RIKEN, Wako, Japan

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EoS of Nuclear Matter and Transport Parameters in Neutron Stars

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- ✓ **EoS of Nuclear Matter from Brueckner Theory**
- ✓ **In-medium NN Cross Section and Transport Parameters in β-stable Nuclear Matter**
- ✓ **Some Implications for the Rotational Dynamics of Neutron Stars:**
  - **i) Interplay between gravitational radiation and viscosity in non radial modes of neutron stars**
  - **ii) thermal relaxation of 'newborn *NS*' and long era cooling (vs. urca processes and superfluidity)**

Motivation

gravitational radiation emitted by r-modes in rapidly rotating neutron stars drives the instability of the system (Chandrasekhar 1970)

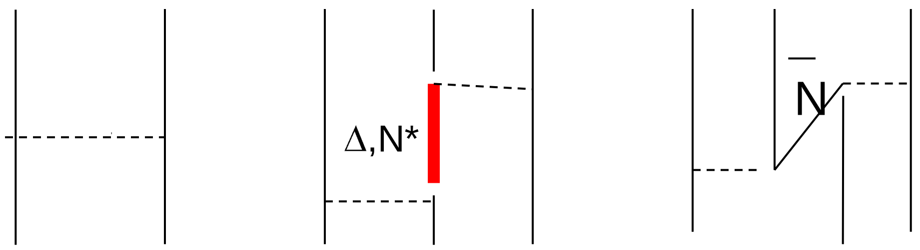
Which dissipation mechanism tend to suppress this instability?
A good candidate is the shear viscosity

Which constituent of NS mainly contribute to the viscosity ?
neutrons, leptons, hyperons, quarks,…

This is a preliminary investigation

ab initio Calculations from Brueckner theory

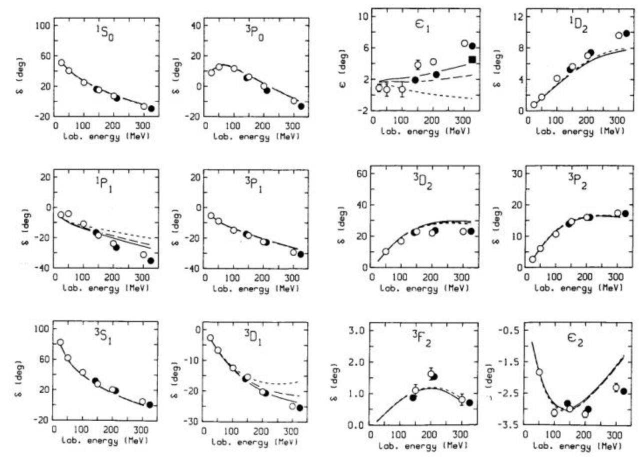
Input: *Bare Interaction V_{NN} (OBEP)*



2BF

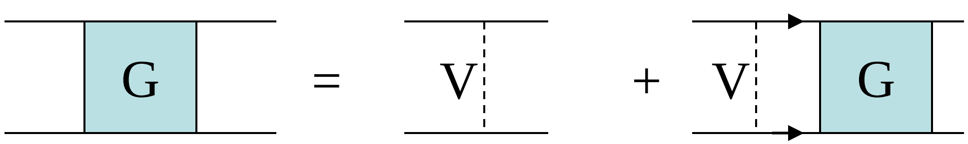
3BF
Fujita-Mihiazawa

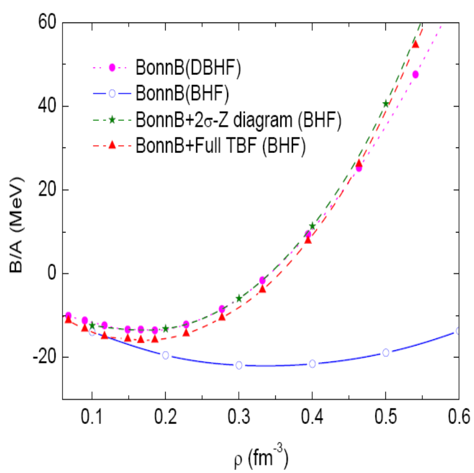
Dirac sea
excitations



fit of experimental NN phase shifts

Output: *In-medium Interaction : G_{NN}*





zero density -> NN scattering amplitude

Properties of G-matrix:

- ***EoS of Nuclear Matter   E=E(ρ,T)***
- ***Symmetry energy Esym(ρ,T)***
- ***MD mean field → effective masses m^*_p and m^*_n***
- ***in-medium NN scattering amplitude***

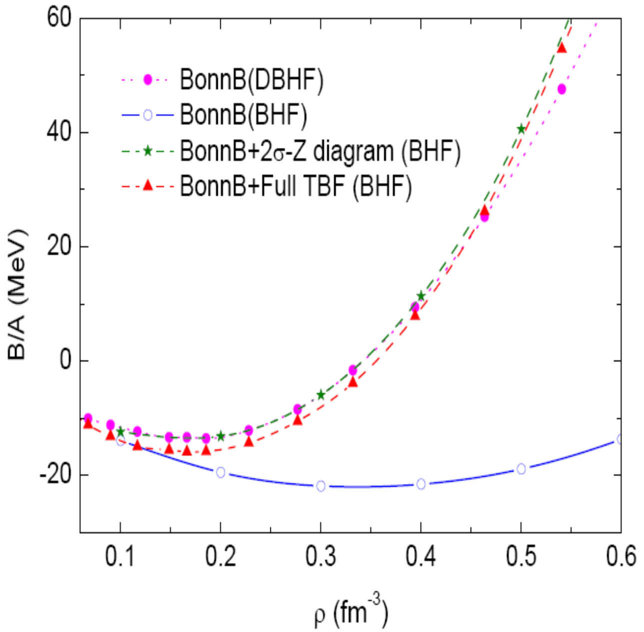
providing a unified framework where to study equilibrium and transport properties of nuclear matter

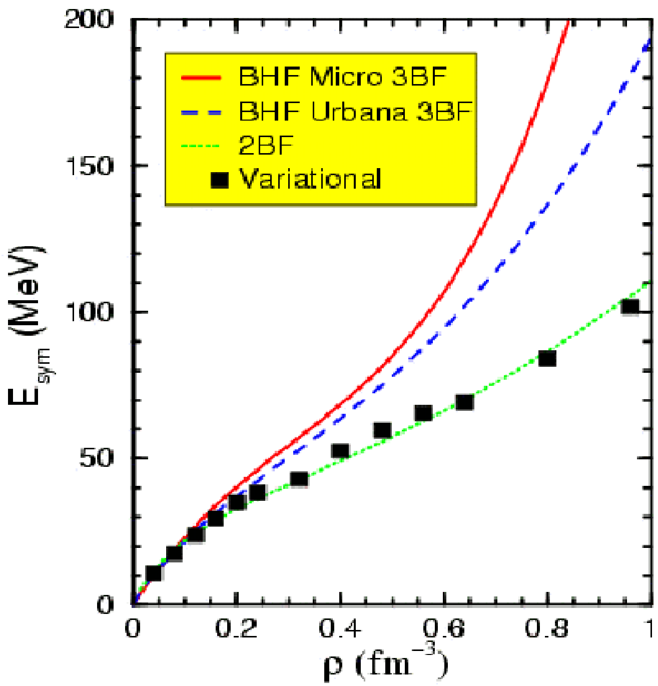
Equilibrium : HJ Schulze talk

EoS of Nuclear Matter

$$E(\rho) = K + \frac{1}{2}\sum_{ST}\sum_{p,p'<p_f} < pp'|G^{ST}(\rho)|\, pp' >_a$$

$$E_{sym}(\rho) = E_{PNM}(\rho) - E_{SNM}(\rho)$$





NS internal structure

nucleons,hyperons, kaons, quarks

in beta-equilibrium with leptons

- **beta-equilibrium with electrons and muons** : $p + e^{-} \rightarrow n + \nu$
- **hyperonized matter:** $n + n \rightarrow n + \Lambda \; (p + \Sigma^{-})$ at $\rho > 2\rho_o$
- **kaon condensation** $n \rightarrow p + K^{-}$ at $\rho > 2\text{-}3\rho_o$
- **transition to quark matter** HP $\rightarrow$ QP (u,d,s) at $\rho \sim 6\rho_o$

Any new degree of freedom makes the EoS to be softer and the maximum mass turns out to be lowered

baryons are in weak-coupling with leptons (electrons,muons,…)

$$\boxed{\begin{array}{l} p + e^- \to n + \nu_e \\ n \to p + e^- + \nu_e \end{array}} \qquad \boxed{\begin{array}{l} p + \mu^- \to n + \nu\mu \\ n \to p + \mu^- + \nu\mu \end{array}}$$

$$\textit{charge conservation}: \ \rho_p = \rho_e \ \to \ k_p^F = k_e^F$$

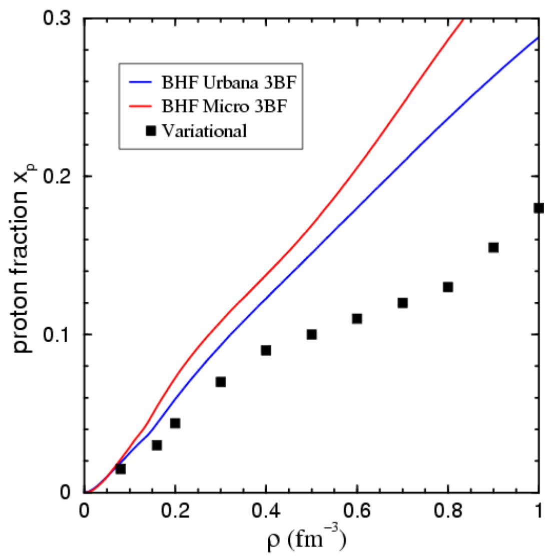
$$\textit{chemical equilibrium}: \ \mu_n - \mu_p = \mu_e$$

$$\mu_n - \mu_p = 4E_{sym}\beta$$

energy loss replacing a proton with a neutron,

namely the symmetry energy (a_A *in* B-W *mass formula*, $N \neq Z$)

$$\rho_p \approx \frac{1}{2}\left(\frac{4E_{sym}}{\hbar c k_F}\right)^3$$



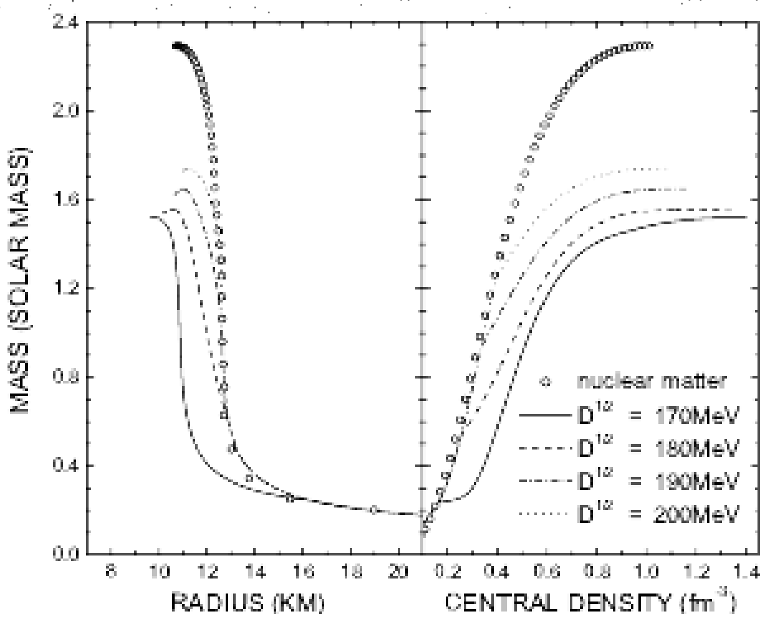
Gravitation vs Nuclear force: a new game
Tolman-Oppenheimer-Volkov (TOV) and nuclear EoS

Input → **Equation of State P=P(ρ, ρ_p)**

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{G\rho m}{r^2}\left(1+\frac{P}{\rho c^2}\right)\left(1+\frac{4\pi P r^3}{mc^2}\right)\left(1-\frac{2Gm}{c^2 r}\right)^{-1},$$

$$\frac{\mathrm{d}m}{\mathrm{d}r} = 4\pi r^2 \rho,$$

Output → **Mass-Radius plot**



Transport Kinetic Equations

-----multicomponent system-----

$$\frac{\partial f_i}{\partial t} + \nabla \varepsilon_i \cdot \nabla_p f_i - \nabla f_i \cdot \nabla_p \varepsilon_i = \sum_j I_{ij} = \sum \left(\frac{d\sigma_{ij}}{d\Omega} \right)_{med} \cdot \underbrace{(f_i' f_j' \tilde{f}_i \tilde{f}_j}_{\text{gain}} - \underbrace{f_i f_j \tilde{f}_i' \tilde{f}_j')}_{\text{loss}}$$

$$\varepsilon^{\tau}{}_p = \frac{p^2}{2m} + \sum_{\tau'} \sum_{p' < p_f} < pp' | G^{\tau\tau'} | pp' >_a = \frac{p^2}{2m_\tau^*} + U_\tau$$

$$\frac{d\sigma_{np}(\vartheta)}{d\Omega} \sim \frac{m^{*2}}{4\pi^2 \hbar^4} \sum_{SS_z S_z'} |\langle p | G^S{}_{S_z S_z'}(\vartheta) | p' \rangle|^2$$

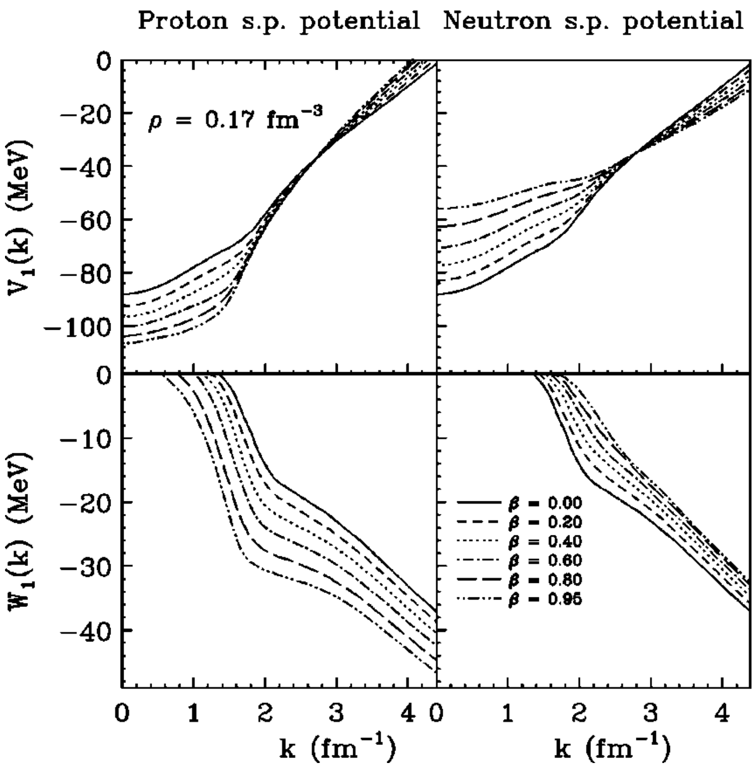
medium effects:

✓Pauli blocking: nucleons scatter into unoccupied states

✓Strong mean field between two collisions

✓Compression of the level densities in entry and exit chennels

Mean field and Effective Mass



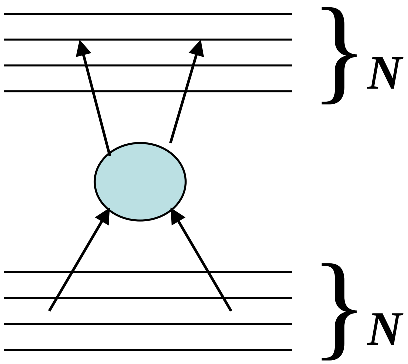
$$U_{sym}(E) = \frac{U_n(E) - U_p(E)}{2\beta}$$

$$2\beta \frac{dU_{sym}(E)}{dE} = \frac{m_p^* - m_n^*}{m}.$$

empirical OMP data support the prediction **$m^*_n > m^*_p$**

Many-Body Effects on

-------------------------Finite Range Interaction--------------------------

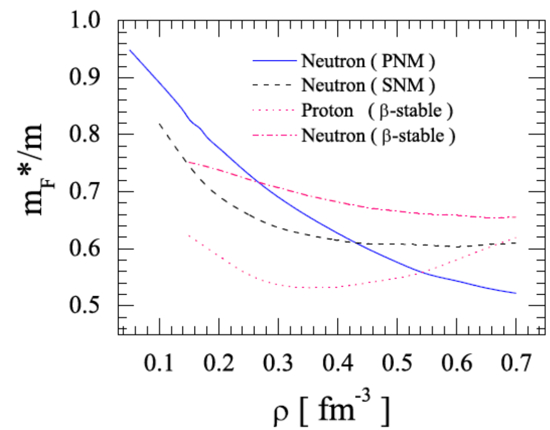


level density:

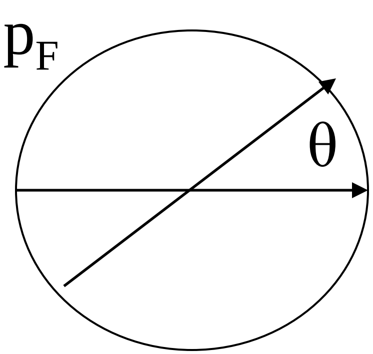
$$N = \left(\frac{d\varepsilon_p}{dp} \right)^{-1} = \frac{m^*}{p}$$

$$\varepsilon_p = \frac{p^2}{2m} + U_p = \frac{p^2}{2m^*} + U_0$$

p-dependent potential



-------------------------Pauli blocking---------------------------------------



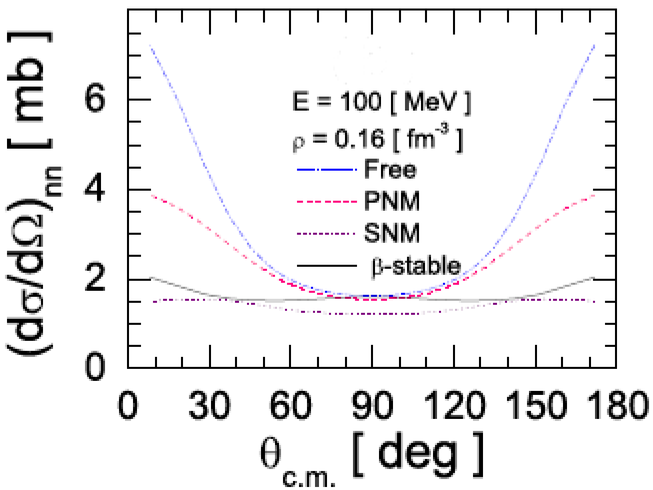
Fermi sphere
In CM frame

$$\Delta p = 2p_F \sin(\theta/2) > 0$$

backward and forward scatterings sizably suppressed

In Medium Cross Sections

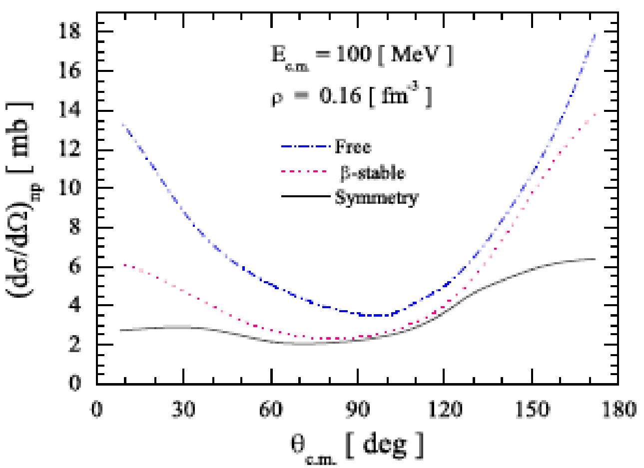
neutron matter



like particles

$$\sigma_{nn}(\vartheta) \sim \frac{m^{*2}}{16\pi^2\hbar^4} \sum_{SS_zS_z'} |\langle p | G^S{}_{S_zS_z'}(\vartheta) | p' \rangle + (-)^S \langle p | G^S{}_{S_zS_z'}(\pi - \vartheta) | p' \rangle|^2$$

nuclear matter



unlike particles

$$\sigma_{np}(\vartheta) \sim \frac{m^{*2}}{4\pi^2\hbar^4} \sum_{SS_zS_z'} |\langle p | G^S{}_{S_zS_z'}(\vartheta) | p' \rangle|^2$$

In Medium Cross Sections

$$\sigma(\Omega) = N_0^{\,2} \,|\langle q | V_{nn}^{(2)} | q \rangle|^2 \Rightarrow N^2 \,|\langle q | G_{nn}^{(2+3)} | q \rangle|^2$$

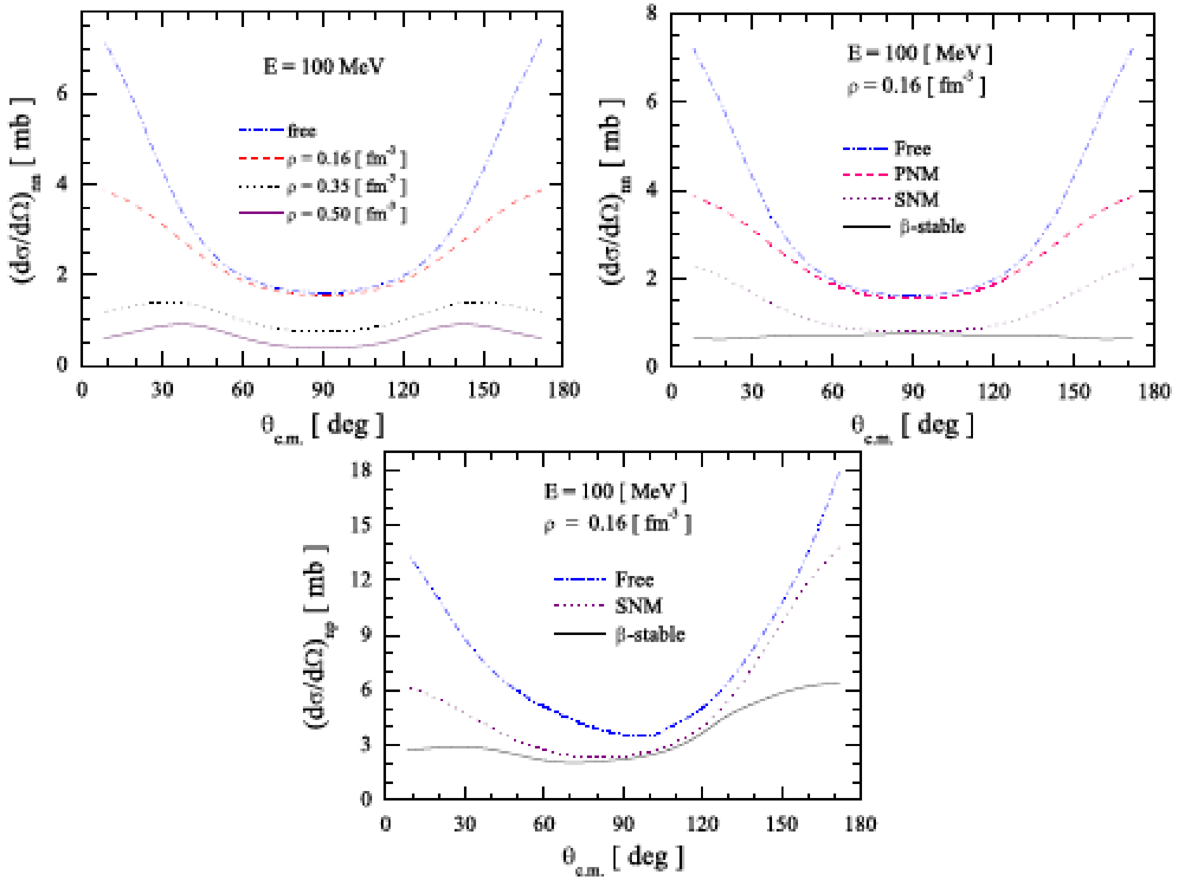


FIG. 3: (color online). Upper panel: nn differential cross sections in pure neutron matter (left) nn differential cross section in pure neutron matter, symmetric and β-stable nuclear matter (right). Lower panel np differential cross section in pure neutron matter, symmetric and β-stable nuclear matter. The free cross section is also plotted for comparison.

Transport Parameters

Collaboration:

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Transport parameters in neutron stars from in-medium NN cross sections

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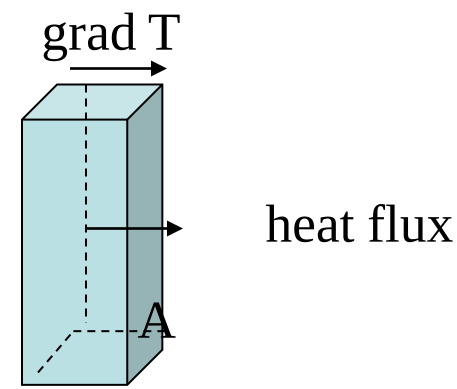
transport equations

heat conduction

$$\frac{2}{3}\rho\left(\frac{\partial}{\partial t}+u\cdot\nabla\right)\theta = -\rho(\nabla\cdot u)\theta + K\Delta\theta$$



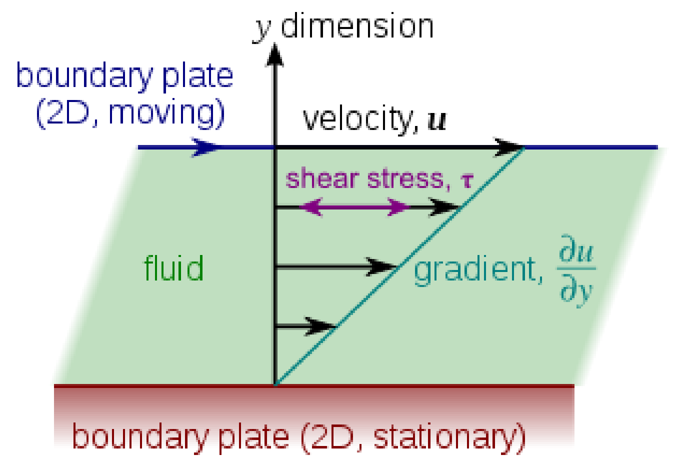
(K **thermal conductivity)**

Navier-Stokes Eq.

bulk and shear viscosity

$$\rho\left(\frac{\partial}{\partial t}+u\cdot\nabla\right)u = f-\nabla\left(p-\frac{\mu_V}{3}\nabla\cdot u\right)+\mu_T\Delta u$$

compression modes



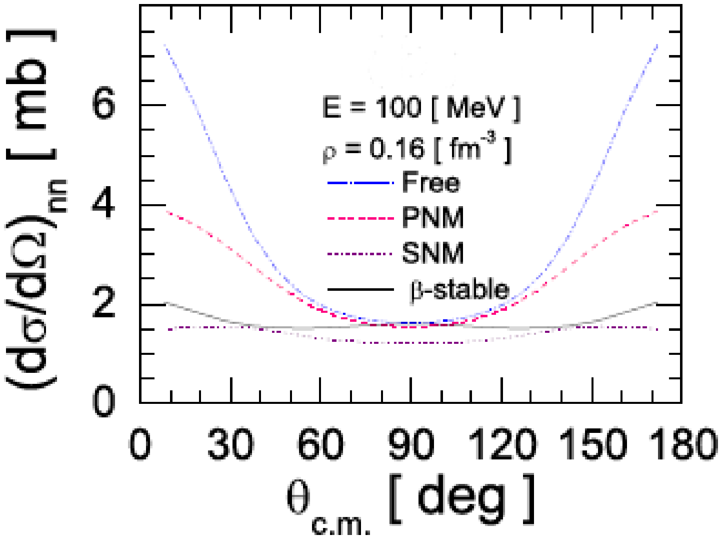
Cross Sections in β-stable matter

ANM with $\beta = \beta(\rho)$

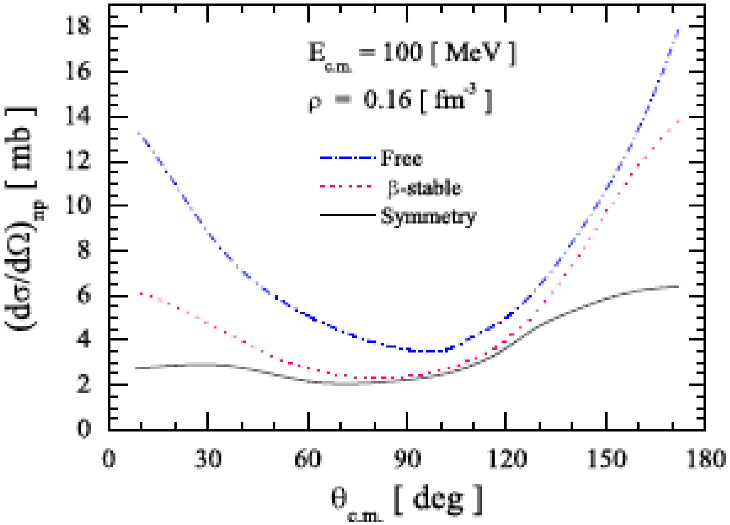
$p + e^- \rightarrow n + \nu_e$

$n \rightarrow p + e^- + \nu_e$

nn collisions



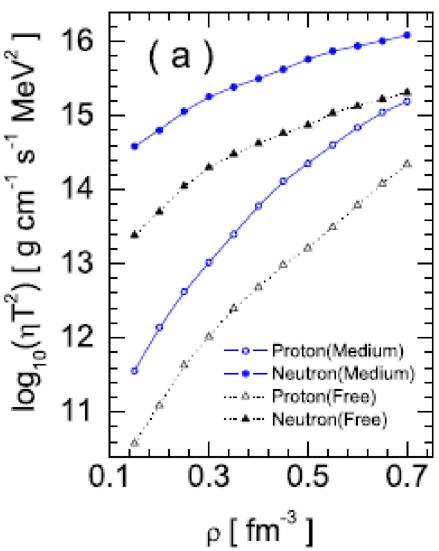
np collisions

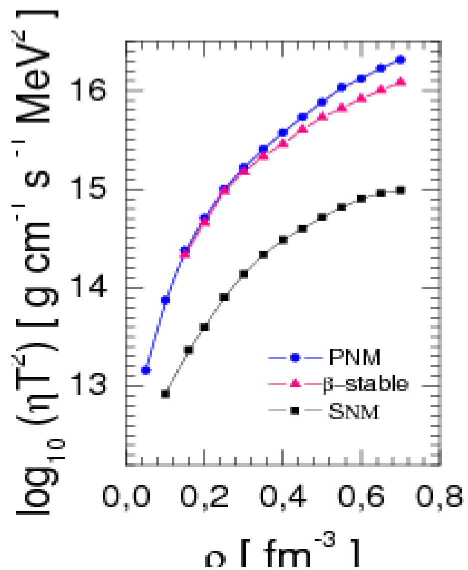


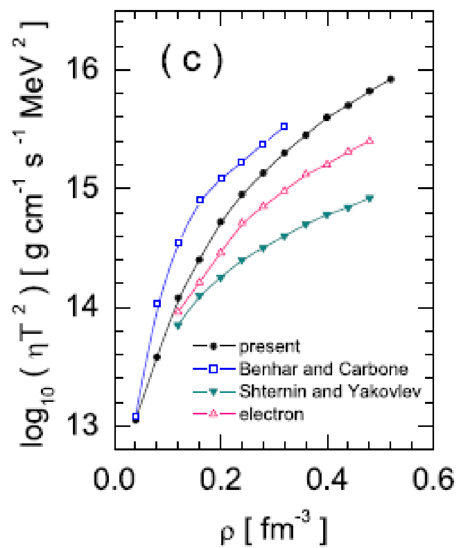
non linear behaviour of proton mean field and effective mass

shear viscosity

$$\eta T^2 = \frac{1}{20}\rho v_F^2 C(\lambda)\int_0^{4\varepsilon_F}\frac{dE}{2\varepsilon_F}\int_0^{2\pi}\frac{d\vartheta}{2\pi}\left(1-E/4\varepsilon_F\right)^{-\frac{1}{2}}\sigma(E,\vartheta)$$







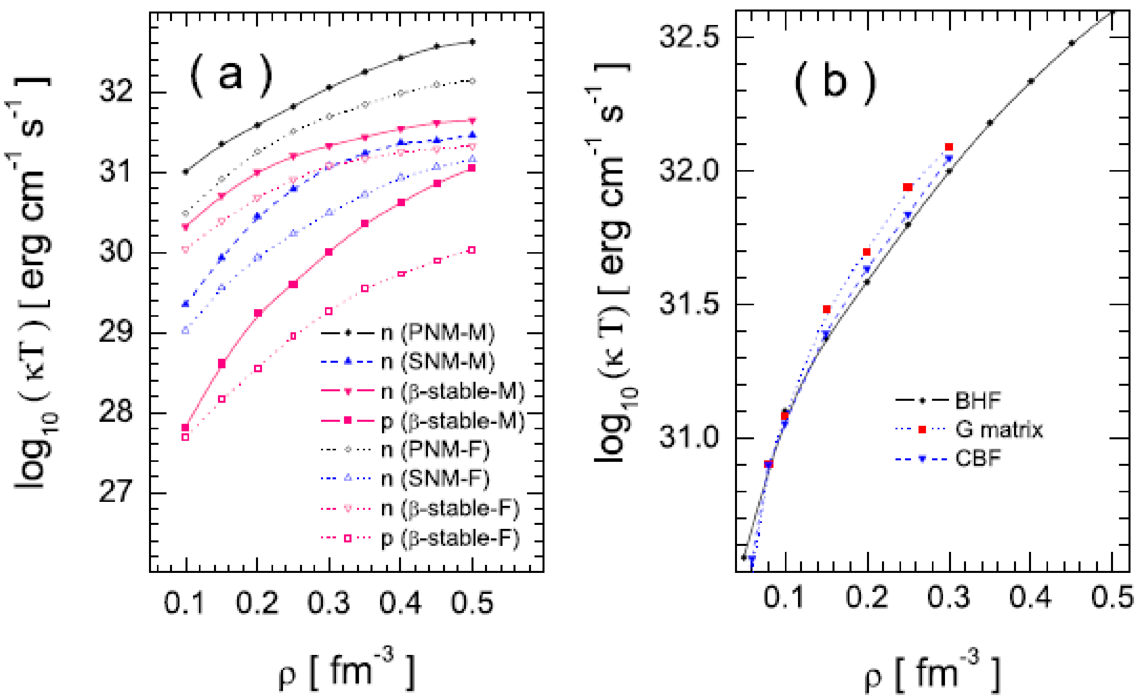
Shternin & Yakovlev PRD(2008), APR
Benhar & Carbone, arXiv09112.0129,CBF

$\sigma_o(\Omega) \rightarrow \sigma(\Omega)$: η~$10\cdot\eta_0$ Flowers & Itoh, ApJ (1979)
isospin effect: η(proton) $\rightarrow$ η(neutron) at higher density
m $\rightarrow$ m* : $\eta_n >> \eta_e$ Shternin & Yakovlev PRD(2008)

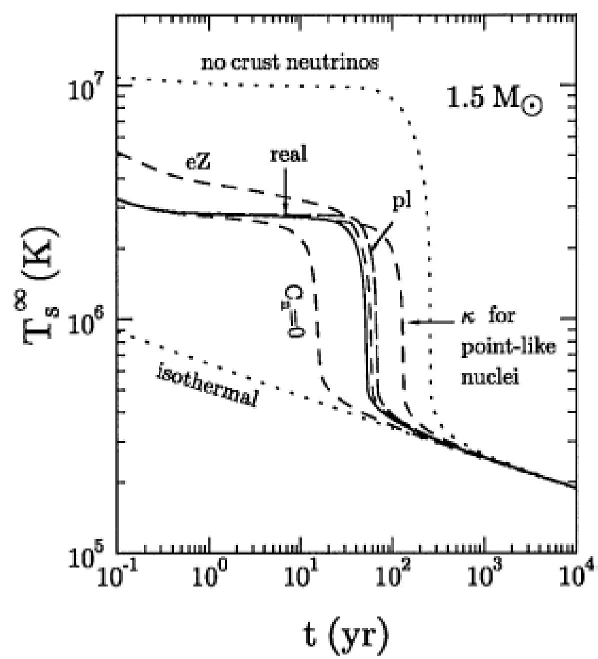
$\longrightarrow$ no r-mode damping

thermal conductivity

$$\kappa T = \frac{1}{12} p_F v_F^2 H(\mu) \int_0^{4\varepsilon_F} \frac{dE}{2\varepsilon_F} \int_0^{2\pi} \frac{d\vartheta}{2\pi} \left(1 - E/4\varepsilon_F\right)^{-\frac{1}{2}} \sigma(E,\vartheta)$$



NS Cooling



Yakovlev et al, Phys.Reports 354 (2001) 1

Dissipation of r-modes

velocity perturbation: $\delta\vec{v} \sim R\,\Omega \left(\frac{r}{R}\right)^m \vec{Y}_{mm}\, e^{i\omega t}$

collective energy: $E = \frac{1}{2}\int dV \rho\, |\delta\vec{v}\,|^2$

dissipation time scale:

$$\frac{1}{\tau} = -\frac{1}{\tau_{GR}} + \frac{1}{\tau_{v}} = -\frac{1}{2E}\,\frac{dE}{dt} = -\left(\frac{1}{2E}\,\frac{dE}{dt}\right)_{GR} - \left(\frac{1}{2E}\,\frac{dE}{dt}\right)_{v}$$

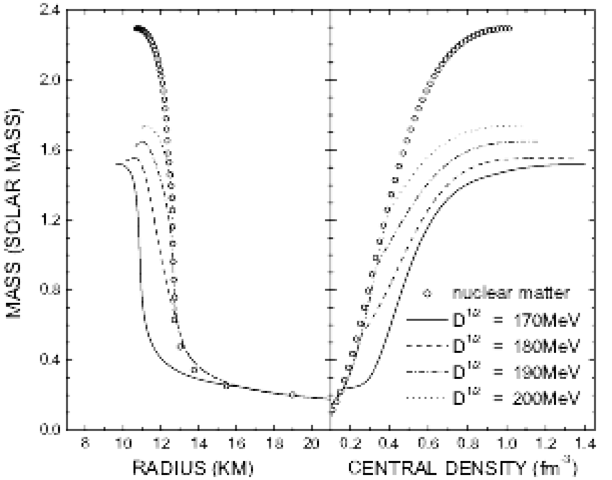
$\Omega \sim 1000$ Hz $\rightarrow$ $\tau_{GR} \sim 100$ sec (depending only on rotation)

Time scale of nonradial modes damping from shear viscosity

constant mass approximation

$$\frac{1}{t_V} = \; l-1 \;\; 2l+1 \; \frac{\eta}{\rho R^2} \qquad\qquad \rho = \frac{M_\odot}{\frac{4\pi}{3} R^3} \; = const$$

Li,Lombardo,Peng, PRC (2008))



M-R in NS stable configurations from TOV eqs
constant density approximation: ρ = M/(4π/3 R^3)

$M/M_\odot$	R(Km)	$\rho(g/cm^3)\times 10^{14}$	t_V (s)
0.8	12.8	1.8	1865
1.4	12.5	3.4	1680
1.8	12.2	4.7	883
2.3	11.0	8.25	317

t_k (s)
$0.54\ 10^{16}$
$1.5\ 10^{16}$
$3.0\ 10^{15}$
$6.0\ 10^{15}$

thermal cond.

Conclusions

- *Brueckner theory provides a unified treatment of equilibrium and transport properties of nuclear matter*

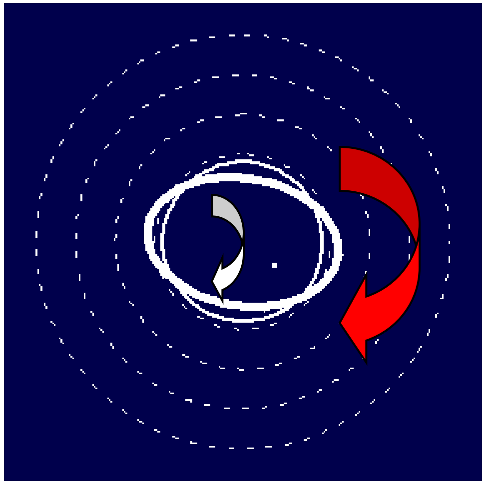
- *The medium effects in the* $\sigma_{NN}(\Omega)$ *were calculated within the BHF+3BF theory, showing that Pauli principle mainly suppress the forward and backward angles whereas the mass renormalization plays the most important role*

- *Transport parameters, shear viscosity and thermal conductivity were calculate in different configuations of nuclear matter, including beta-stable nuclear matter and neutron stars*

- *Preliminary estimates of the time scale for the neutron viscosity damping of nonradial modes is comparable with the gravitational radiation damping*

Chandrasekhar Instability (1970)

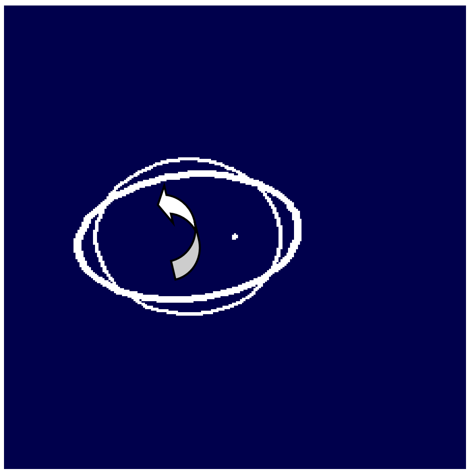
Y_{22} - nonradial mode: $\nu \sim \omega_0$ (Coriolis force)



Inertial frame

$\omega_0 - \omega_{22} >> 0$

Y_{22} is a source of gravitational radiations, that extract ΔL_0 and the star spins down



Corotating frame

$\omega_{22} < 0$

L_{22} is increasingly negative → large frequency and amplitude osc.

The amount of gravitational radiations is increasingly large (expected to be detected in terrestrial labs (LIGO,VIRGO,…)

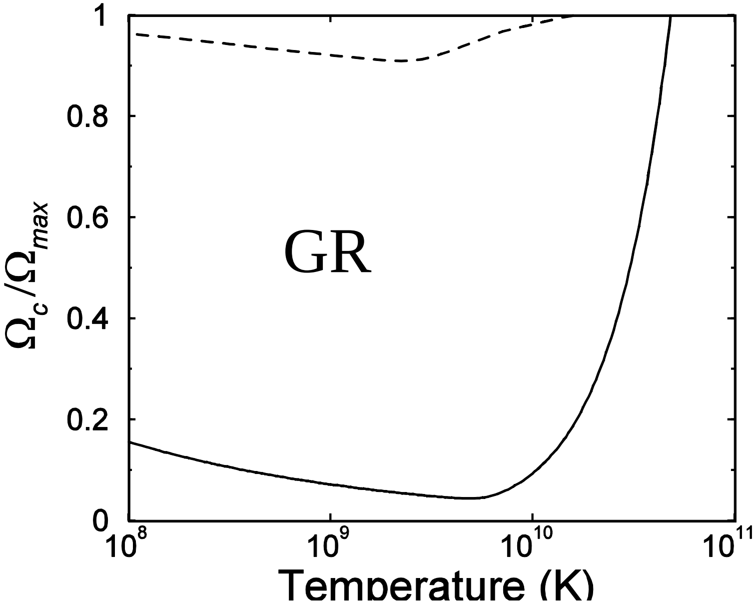
interplay between GR driving instability and viscosity damping

Critical velocity:

$$\frac{1}{\tau_{GR}(\omega_c)} + \frac{1}{\tau_V(\omega_c)} = 0$$

Astrophysical Implications

Lindblom 2000



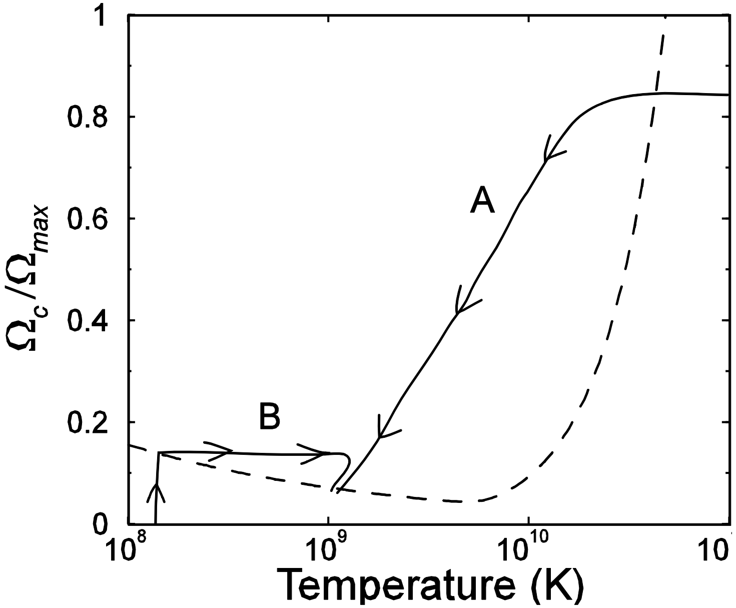
$$\frac{1}{\tau_{GR}(\omega_c)} + \frac{1}{\tau_V(\omega_c)} = 0$$

$$\frac{1}{\tau_{GR}} \sim \Omega^6 \qquad \frac{1}{\tau_V} \sim T^{-2}$$

The critical frequency gives an upper limit to the observed stars

Astrophysical Implications

Lindblom 2000



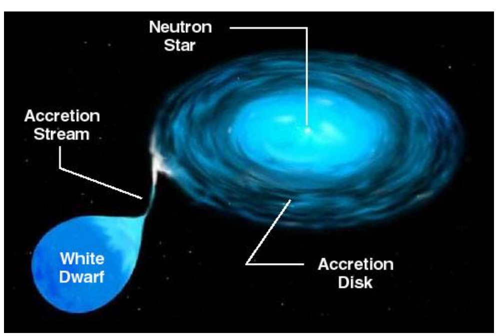
<u>Scenario A</u>

a) NS from progenitor gravitational collaps
   high Ω and T ($\geq$ 10 MeV)

b) r-modes are driven unstable by GR ($\Omega > \Omega c$)
   low Ω and T ($\leq$ 0.1 MeV)

c) NS gets stable at $\Omega = \Omega c$

Observation: no rapidly rotating NS in young supernovae remnants



<u>Scenario B</u>

a) NS accreting in a binary system
   low Ω and low T ($\leq$ 0.01 MeV)

b) Viscous dissipation of growing r-modes
   until viscous heating ~ neutrino cooling
   Ω=const T$\rightarrow$ 0.1 MeV

c) GR spin down
   $\Omega \rightarrow \Omega c$ T=const

Observation: narrow frequency range in low mass x-ray binarues

World Network of G.W. Detectors