2007)

Several types of cooling:	***Time-scales:***
• *Convective*	*s*
• *Conductive*	*1-100 y*
• *Radiative*	*>10^5 y*



I: Crust thermalisation epoch
II: ν cooling epoch
III: Photon epoch

Cooling of Neutron stars



URCA process (cf talk of Nakatsuka-san):

$$n \rightarrow p + e^- + \bar{\nu}_e, \quad p + e^- \rightarrow n + \nu_e$$

Gamow & Schoenberg, PR 59 (1941)

If proton fraction Z/A > 14% (large L, cf talk of Van Giai):
fast cooling by URCA process.

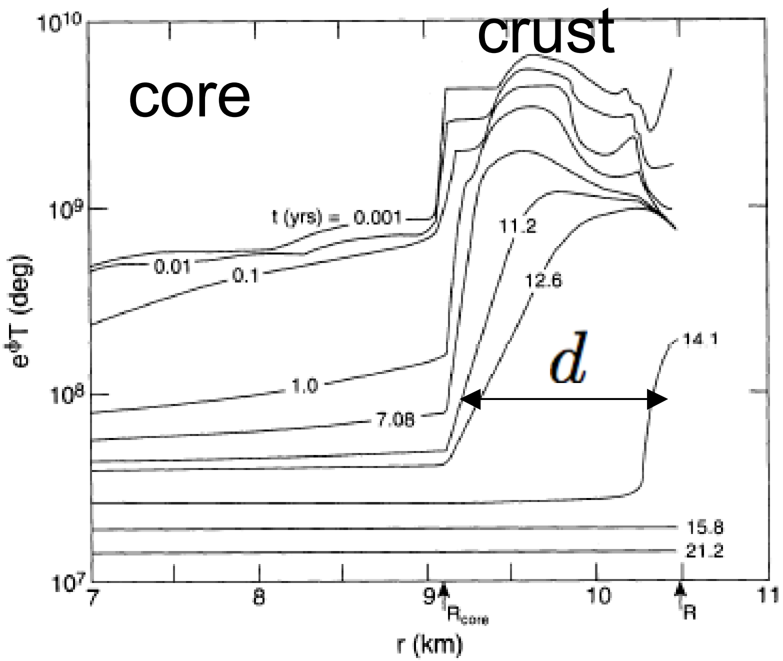
Lattimer et al., PRL 66 (1991)

Fast cooling:

→ after ~1 year: Tcore << Tcrust~0.5 MeV,
→ next ~10-100 years: **thermalisation** of the crust.

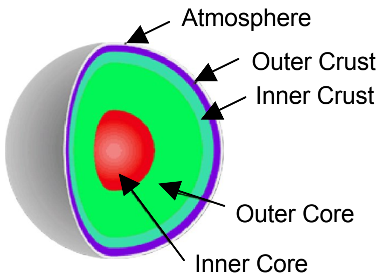
$$\tau \propto \frac{d^2}{D}$$

with $D = \dfrac{K}{\sum_i C_{v,i} \approx C_{v,n}}$



Lattimer et al., APJ 425 (1994)

Heat transport equations



GR equation for radiative transport (Thorne 1977):

$$e^{\phi(r)}L = -K4\pi r^2\sqrt{1-\frac{2Gm(r)}{rc^2}}\,\frac{\mathrm{d}(Te^{\phi(r)})}{\mathrm{d}r}, \quad \frac{\mathrm{d}(Le^{2\phi})}{\mathrm{d}r} = \frac{4\pi r^2}{\sqrt{1-2Gm(r)/rc^2}}e^{2\phi}\left(-Q_\nu - ne^{-\phi}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\varepsilon}{n}\right)\right)$$

Heat transport equation:

$$\frac{\partial}{\partial r}\left[\frac{Kr^2}{\Gamma(r)}e^{\phi}\frac{\partial}{\partial r}(e^{\phi}T)\right] = r^2\Gamma(r)e^{\phi}\left(C_V\frac{\partial T}{\partial t} + e^{\phi}Q_\nu\right)$$

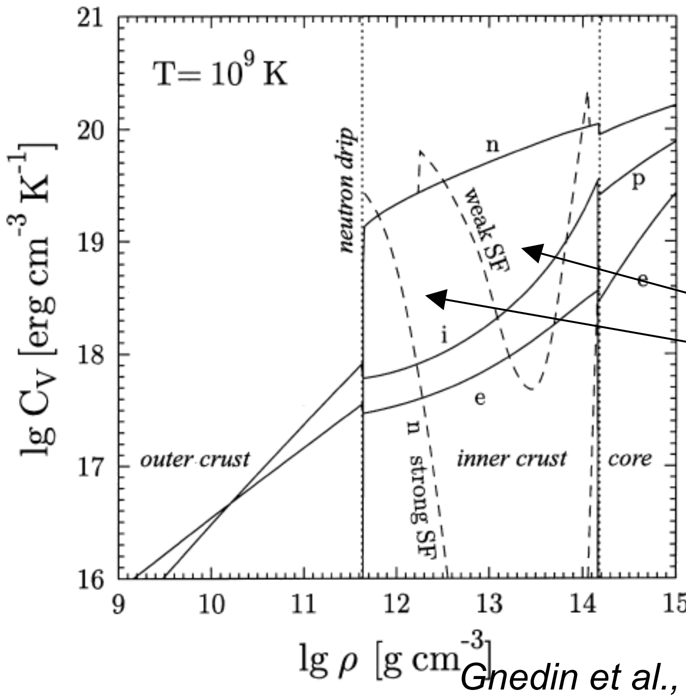
Neutrinos
emissivity

Solved with the cooling code of D. Page.

Microscopic inputs:

- Specific heat: $C_v(T,\ldots)$
- Conductivity: $K(T,\ldots)$

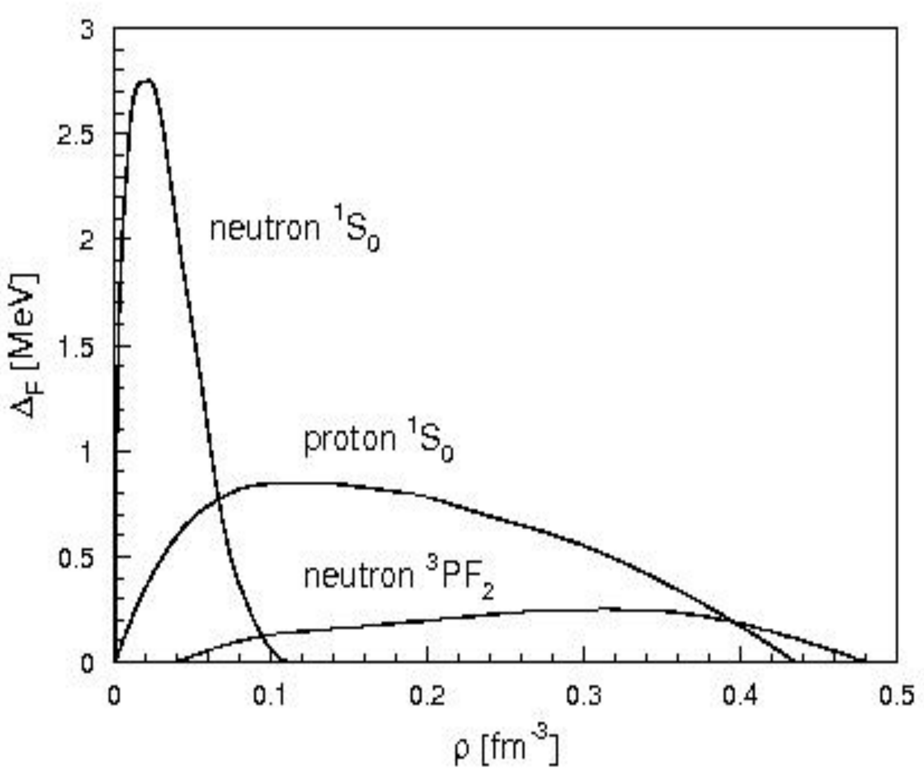
Depends on the properties of the crust (composition, superfluidity, ...).



Weak or
Strong neutron
pairing ?

Gnedin et al., MNRAS 324 (2001)

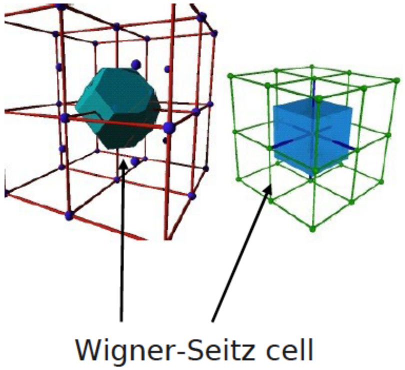
Superfluidity in Neutron Stars



(cf talk of Nakatsuka-san):

- ***Crust*** : - neutron $^1S_0$ superfluidity
- ***Core*** : - neutron $^3PF_2$ superfluidity
  - proton $^1S_0$ superconductivity
  - "exotic" superfluidity
- ***Consequences*** : - giant glitches
  - cooling

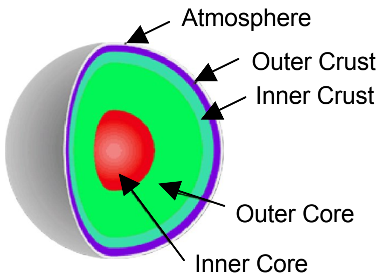
Inner crust:
Lattice of nuclear clusters + unbound particles (e, n)



Neutrons are superfluid in the $^1S_0$ channel, acting inside the nuclear cluster and in the gas.
➜ non-uniform superfluid matter treated in the **HFB** theory.

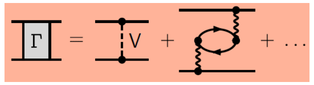
${}^1S_0$ Pairing in uniform matter

(BCS and beyond)



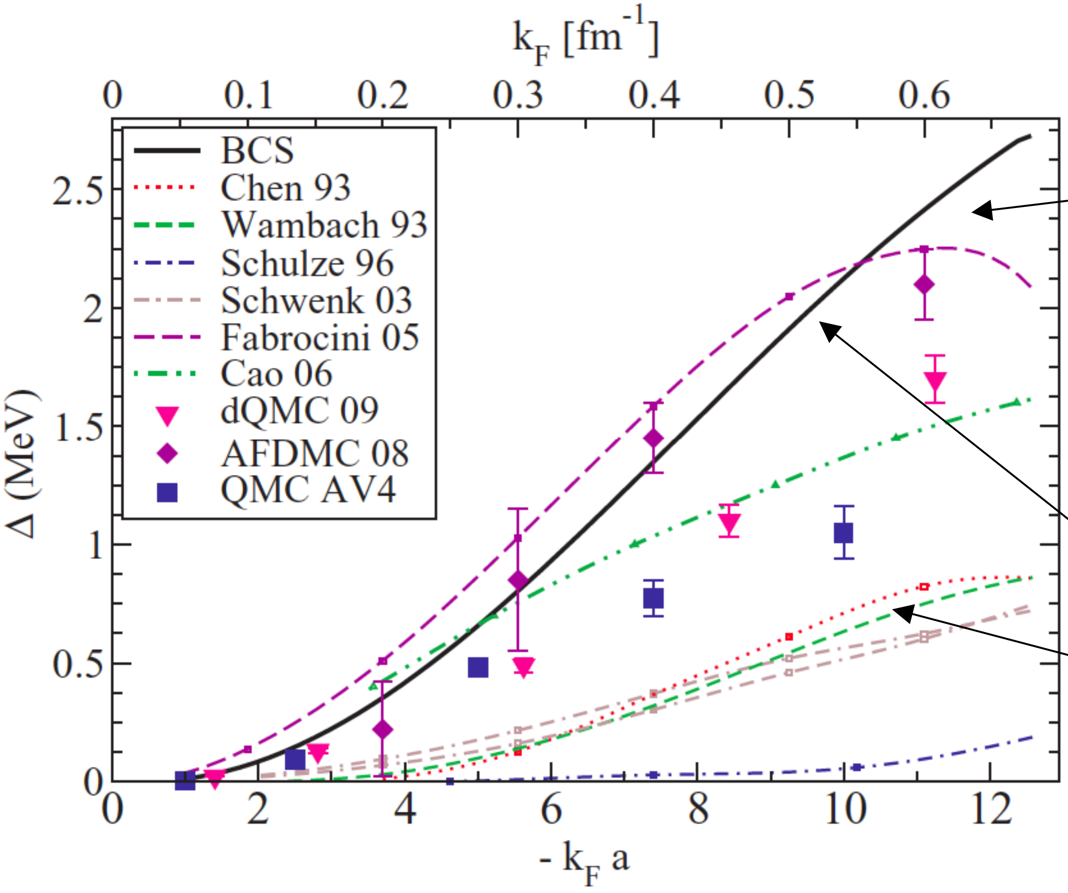
Theories for uniform matter:

- BCS,
- beyond BCS: BCS+ screening, QMC, AFDMC, ...



Gezerlis, Carlson, Phys. Rev. C 81 (2010)

Lombardo, Schulze, Lect. Notes Phys. 578 (2001)



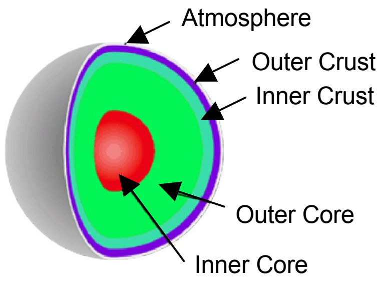
BCS

We assume two models for pairing:

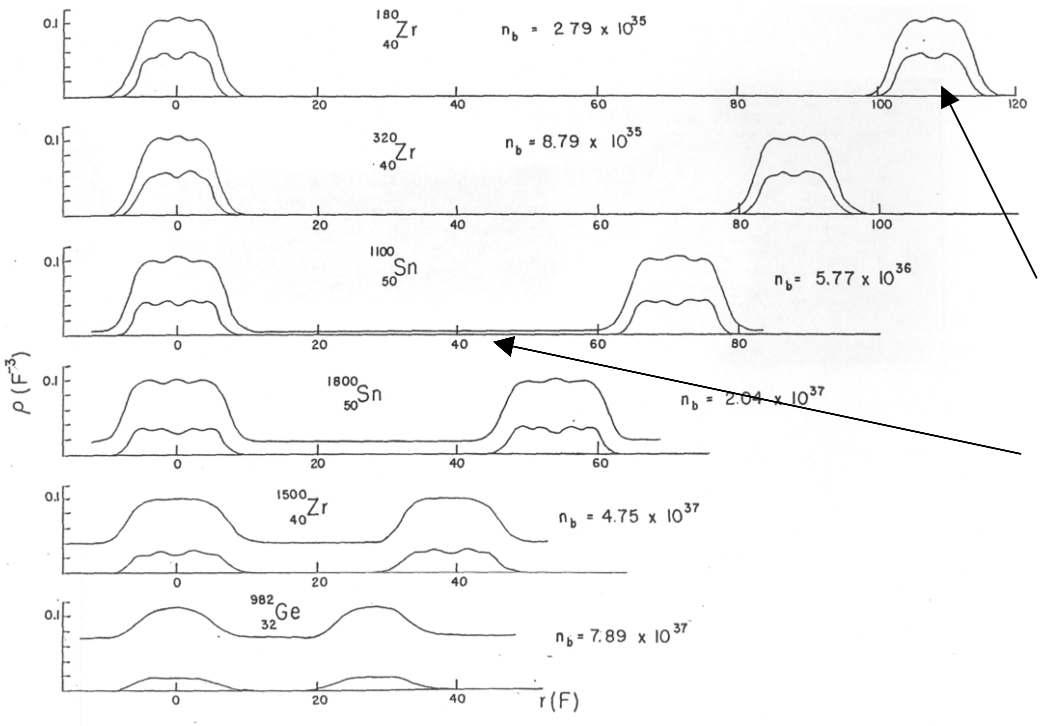
Strong (max at 3 MeV),

Weak (max at 1 MeV).

Skyrme Self-Consistent H-F Bogoliubov in coordinate space



Approx.: 1 single nuclei + n & p (no NSE, cf talk of Gulminelli, Typel)



Nuclear clusters

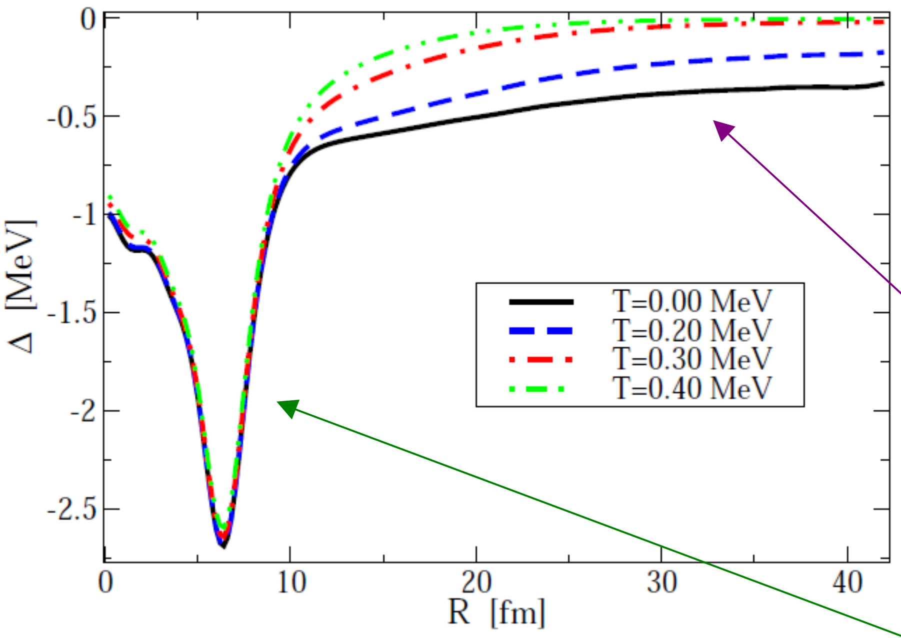
Unbound & Superfluid neutrons

Negele & Vautherin NPA 207 (1973)

Neutrons specific heat in ^{500}Zr

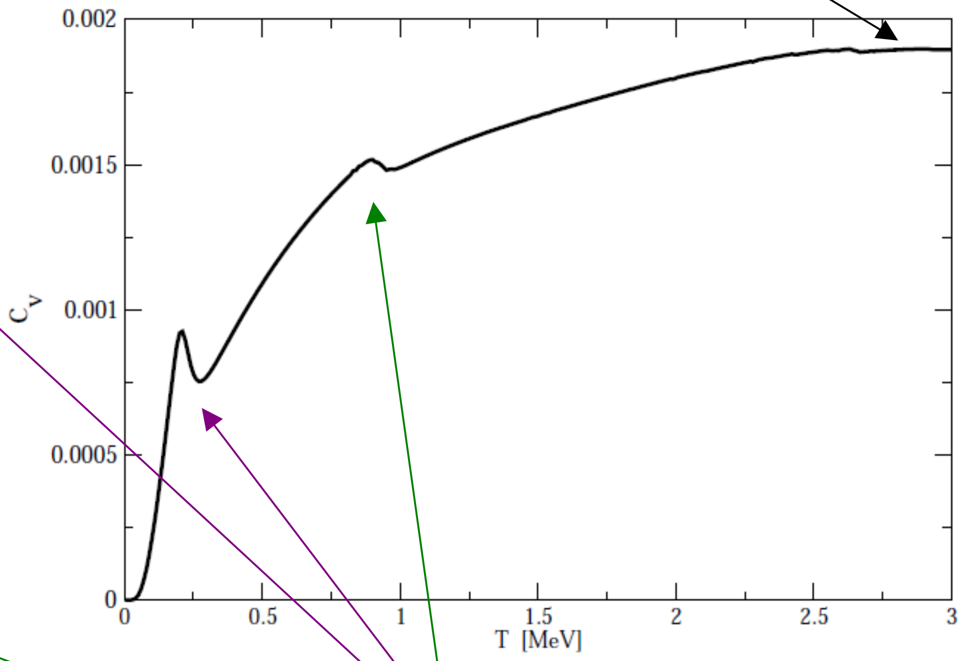
N=460, Z=40

Pairing field profile
at various temperatures:



Neutron specific heat:

Classical regime



Disappearance of the pairing:

in the neutron gas

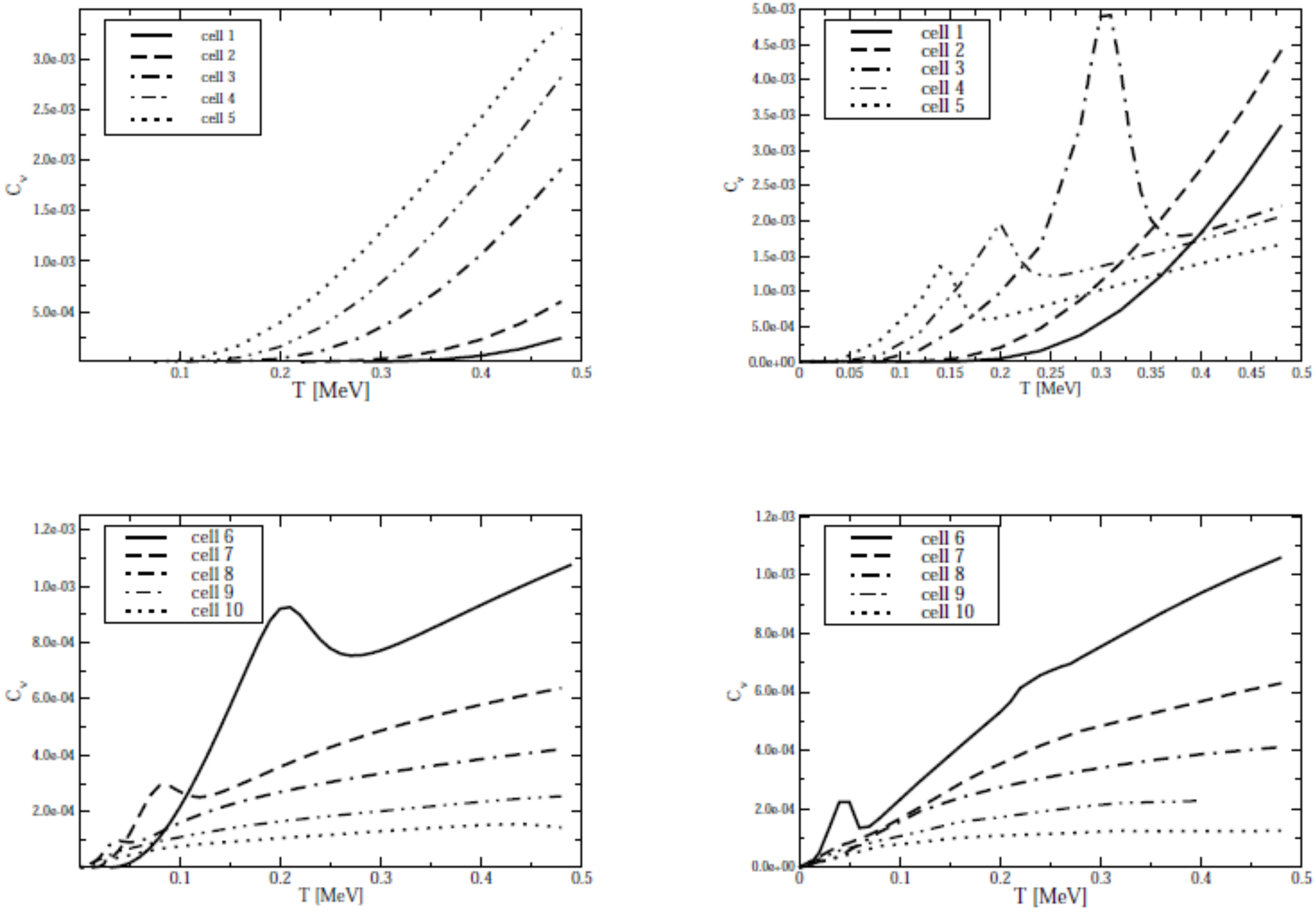
in the cluster

M. Fortin, F. Grill, J.M., D. Page, N. Sandulescu, arXiv/nucl-th/0910.5488

Neutrons specific heat (HFB)

Strong pairing

Weak pairing



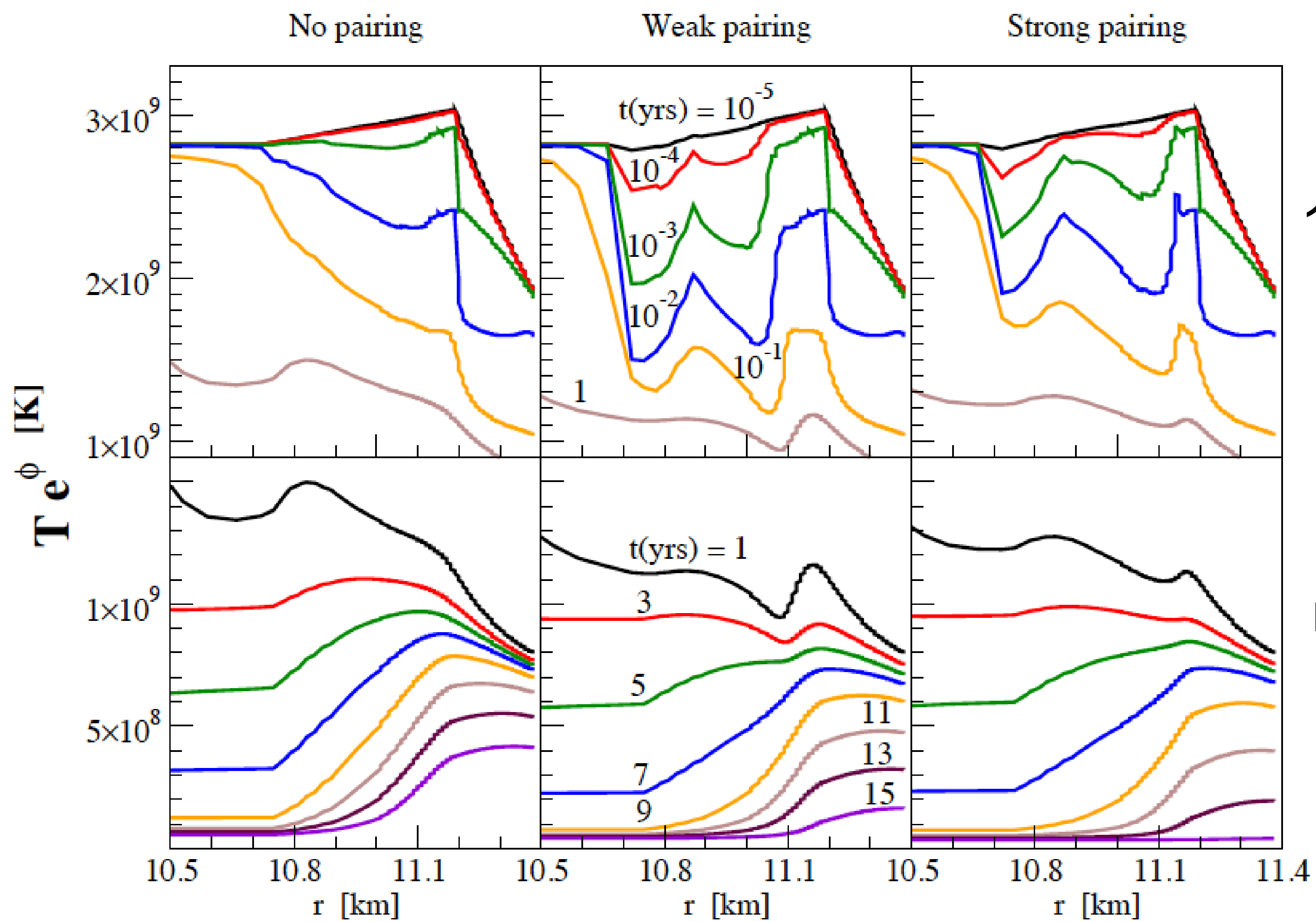
We propose a general formula fitting these results.

↓

Simple implementation in the cooling calculation.

M. Fortin, F. Grill, J.M., D. Page, N. Sandulescu, arXiv/nucl-th/0910.5488

Temperature profiles in the crust



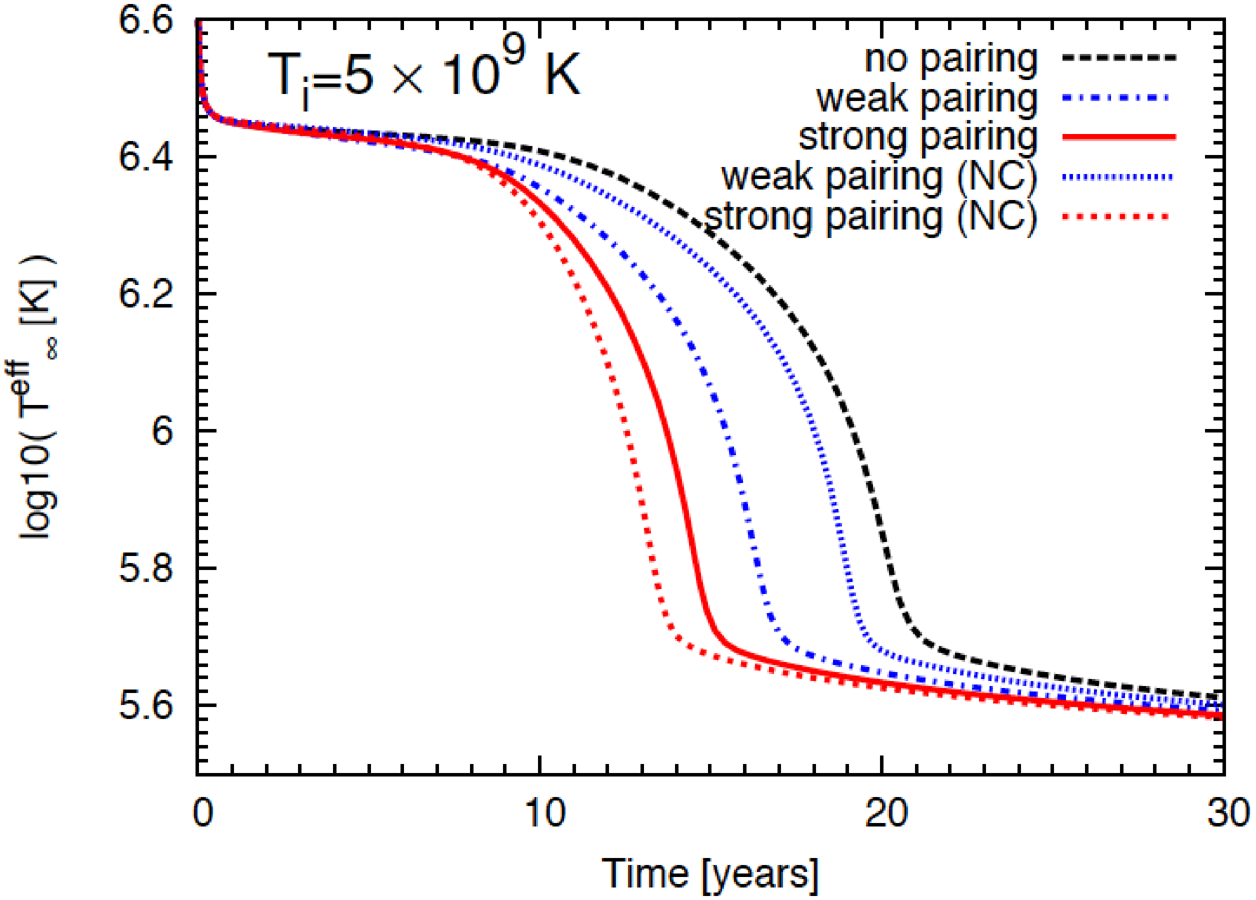
1.6 M_o
Fast cooling

1st year

next 15 yrs

M. Fortin, F. Grill, J.M., D. Page, N. Sandulescu, arXiv/nucl-th/0910.5488

Surface temperature



1.6 M_o
Fast cooling

NC=No clusters

The presence of non-uniform matter reduces the difference between strong and weak pairing.

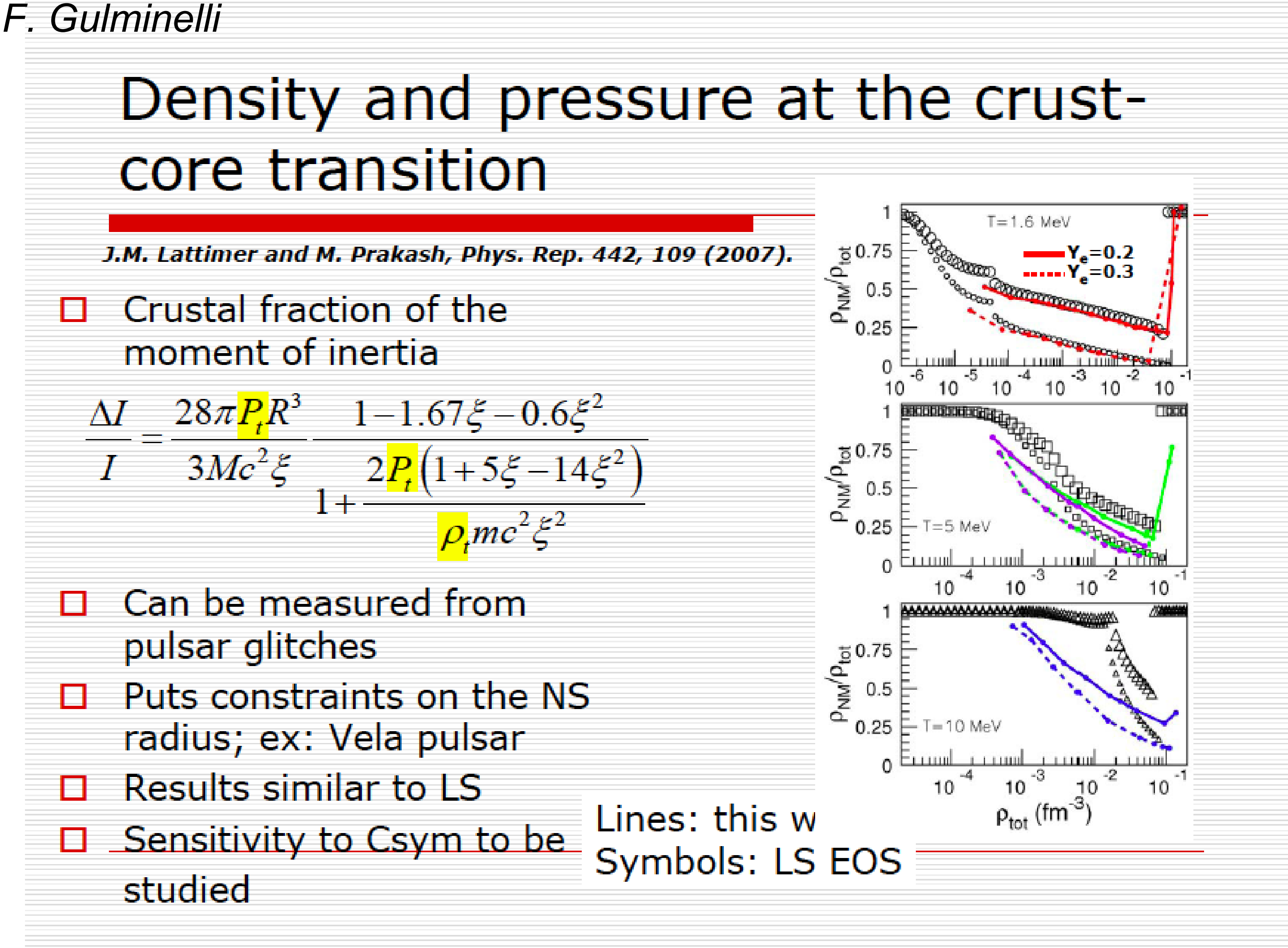
M. Fortin, F. Grill, J.M., D. Page, N. Sandulescu, arXiv/nucl-th/0910.5488

Conclusions

- We have described **pairing correlations** in **non-uniform** nuclear matter using HFB theory & calculated C_v.
- We propose a formula for the C_v in the crust.
- The C_v have been used in a model for **thermal relaxation** of the crust (fast cooling).
- The crust thermalisation is influenced by the pairing correlations, the non-uniform matter induces some effects: the **difference** of cooling time between strong and weak pairing interaction **is reduced** compared to a calculation in uniform matter.

Nuclear symmetry energy and core-crust transition in neutron star: a critical study

Talk of F. Gulminelli



Density and pressure at the crust-core transition

J.M. Lattimer and M. Prakash, Phys. Rep. 442, 109 (2007).

- ☐ Crustal fraction of the moment of inertia

$$\frac{\Delta I}{I}=\frac{28\pi P_t R^3}{3Mc^2\xi}\,\frac{1-1.67\xi-0.6\xi^2}{1+\dfrac{2P_t\left(1+5\xi-14\xi^2\right)}{\rho_t mc^2\xi^2}}$$

- ☐ Can be measured from pulsar glitches
- ☐ Puts constraints on the NS radius; ex: Vela pulsar
- ☐ Results similar to LS
- ☐ Sensitivity to Csym to be studied

Lines: this w
Symbols: LS EOS

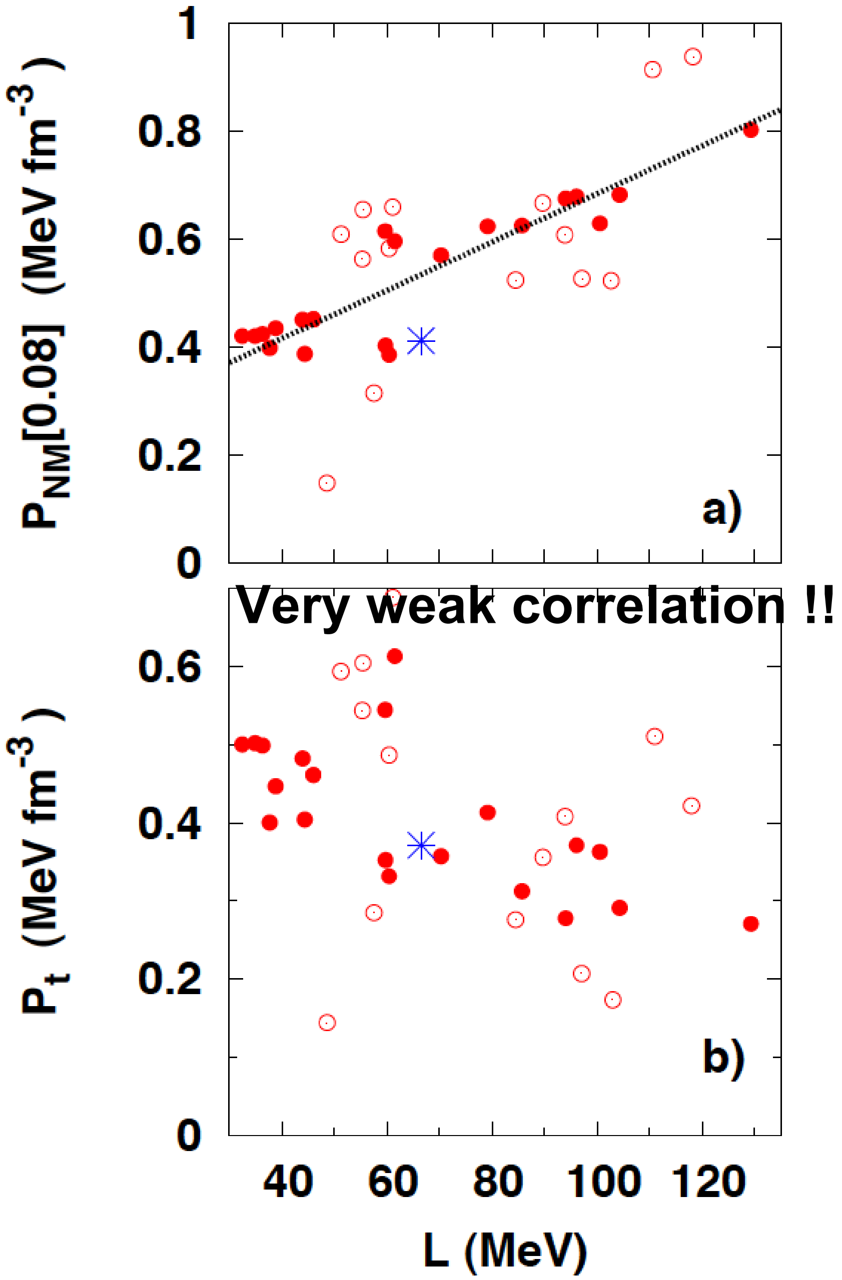
Correlation of P_t versus L

There is a difference between the pressure in neutron matter and at fixed density:

and the transition pressure P_t:

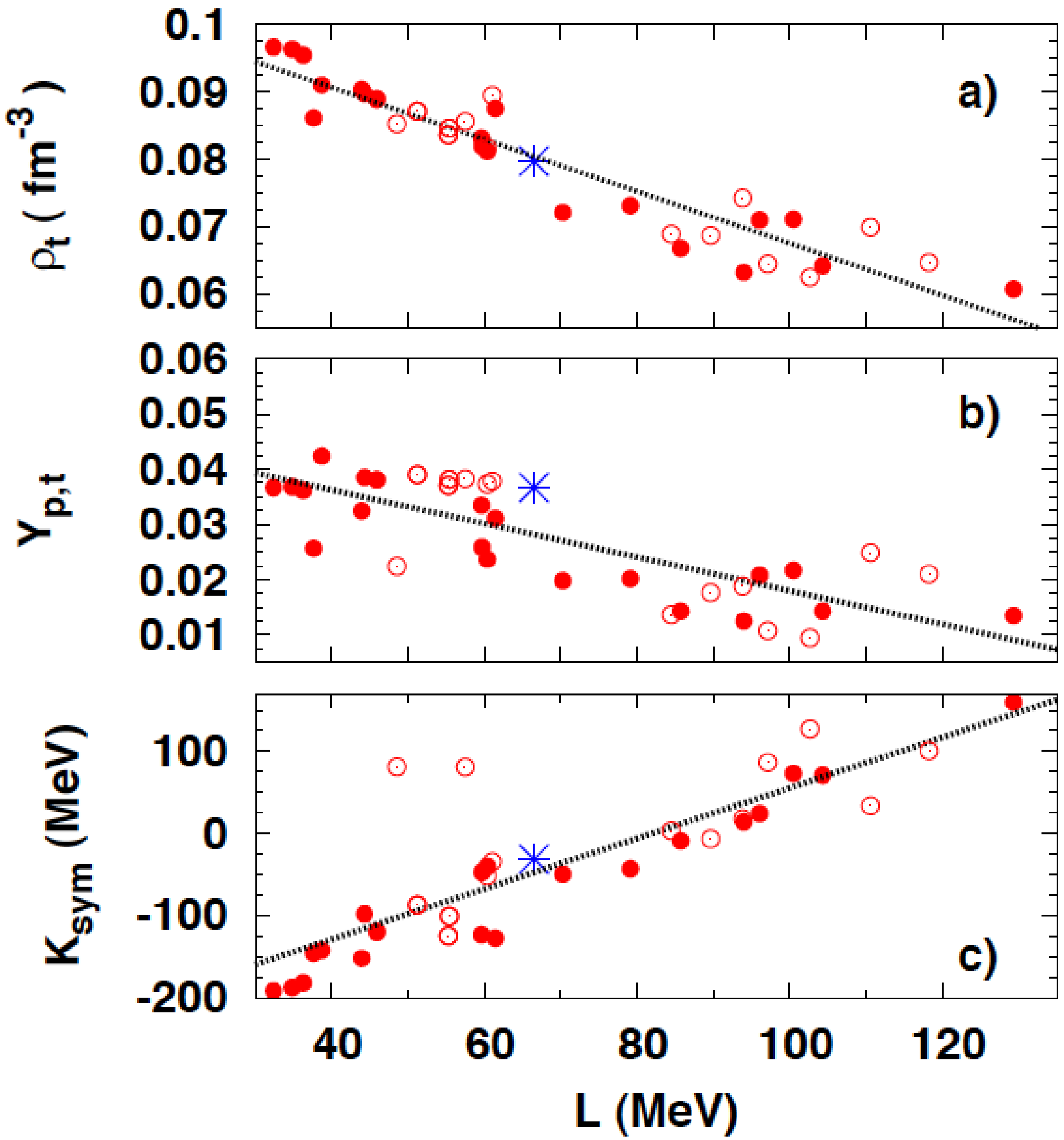
One has to define the transition point.
Here we choose the intersection of the β-eq. with the spinodal contour.

Filled: 21 Skyrme interactions
Empty: 7 NL RMF, 4 DD RMF
Cross: BHF.



Very weak correlation !!

Correlations of ρ_t and $Y_{p,t}$ versus L



Generalized Liquid-drop model:

$$E(\rho, y) \;=\; \sum_{n\geq 0} \left(c_{\mathrm{IS},n} + c_{\mathrm{IV},n} y^2\right) \frac{x^n}{n!}$$

where

$$y = (\rho_n - \rho_p)/\rho$$

$$x = (\rho - \rho_0)/(3\rho_0)$$

$$c_{\mathrm{IS},0} = E_0 \equiv E(\rho_0)$$

$$c_{\mathrm{IV},0} = J \equiv S(\rho_0)$$

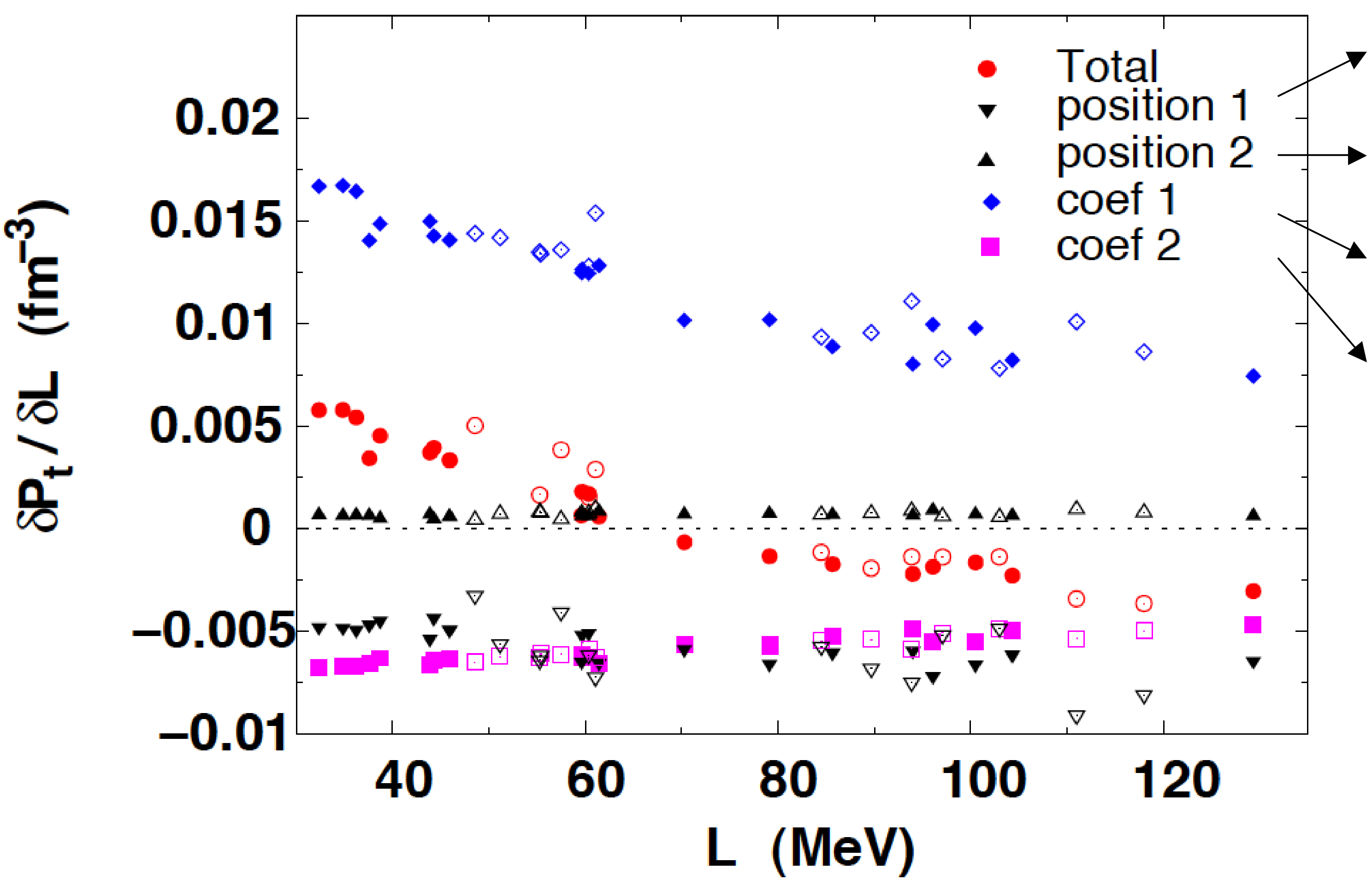
$$c_{\mathrm{IS},1} = 0 \qquad c_{\mathrm{IS},2} = K_\infty$$

$$c_{\mathrm{IV},1} = L \qquad c_{\mathrm{IV},2} = K_{\mathrm{sym}}$$

Contributions to $\delta P_t/\delta L$

GLDM:

$$E(\rho, y) = \sum_{n\geq 0} \left(c_{\mathrm{IS},n} + c_{\mathrm{IV},n} y^2\right) \frac{x^n}{n!} \qquad P(\rho, y) = \frac{\rho^2}{3\rho_0}\left[Ly^2 + \sum_{n\geq 2} \left(c_{\mathrm{IS},n} + c_{\mathrm{IV},n} y^2\right) \frac{x^{n-1}}{(n-1)!}\right]$$



$$\frac{\partial P}{\partial \rho}\frac{\delta \rho_t}{\delta L}$$

$$\frac{\partial P}{\partial y}\frac{\delta y_t}{\delta L}$$

$$\frac{\partial P}{\partial L}$$

$$\frac{\partial P}{\partial K_{\mathrm{sym}}}\frac{\delta K_{\mathrm{sym}}}{\delta L}$$

Due to the cancelation between the terms, $\delta P_t/\delta L$ is close to 0 ➜ very weak correlation.

Collaboration with:

C. Ducoin (Univ. Of Coimbra)
M. Fortin (Obs. of Meudon & CAMK Warsaw)
F. Grill (Univ. of Milano)
D. Page (UNAM Mexico)
C. Providenca (Univ. of Coimbra)
N. Sandulescu (NIPNE Bucharest)

Thank you !